Circle

01. Using first principle find the radius and centre of the following circles.

i. $x^2+y^2+3x-5y+1=0$

ii. $3x^2+3y^2-x+2y-1=0$

iii. $2x^2+2y^2+2x-4y-3=0$

iv. $x^2+y^2+4x+2y-2=0$

v. $x^2+y^2-\frac{x}{2}-3y-1=0$

02. Find the equations of the circles with the following points as its a diameter.

i. $\left(-4a,\ \sqrt{1-a^2}\right), \left(-4a,\ -\sqrt{1-a^2}\right)$

ii. $(-4,\ -1), (3,\ -3)$

iii. $(-1,\ 4), (3,\ -2)$

iv. $\left(3a,\ -\sqrt{2-a^2}\right), \left(-3a,\ \sqrt{2-a^2}\right)$

03. (*a*). Find the equations of the circles passing through the given points and find its centre and radius.

i. (1, 0), (0, 1), (1, 1)

ii. (1, 2), (2, 1), (0, 0)

iii. (0, 0), (1, 1), (-1, 2)

(*b*). If the points (2, 3), (0, 2), (4, 5) and (0, t) are on a circle, show that $t=2$ or $t=17$.

04. Find the position of the point(s) given below relative to the given circle.

i. $x^2+y^2-3ax-ay-2a^2=0$ $(a,\ a)$

ii. $2x^2+2y^2-ax-ay-a^2=0$ $(-a,\ 2a)$

iii. $x^2+y^2-2x-4y-1=0$ $(1,\ 3), (-4,\ -2)$

iv. $3x^2+3y^2-4x-3y-1=0$ $(5,\ 2), (-1,\ -3)$

05. Find the position of the line given below relative to the circle given.

i. Position of $x+2y-4=0$ relative to $x^2+y^2-2x-3y-1=0$

ii. Position of $x-y-1=0$ relative to $x^2+y^2-x+y-4=0$.

iii. Position of $x+2y-1=0$ relative to $x^2+y^2+3x+2y+1=0$.

06. i. Show that the line $3x+2y=30$ is a tangent to the circle $x^2+y^2-10x-2y+13=0$ and find the point of contact.

ii. Show that the line $3x+2y=12$ intersects the circle $x^2+y^2=13$ and find the coordinates of points of intersection. Find for what value of *c* the line $3x+2y=c$ becomes a tangent to the circle.

iii. Find the equations of the tangents drawn to the circle $x^2+y^2-4x-3y+5=0$ parallel to the line $x+y=0$.

07. Find the equation of the tangent to the circle at given point below.

i. to the circle $x^2+y^2+2gx+2fy=0$ at (0, 0)

ii. to the circle $x^2+y^2+2x+2y-3=0$ at (-3, -2)

iii. to the circle $x^2+y^2+3x-3y+2=0$ at (-1, 3)

iv. to the circle $x^2+y^2-6x+4y-12=0$ at (6, 2)

v. to the circle $x^2+y^2-4x-y+1=0$ at (1, 2)

08. Find the equation of the perpendicular drawn to the circle at the given point below.

i. to the circle $x^2+y^2-7x-5y+18=0$ at $(3, 2)$.

i. to the circle $x^2+y^2-2x-6y-15=0$ at $(-2, 7)$

09. Find the length of the tangent drawn to the circle from the point given below.

i. from the point $(1, -4)$ to the cicle $2x^2+2y^2-x-6y-2=0$

ii. from the point $(3, 1)$ to the circle $x^2+y^2+x-2y-1=0$.

iii. from the point $(5, 3)$ to the circle $x^2+y^2-2x-3y-4=0$.

10. Find the equation of chord of contact of the tangents drawn from thepoints to the circle given below.

i. from $(-3, -4)$ to the circle $x^2+y^2-6x-2y-1=0$.

ii. from $(2, 3)$ to the circle $2x^2+2y^2+4x+3y-2=0$.

iii. from $(2, -2)$ to the circle $x^2+y^2+3x-2y-1=0$.

iv. from $(1, 2)$ to the circle $2x^2+2y^2-2x-y-2=0$.

