MATRICES

1). Let $A=\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ and $\lambda, \mu \in \Re$. Find the values of λ and μ such that $A(\lambda A+\mu I)=I$, where I is the 2×2 identity matrix. Hence find A^{-1}.

2). Let $A=\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$ be a 2×2 matrix.

Show that $A^2-3A+2I=O$, where I is the 2×2 identity matrix and O is the 2×2 zero matrix.

Hence, find A^{-1}.

Let $B=\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ be a 2×2 matrix.

Show that $BA=B$.

Hence, or otherwise find a non-zero 2×2 matrix C such that $BC=O$.

3).Let $Q=\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

Find the value of $\lambda \in \Re$ such that $Q^T Q=\lambda I$, where Q^T is the transpose of Q and I is the 2×2 identity matrix.

Hence, find the inverse of the matrix $P=\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.

Let A be a 2×2 matrix such that $AP=PD$, where $D=\begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$. Find A.

4).Let $a,b\in\Re$ and let $A=\begin{pmatrix} 1 & 0 \\ 0 & a \\ 1 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} b & 1 \\ 1 & 1 \end{pmatrix}$. Find the values of a and b such that $A^T A=B$. where

A^T denotes the transpose of the matrix A.

Let $C=\begin{pmatrix} 7 & 5 \\ 5 & 3 \end{pmatrix}$ and $X=\begin{pmatrix} u \\ u+1 \end{pmatrix}$, where $u\in\Re$. Also , let $CX=\lambda BX$, where $\lambda\in\Re$.

Find the value of λ and the value of u.

For this value of λ, find the matrix $C-\lambda B$ and show that its inverse does not exist.

5).Three matrices A, B and C are given by

$A=\begin{pmatrix} 0 & 2 & -3 \\ 0 & -1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} a & b & 0 \\ c & d & 0 \end{pmatrix}$ and $C=\begin{pmatrix} 3 & 4 \\ 2 & 3 \\ 1 & 2 \end{pmatrix}$

(i) Show that $AC = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Also find the product CA.

(ii) Find the values of a, b, c and d such that $BC = I_2$.

(iii)If $(\lambda A + \mu B)C = I_2$. obtain an equation connecting λ and μ. Express the matrix $D = \begin{pmatrix} -3 & 8 & -6 \\ 2 & -5 & 4 \end{pmatrix}$ in terms of A and B, and hence find the product DC.

6).Let $A = \begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix}$, $X = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $Y = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Find real constants λ and μ such that $AX = \lambda X$ and $AY = \mu Y$.

Let $P = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$. Find P^{-1} and AP, and show that $P^{-1}AP = D$. Where $D = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$.

7).Let $A = \begin{pmatrix} 7 & 8 \\ -6 & -7 \end{pmatrix}$. Find A^2 and hence obtain A^{-1}.

Determine the matrix X such that $A^{2017}X = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$.

8).Let $A = \begin{pmatrix} 3 & p \\ -2 & -3 \end{pmatrix}$, where $p \in \Re$. Find the non-zero values of p for which the matrix A has an inverse.

Find the value of the constant p for which $A^{-1} = A$, and hence obtain two non-zero matrices B and C of order 2 such that $BC = O$. Here, O is the zero matrix of order 2.

9).Let $P = \begin{pmatrix} -5 & 3 \\ 6 & -2 \end{pmatrix}$. Find the two distinct real values of λ such that $\det(P - \lambda I) = 0$. Here I is the unit matrix of order 2×2.

For each value of λ, find the column matrix $X = \begin{pmatrix} x \\ y \end{pmatrix}$ which satisfies $PX = \lambda X$.

10). Three matrices are given as $A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 5 \\ -4 & k \end{pmatrix}$.

Find the value of k such that $A(2B - C) = 6I$. Where I is the unit matrix. Hence deduce A^{-1}.

11)Let $A = \begin{pmatrix} 2 & a & 3 \\ -1 & b & 2 \end{pmatrix}$. $B = \begin{pmatrix} 1 & -1 & a \\ 1 & b & 0 \end{pmatrix}$ and $P = \begin{pmatrix} 4 & 1 \\ 2 & 0 \end{pmatrix}$, where $a, b \in \Re$.

It is given that $AB^T = P$, where B^T denotes the transpose of the matrix B. Show that $a = 1$ and $b = -1$, and with these values for a and b, find $B^T A$.

Write down P^{-1}, and using it, find the matrix Q such that $PQ = P^2 + 2I$, where I is the identity matrix of order 2.

12. Matrices A and B satisfy $AB = B^{-1}$,where $B = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

i.Without finding B^{-1} find k such that $kA - 2B^{-1} + I = 0$

ii.Without finding A^{-1} find X such that $A^{-1}XA = B$

13. Let $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ and $\lambda, \mu \in \Re$. Find the values of λ and μ such that $A(\lambda A + \mu I) = I$, where I is the 2×2 identity matrix. Hence find A^{-1}.