11. If the circles $x^2+y^2+px+py-7=0$ and $x^2+y^2-10x+2py+1=0$ intersect orothogonally. Show that $p=2$ or $p=3$.

12. A circle passes through the point (0, 0) and its centre lies on the line $x+y=4$. If this circle intersects the circle $x^2+y^2-4x+2y+4=0$ orthogonally, show that the equation of this circle is $x^2+y^2-4x-4y=0$.

13. Find the equation of the circle which intersects the circles $x^2+y^2=1$, $x^2+y^2-x-3=0$ and $x^2+y^2-y-4=0$ orthogonally.

14. Find the equation of the circle whose centre is on the line $x-y+1=0$ and intersects the circles $x^2+y^2=2$ and $x^2+y^2+2x+4=0$ orthogonally.

15. Find the smallest circle which passes through the intersection of $x^2+y^2+2x-8=0$ and $x+y-1=0$.

16. The circle passing through the intersection of $x^2+y^2-3=0$ and $x-y-1=0$ touches the line $x+y-3=0$. Show that there are two circles and find their equations.

17. Find the equation of the circle which passes through the intersection of $x^2+y^2+2x+8=0$ and $x-y-1=0$ intersecting the circle $x^2+y^2+2x-1=0$ orthogonally.

18. Find the equations of the common tangents drawn to the circles $x^2+y^2+10x+21=0$ and

$x^2+y^2-10x-3y=0$.

19. Find the equation of the circle with the points (15, 5) and (7, -1) as ends of a diameter. If the line $y=mx$ cuts this circle, show that x – values at the points of intersection are given by the equation. $(1+m^2)x^2-2(2m+11)x+100=0$ Hence find the equation of the tangents drawn to this circle from the origin.

20. For all values of the constants p,q show that the circle $(x-a)(x-a+p)+(y-b)(y-b+q)=r^2$ bisects the circumference of the circle $(x-a)^2+(y-b)^2=r^2$ Find the equation of the circle which bisects the circumference of $x^2+y^2+2y=3$ and touches the line $x-y=0$ at the origin.

21. Find the common equation for all circles which pass through the points $(1,0)$ and $(-1,0)$. Prove that two circles of them touch the line $2x-y=3$ and find their equations.

22. The circle passing through the distinct points $A(2a,0), B(0,2b)$ and $c(a+b,a+b)$ is S. Where a and b are positive numbers. Show that S passes through . $P(2a,2b)$ If the tangents drawn to S at B and P meet at Q, show that $PQ=\frac{a}{b}\sqrt{a^2+b^2}$ Find the equation of the path of the centre of the circle which touches the circle S externally and touches the line $ax+by+c=0$ $(c>0)$

23. i.A variable chord of the circle $x^2+y^2-4=0$ subtends right angle at its centre show that the path of the mid-point of the chord is $x^2+y^2-2=0$

ii. A variable chord is drawn to the circle $x^2+y^2-2ax=0$ through $(0,0)$ Show that the equation of the locus of the centre of the circle with this chord as a diameter is $x^2+y^2-ax=0$

iii. Show that the equation of the path of the centre of the circle which passes through the fixed point (a,b) and intersects the circle $x^2+y^2-k^2=0$ orthogonally is $2ax+2by=a^2+b^2+k^2$

24. *i.* Find the equations of the circles radius $\sqrt{2}$ passing through (0, 1) and centre is on the line $2x-y+4=0$.

ii. Find the circle which touches the axes passing through (2, 1)

iii. Find the equations of the circles which touch the y - axis, cut off an intercept 4 units on x - axis with the centre on the line $2x-y+4=0$.

iv. Find the equation of the circle with (1, 2), (3, -4) are at the extremities of a diameter.

v. A circle with centre (2, -3) passing through the origin cut off a chord of 4 units on the line $3y-4x=\alpha$, find α.

25. Find the length of the tangent from the given points to the given circles.

i. From (1, -4) to the circle $2x^2+2y^2-x-6y-2=0$

ii. From (3, 1) to the circle $x^2+y^2+x-2y-1=0$

26. i. Show that the point (1, 2) is on the circle $x^2+y^2-4x-y+1=0$ and find the perpendicular and tangent drawn to the circle at this point.

ii. Find the equation of the tangent at (6, 2) on the circle $3x^2+3y^2-18x+12y-36=0$.

iii.Show that the points (4, 3) and (3, 2) lie on the circle $x^2+y^2-7x-5y+18=0$ and find the equations of the tangents at these points.

iv. Find the equation of the perpendicular at (-2, 7) on the circle $x^2+y^2-2x-6y-15=0$.

27. Find the equation of chord of contact from the given points to the given circles below,

i. $x^2+y^2-6x-2y-1=0$ (-3, -4)

ii. $2x^2+2y^2+4x+3y-2=0$ (2, 3)

iii. $x^2+y^2+3x-2y-1=0$ (2, -2)

iv. $2x^2+2y^2-2x-y-2=0$ (1, 2)

v. The chord of contact drawn from a point P to the circle $x^2+y^2+2x+2y-2=0$ always passes through (-3, 3). Find the path of P.

28. i.Find the equation of the circle passing through the point (1, 0) and intersection of $x^2+y^2-4x+8y+10=0$ and $x+2y+1=0$.

ii.Find the equation of the smallest circle passing through the intersection of $x^2+y^2+2x-8=0$ and $x+y-1=0$.

iii. The circle passing through the intersection of $x^2+y^2-3=0$ and $x-y-1=0$ touches the line $x+y-3=0$. Find the equations of the circles.

29. i.Find the equation of the circle passing through the point (1, -1) and intersection of the circles $x^2+y^2-14x+6y+42=0$ and $x^2+y^2-2x-4y-11=0$.

ii.Find the equation of the circle passing through the intersection of the circles $x^2+y^2-2x-4y-4=0$ and $x^2+y^2+8x-4y+6=0$ and through origin.

iii. Find the smallest circle passing through the intersection of the circles $x^2+y^2-4x-4y-1=0$ and $x^2+y^2+2x+2y-7=0$

30. i. Show that the circles $x^2+y^2-4x-2y-4=0$ and $x^2+y^2-6x-8y+21=0$ intersect each other and find their common chord.

ii. Three circles S_1, S_2, S_3 are represented by $S_1 \equiv x^2+y^2-4x-2y-4=0$, $S_2 \equiv x^2+y^2-\lambda x-8y+21=0$ and $S_3 \equiv x^2+y^2-8x+\mu y+46=0$. If the common chord of S_1 and S_3 coincides with the common chord of S_2 and S_3 find λ and μ. For these values of λ and μ . Show that the common chords of S_1, S_3 and S_1, S_2 also coincide.

31. i. Show that the circles $x^2+y^2+10x-2y+22=0$ and $x^2+y^2+2x-8y+8=0$ touch each other find,

a. Coodinates of point of contact

b Equation of the common tangent at the point of contact.

c. The area bounded by common tangent line joining the centres and y axis.

ii. Show that the cricles $x^2+y^2-2x+4y=0$ and $x^2+y^2-10x+20=0$ touch each other externally.

iii. Find μ such that the circles $x^2+y^2+\mu x-2y=0$ and $x^2+y^2-2x+2y=0$ touch each other externally.

iv. Show that the circles $x^2+y^2=4$ and $2x^2+2y^2-3x+y-3=0$ intersect eachother. Find the equation of the common chord.

v. Show that the circles $5x^2+5y^2-4x+8y-16=0$ and $x^2+y^2+4x-2y-20=0$ touch each other internally.

vi. Find the values of k for which the circles $x^2+y^2+4x-2y-5=0$ and $x^2+y^2+x-3y+k=0$ touch each other. In each case, determine whether they touch internally or externally.

32. i. If the circles $x^2+y^2+px+py-7=0$ and $x^2+y^2-10x+2py+1=0$ intersect orthogonally, show that $p=2$ or $p=3$.

ii. Find the equation of the circle passing through (-3, 2) and intersecting the circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-3x+6y-2=0$ orthogonally.

iii. Show that the equation of the circle passing through (0, 0) whose centre is on the line $x+y=4$ and intersecing the circle $x^2+y^2-4x+2y+4=0$ orthogonally is $x^2+y^2-4x-4y=0$

iv. Find the equation of the circle which intersects the three circles $x^2+y^2=1$, $x^2+y^2-x-3=0$ and $x^2+y^2-y-4=0$ orthogonally.

v. Find the equation of the circle whose centre is on the line $x-y+1=0$ and intersects the circles $x^2+y^2=2$ and $x^2+y^2+2x+4=0$ orthogonally.