14. Let $A = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$ be a 2×2 matrix.

Show that $A^2 - 3A + 2I = O$, where I is the 2×2 identity matrix and O is the 2×2 zero matrix. Hence, find A^{-1}.

Let $B = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ be a 2×2 matrix.

Show that $BA = B$. Hence, or otherwise find a non-zero 2×2 matrix C such that $BC=O$.

15. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 4 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2a \\ -1 & 0 \\ 1 & 3a \end{pmatrix}$, where $a \in \Re$.

Find the matrix $\boldsymbol{P}$ defined by $P = AB$ and show that P^{-1} **does not exist** for any value of a.

Show that if $P\begin{pmatrix} 1 \\ 2 \end{pmatrix} = 5\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, then. $a = 2$

With this value for a, let $Q = P + I$, where I is the identity matrix of order 2. Write down Q^{-1} and find the matrix R such that $AA^T - \frac{1}{2}R = \left(\frac{1}{5}Q\right)^{-1}$.

16.Let $A = \begin{pmatrix} a & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -a & 4 \end{pmatrix}$ and $C = \begin{pmatrix} b & -2 \\ -1 & b+1 \end{pmatrix}$ be matrices such that $AB^T = C$, where $a, b \in \mathbb{R}$

Show that $a = 2$ and $b = 1$.

Show also that, C^{-1} does not exist.

Let $P = \frac{1}{2}(C - 2I)$. Write down P^{-1} and find the matrix Q such that $2P(Q + 3I) = P - I$, where I is the identy matrix of order 2.

17.Let $A = \begin{pmatrix} a+1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a & 2 \end{pmatrix}$ and $C = \begin{pmatrix} a & 1 \\ a & 2 \end{pmatrix}$, where $a \in \mathbb{R}$.

Show that $A^T B - I = C$; where I is the identity matrix of order 2.

Show also that C^{-1} exists if and only if $a \neq 0$.

Now, let $a = 1$. Write down C^{-1} .

Find the matrix P such that $CPC = 2I + C$.

18. Let $A=\begin{pmatrix} a & -2 \\ 1 & a+2 \end{pmatrix}$. Show that A^{-1} exists for all $a \in \mathbb{R}$.

The matrices $P=\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix}$, $Q=\begin{pmatrix} 2 & 3 & 2 \\ -1 & 7 & 4 \end{pmatrix}$ and $R=\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ are such that $A=PQ^T+R$. Show that $a=1$.

For this value of *a*, write down A^{-1} and **hence**, find the values of *x* and *y* such that $A\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -5 \\ 10 \end{pmatrix}$.

19. Let $A=\begin{pmatrix} a & 0 & 3 \\ 0 & a & 1 \end{pmatrix}$ and $B=\begin{pmatrix} a & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, where $a \in \mathbb{R}$.

Also, let $C=AB^T$. Find C in terms of *a*, and show that C^{-1} exists for all $a \neq 0$.

Write down C^{-1} in terms of *a*, when it exists.

Show that if $C^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix}=\frac{1}{8}\begin{pmatrix} 9 \\ -11 \end{pmatrix}$, then $a=2$.

With this value for *a*, find the matrix *D* such that $DC-C^TC=8I$, where I is the identity matrix of order 2.

20.The matrix $A=\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$. Show that $A^2-4A+5I=0$ and hence find A^{-1}. using matrices, solve the equations

$$x-y=4$$
$$2x+3y=1\cdot$$

21. $A=\begin{bmatrix} -1 & 2 \\ 4 & 1 \end{bmatrix}$ and $P=\begin{bmatrix} 1 & 2 \\ \lambda & \mu \end{bmatrix}$; where $\lambda<\mu$. If *P* is a non singular matrix, find λ and μ such that $P^{-1}AP$ is a diagonal matrix.Hence evaluate $P^{-1}A^2P$, and show that $(P^{-1}AP)^{-1}=\frac{1}{9}(P^{-1}AP)$

22.Given that $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, Show that $A^2-5A-2I=O$. Hence find A^{-1}. Find *a* and *b* such that $A+aA^{-1}=bI$.

Find a matrix *B* such that $B\left[A-2A^{-1}\right]C=D$. Where $C=\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ and $D=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

23. $A=\begin{bmatrix} 0 & -2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Find the matrix *C* such that $C=AB$ and write down C^{-1}.

Find a matrix *D* such that $CDC^{-1}=2C^2+3C$.

Justify that $(CD)^{-1}=D^{-1}C^{-1}$. Also find the matrix $P=BA$.

Let $X=\begin{bmatrix} a \\ 2 \\ b \end{bmatrix}$ where , $a,b \in \Re$. Find *a* and *b* such that $PX=\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$.