33. i. Show that the equation of the circle which is orthogonal to $x^2+y^2+4x+2y-4=0$ and bisects the circumference of the circle $x^2+y^2-4x=0$ can be written as $2x^2+2y^2-(8+\lambda)x+4(\lambda+2)y+2\lambda=0$ where λ is a parameter show also that this circle passes through two fixed points.

ii. Let $S\equiv x^2+y^2+2gx+2fy+C=0$ and $S_1\equiv x^2+y^2+2g_1x+2f_1y+C_1=0$ be two circles. Show that the condition required for bisection of circumference of S_1 by S is $2g_1g_2+2f_1f_2-C=2g_1^2+2f_1^2-C_1$.

The centre of a circle S is on the common chord of the circles, $S_1\equiv x^2+y^2+4x+8y+8=0$ and $S_2\equiv x^2+y^2-2x+6y+2=0$ and S bisects the circumference of S_2. If S passes through the origin find S.

iii. Show that for all values of p and q the circle $(x-a)(x-a+p)+(y-b)(y-b+q)=r^2$ bisects the circumference of the circle $(x-a)^2+(y-b)^2=r^2$.

Find the equation of the circle which bisects the circumference of $x^2+y^2+2y-3=0$ and touches the line $x-y=0$ at the origin.

iv. The circle $S\equiv 2x^2+2y^2-3x+6y-2=0$ cuts orthogonally the circle $S'=0$ which passes through (0, -1) and centre lying on the line $y=2$. Find the equation of the circle S' and show that it bisects the circumfer ence of the circle $x^2+y^2=5$.

34. i. Find the four common tangents drawn to the circles

$x^2+y^2-20x+6y+84=0$

$x^2+y^2+24x-2y-80=0$

ii. Show that the equations $x^2+y^2+2x-4y-8=0$ and $x^2+y^2-6x+8y+12=0$ represent two contact circles with equal radii. Find the common tangent at their point of contact and find the other two tangents.

35. If the line $lx+my+n=0$ touches the circle $(x-a)^2+(y-b)^2=r^2$ then prove that

$(al+bm+n)^2=(l^2+m^2)r^2$.

Find the equations of the two tangents drawn to the circle $S=(x+1)^2+(y+2)^2-1=0$ parallel to the line $3x+4y=0$. Find the equations of the two circles which touch these tangents and the circle $S=0$.

36. If p,m are parameters, show that $x^2+y^2-a^2+p(y-mx)=0$ is a circle which bisects the circumference of the circle $x^2+y^2=a^2$. A circle S bisects the circumference of the circle $3x^2+3y^2=5$ and the tangents drawn from $P(1, 2)$ to S are perpendicular each other. Show that the locus of the centre of *S* is $3x^2+3y^2+6x+12y-5=0$.

37. Find the equation of the common chord of the circles $x^2+y^2+2g_1x+2f_1y+C_1=0$ and $x^2+y^2+2g_2x+2f_2y+C_2=0$. If they intersect, at two points on a diameter of the first circle, show that $2g_1^2+2f_1^2-C_1=2g_1g_2+2f_1f_2-C_2$. $S_1\equiv x^2+y^2-4x-6y+6=0$ and $S_2\equiv x^2+y^2-4x+6y-22=0$ are two circles. The circle *S* bisects the circumferance of S_1 and S_2 bisects the circumference of *S*. Show that the locus of the centre of S is $x^2+y^2-4x-1=0$.

38.Show that for all values of θ the locus of the point $P(3+5\cos\theta, -4+5\sin\theta)$ is a circle through the origin. Find the equation to the tangent at (0, 0). Find the coordinates of points of contact of the tangents parallel to $y=x$.

39. Find the path of the point at which the circle $x^2+y^2-4x-6y+4=0$. Subtends a right angle.
(locus $x^2+y^2-4x-6y-5=0$)

40. Find the equation of the circle with centre (4, 5) passing through the centre of the circle $x^2+y^2+4x-6y-12=0$

41.A curve is given by $x=3+2\sin\theta,\ y=2(1+\cos\theta)$ θ is a parameter. Show that the curve is a circle.

42. A and B are two points *A*(1, 1) and *B*(5, 7)
   i. Find the equaiton to the circle with *AB* as a diameter.
   ii. Find the coordinates of the extremities of the diameter perpendicular to *AB*.

43. Find the equation to the circle which touches the *X*-axis at (3,0) and making an intercept of 8(units) on the *Y*-axis.

44. Show that the circles $x^2+y^2-4x-4y+7=0$ and $x^2+y^2-10x-12y+45=0$ touch externally.

45. i.Find the circle passing through the point (0, 0), (0, 1), (2,3)

ii.Find the two circles which touch the lines $4x+3y=1$, $x=0$, $y=0$ with centres lying on the line $x=y$.

46. Find the circle which touches the *X*-axis at $x=2$ and cuts the *Y*-axis at $y=1$. What are the centre and the radius ? Show that this circle intersect the *Y*-axis at $y=4$.

47. Show that the circle, the common chord of the circles $x^2+y^2+2x-5y=0$ and $x^2+y^2+6x-8y-1=0$ is as a diameter touches the axes.

48. Show the the circles $x^2+y^2+x+y-5=0$ and $x^2+y^2-7x-16y+20=0$ touch each other externally at the point (-1, 2). The line joining the centres of these circles meet the circles again at *P* and *Q*. Find the equation of the circle such that *PQ* is a diameter.

49. i.Find the point on the circle $x^2+y^2-10x-2y+25=0$ which is closest to the line $3x-4y+7=0$.

   ii. Find, for which values of m the line $y-4=m(x-7)$ and the circle $x^2+y^2=64$ touch each other. Deduce the equations of the tangents drawn from (4, 7) to the circle.

50. Find the equation of the circle whose two extermities of a diameter are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$. The vertices of a triangle are *A*(1,2), *B*(2,-1), *C*(-1,-1). Find the orthocentre *H* of the triangle. Find the equations of the circles. So that *AH* and *BC* are diameters and show that the are orthogonal.

51.Show that there exist four circles passing through the origin and cut off chords of length a on the lines $y - x = 0$ and $y + x = 0$ and their equations are $x^2 + y^2 \pm \sqrt{2}ax = 0$, $x^2 + y^2 \pm \sqrt{2}ay = 0$.

52. Find the equations to the common tangents of the circle.

$S \equiv x^2 + y^2 - 2x - 6y + 9 = 0$ and

$S' \equiv x^2 + y^2 + 6x - 2y + 1 = 0$

53. Obtain the condition required to subtend a right angle at the origin by the chord lx+my+n=0 of the circle $(x-c)^2 + y^2 = a^2$ A variable chord *PQ* of the circle with centre *C*, subtends a right angle at an interior fixed point *O*. Show that the locus of the foot of the perpendicular drawn from *O* to *PQ* is a circle and its centre is the mid point of *OC*.

54. If the two circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ intersect at right angles then show that $2gg' + 2ff' = c + c'$. Show that there are two circles passing through the origin., touching the line $x + 2y + 1 = 0$ and intersecting the circle $x^2 + y^2 - x + 3y - 1 = 0$ orthoganally and find their equations.

55. Show that the circles $x^2 + y^2 = 1$, $x^2 + y^2 - 8x + 7 = 0$ and $x^2 + y^2 - 6y + 5 = 0$ touch eachother. Find the equations of the three common tangents passing through the point of contacts and show that they concurrent. Find the equation of the circle which is orthogonal to the given circles.

56.Show that the circles $x^2 + y^2 + 8x + 2y + 8 = 0$ and $x^2 + y^2 - 8x + 2y - 2 = 0$ have three common tangents and find their equations.

57. Find the condition required for the circles $S = 0$ and $S' = 0$ to be intersect orthogonally. A circle $S' = 0$ passing through (0,-1) whose centre is on the line $y = 0$ and intersets the circle $S \equiv 2x^2 + 2y^2 - 3x + 6y - 2 = 0$ at right angles. Find the equations of $S' = 0$.

58. Find the condition required for the circles $x^2 + y^2 + 2gx + 2fy + c = 0$, $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ to be touch each other. If they touch, then show that the point of contact lies on $2(g - g')x + 2(f - f')y + c - c' = 0$ and $(f - f')x - (g - g')y - f'g + fg' = 0$. Hence find *k*. So that the circles $x^2 + y^2 + 2x + 4y + 1 = 0$ and $x^2 + y^2 - 4x + 4y + k = 0$ touch each other. In each case explain whether they touch internally or externally.

59. Show that the coordinates of point *P* on the circle $(x-a)^2 + (y-b)^2 = r^2$ can be written of the form $(a + r\cos\theta, b + r\sin\theta)$ and the tangent at *P* is $(x-a)\cos\theta + (y-b)\sin\theta = r$. Show that the coordinates of the foot of the perpendicular *N* drawn from the origin to this tangent satisfy the equation $y\cos\theta = x\sin\theta$. Show that as *P* moves on the circle the path of *N* is $[x(x-a) + y(y-b)]^2 = r^2(x^2 + y^2)$.

60. Show that when λ is a parameter, the equation $S + \lambda S' = 0$ gives the circles passing through the intersection of the circles $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and $S' \equiv x^2 + y^2 + 2g'x + 2f'y + c' = 0$. Where $\lambda \neq -1$.

Find the circle passing through the intersection of the circles $x^2 + y^2 - 10x = 0$ and $x^2 + y^2 - 4x - 8y - 30 = 0$ and the point (15,-5) . Show also that,

i. Two circles out of the three circles intersect orthogonally.
ii. The common chord of the three circles is a diameter of a circle.

61.Show that the common tangents drawn to the circle $(x-a_1)^2+(y-b_1)^2=r_1^2$ and $(x-a_2)^2+(y-b_2)^2=r_2^2$ intersect at the points $\left[\frac{a_1r_2+a_2r_1}{r_1+r_2}, \frac{b_1r_2+b_2r_1}{r_1+r_2}\right]$ or $\left[\frac{a_1r_2-a_2r_1}{r_2-r_1}, \frac{b_1r_2-b_2r_1}{r_2-r_1}\right]$. Find the equations of the common tangents drawn to the circles $x^2+y^2-20x+6y+84=0$ and $x^2+y^2+24x-2y-80=0$

62. Show that if the circles $S=x^2+y^2+2gx+2fy+c=0$ and $S'=x^2+y^2+2g^{/}x+2f^{/}y+c^{/}=0$ intersect. Then the common chord of the circles is s-s/=0. Show that the circles $C=x^2+y^2-4x+6y+10=0$ and $C^{/}=x^2+y^2-2x+2y=0$ intersect. Find the equation of the cricle C// whose a diameter is centres of the circles C and C/ Show that the circles C, C', C'' have the same common chord and C, C' intersect at right angles.

63. Show that if $2g_1g_2+2f_1f_2=c_1+c_2$ the circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and `
$x^2+y^2+2g_2x+2f_2y+c_2=0$ intersect orthogonally. If the centres of the circles are *A, B* and the points of intersection are *C, D*. Show that the circle passing through *A,B,C,D* is
$2(x^2+y^2)+2(g_1+g_2)x+2(f_1+f_2)y+C_1+C_2=0$. If it is possible to write the circle *CD* as the diameter as

$$x^2+y^2+2g_1x+2f_1y+c_1-\lambda[2(g_1-g_2)x+2(f_1-f_2)y+(c_1-c_2)]=0$$

Prove that $\lambda=\frac{r_1^2}{AB^2}$. Where r_1 is the radius of the first circle.

64. The circle $x^2+y^2=4$ intersects the positive x-axis at *A*. The point B is on the circle so that $A\hat{O}B=\pi/3$ where $A\hat{O}B$ measured in anticlockwise sense and o is the origin. A point *P* is situated on *OB* such that the circle with *P* as the centre touches the first circle internally and the circle *OA* as diameter externally. Show that $OP=8/5$.

65. If lx+my+n=0 touches $(x-x_0)^2+(y-y_0)^2=R^2$ then showthat $(lx_0+my_0+n)^2=R^2(l^2+m^2)$. The circle with radius unity and centre in the first quardrant touches the x-axis and the line $3y=4x$.
Find its equation and show that is touches the line $3x+4y=15$. Find the equation of the circle lying in the first quardrant which touches the x-axis, $3x+4y=15$ and $3y=4x$.

66. Show that the length of the common chord of the circles $(x-a)^2+y^2=a^2$ *and* $x^2+(y-b)^2=b^2$ *is* $\frac{2ab}{\sqrt{a^2+b^2}}$. Show also that the circle with this chord as the diameter is $(a^2+b^2)(x^2+y^2)=2ab(bx+ay)$

67. Show that the equation of the circle with the points (x_1, y_1) and (x_2, y_2) as the extrimities of a diameter is
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$. The coordinates *A,B,C* are (0,-14), (-5,1), (7,-5). The foot of the perpendicular drawn from *A* to *BC* is *L*. Obtain.

i. The equation of the circle passing through *A,B,C*.
ii. $ALC\Delta : ALB\Delta = 5:1$
iii. The angle between *BC* and the tangent to the circle *ALC* at *L*.

68. Show that

$x^2+y^2+2gx+2fy+c+k(xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ is any circle which touches the circle $x^2+y^2+2gx+2fy+c=0$ at (x_1, y_1), where k is a parameter. Find the circle which touches the circle $x^2+y^2+8x+14=0$ at (-5,1) and passing through (1,3).

Find the equation of the largest circle which touches one circle above internally and other externally.

69. If A and B are points (2,0), (-2,0) respectively. Find the equation to the locus of the point P as it varies. So that $AP:PB$=3:4. Show also that this locus is a circle and find its centre and radius. Find the equation of the circle passing A, B and centre at (0,5). Show that the circles intersect at right angles.

70. Show that when $a^2+b^2=c^2$ the circles $x^2+y^2+ax+by=0$ and $x^2+y^2=c^2$ touch each other. Find the point of contact. Two circles pass through (0,0) and (1,0) and touches the circle $x^2+y^2-4=0$. Find the coordinates of points of contact. Find the equation to the circle whose a diameter is the points of contact.

71. Show that the circles $4(x^2+y^2)-12x-16y-11=0$ and $4(x^2+y^2)-6x+48y+173=0$ touch each other and each circle intersects the circle $12(x^2+y^2)-92x+171=0$ at right angles.

72. Find the equations of the tangents drawn from (0,0) to the circle $x^2+y^2-6x-2y+9=0$. Find also the coordinates of points of contact.

73.Show that the circle whose diameter is the common chord of the circles $x^2+y^2+2x-5y=0$ and $x^2+y^2+6x-8y-1=0$ touches the coordinate axes.

74. Using first principles find the centre and the radius of the circle $x^2+y^2-10x-8y+31=0$. Two perpendicular tangents are drawn to the circle from a point on the x-axis. Show that there exist two such points and find the equations of the tangents in each case.

75. Two tangents are drawn to the circles $5x^2+5y^2-24x+32y+75=0$ and $5x^2+5y^2-48x+64y+300=0$ from any point on the circle $15x^2+15y^2-48x+64y=0$. Show that the ratio of the lengths of the tangents is in the ratio of the radii of the circles.

76. Show that the circles $S\equiv x^2+y^2-2x-6y+1=0$, $S^{/}\equiv 3x^2+3y^2-21x+2y+35=0$ lie completely, externally each other. Find the coordinates of the point P on $S^{/}$ which is furthest from S. Show that the equation of a tangent drawn from P to S is $x=4$ and find the equatoin of the other.

77. Find the condition for the chord lx+my+n=0 of the circle $S\equiv x^2+y^2+2gx+2fy+c=0$ subtends a right angle at the origin. Hence if a variable chord PQ of the circle $S\equiv 0$ subtends a right angle at the origin, then show that the locus of the foot of the perpendicular drawn to PQ from the origin is $x^2+y^2+gx+fy+\frac{c}{2}=0$

78. Find the condition required for the circles $S_1\equiv x^2+y^2+2g_1x+2f_1\,y+c=0$, $S_2\equiv x^2+y^2+2g_2x+2f_2\,y+c_2=0$ to be orthogonal. Let the orthogonal circles $S_1=0$ and $S_2=0$ intersect at P and Q. Show that the circle with the line joining centres of the circles as diameter passes through P and Q and find its equation.

79. The tangents at two points Q_1, Q_2 on the circle. $x^2+y^2+2gx+2fy+c=0$ meets at $P_0\equiv(x_0, y_0)$. Show that the

equation of the chord of contact Q_1, Q_2 of the point P_0 is $xx_0 + yy_0 + g(x + y_0) + f(y + y_0) + c = 0$. Prove that the chords of contact of the point (1,-2) with respect to the circles. $x^2 + y^2 + 6y + 5 = 0$ and $x^2 + y^2 + 2x + 8y + 5 = 0$ coincide. Show also that there is another point the chords of contact of which with respect to the circles are the same and find its coordinates.

80. i.The circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and the straight line $l \equiv px + qy + r = 0$ intersect each other at *A* and *B*. Interpret the equation $S + \lambda l = 0$ where λ is a parameter. Find the equation of the circle S' which has *AB* as the diameter when $S \equiv x^2 + y^2 - 6x + 2y - 17 = 0$ and $l = x - y + 2 = 0$. Show that the circle S' and the circle $x^2 + y^2 - 8x + 2y + 13 = 0$ touch externally.

ii. The circle S passes through (2,0) and cuts the circle S'. $x^2 + y^2 = 1$ at diametrically opposite points on S'. Find the equation of *S* if it cuts the circle $x^2 + y^2 - 4y - 5 = 0$ at right angles.

81. Show that the three circles $S_1 \equiv x^2 + y^2 - 9 = 0$, $S_2 \equiv (x-5)^2 + y^2 - 4 = 0$ and $S_3 \equiv (x-5)^2 + (y-12)^2 - 100 = 0$ touch each other externally. Show that the area enclosed by minor arcs of the above circle is $\frac{1}{2}(60 - 52\pi + 91\alpha)$ where $\alpha = \tan^{-1}\left(\frac{12}{5}\right)$

82. The straight line ax+by+c=1 meets the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ in the points P_1, P_2 and 0 is the origin of coordinates. Show that the equations of OP_1, OP_2 are given respectively by $y = m_1 x$ and $y = m_2 x$. Where m_1, m_2 are the roots of the quadratic equation.
$\left(1 + 2fb + cb^2\right)m^2 + \left(2gb + 2fa + 2abc\right)m + ca^2 + 2ag + 1 = 0$. If the circle passes through the origin *O*.

i. Show that the line joining *O* to the centre C of the circle is $y = \dfrac{a(m_1 + m_2) - b(1 - m_1 m_2)}{b(m_1 + m_2) + a(1 - m_1 m_2)}x$

ii. Evaluate $(y - m_1 x)(y - m_2 x)$ in terms of *f, g, a , b* and hence show that the coordinates of any point $P(x, y)$ on either of the lines OP_1, OP_2 satisfy the equation. $(1 + 2fb)y^2 + (2gb + 2fa)xy + (2ga + 1)x^2 = 0$

83.The straight line $ax + by = 1$ intersects the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the points *A* and *B*. If *AB* subtends a right angle at the origin of co-ordinates, show that $c(a^2 + b^2) + 2(ag + bf + 1) = 0$. A variable chord *PQ* of the circle $x^2 + y^2 - 6x - 4y - 3 = 0$ subtends a right angle at the origin. Using the above result or otherwise, show that the locus of the mid point of *PQ* is the circle $2x^2 + 2y^2 - 6x - 4y - 3 = 0$.

84. $S = 0$ is a circle and $U = 0$ is a straight line . Interpret the equation $S + \lambda U = 0$, where λ is a variable parameter. A variable circle Γ
passes through the points of intersection of the circle $x^2 + y^2 = 4$ and the line $x + y = 1$. Γ intersects the circles $x^2 + y^2 - 2x - 1 = 0$ at *P* and *Q*. Show that the line *PQ* passes through a fixed point and find the coordinates of this point. Show also that the mid point of *PQ* lies on the curve given by $2x^2 + 2y^2 - 5x + y + 3 = 0$.

85. i. Let $A \equiv (1,2)$ and $B \equiv (3,2)$. Let $P \equiv (x, y)$ be a variable point such that the angle APB is a constant.

ii. If $A\hat{P}B = 90^0$, prove that P lies on the circle $x^2 + y^2 - 4x - 4y + 7 = 0$. What is the locus of *P*? Justify your answer.

iii. If $A\hat{P}B = 135^0$, prove that *P* lies either on the circle $x^2 + y^2 - 4x - 2y + 3 = 0$ or on the circle

$x^2+y^2-4x-6y+11=0$. What is the locus of *P*? Show that these two circles intersect at right angles.

86. $P(\cos\theta, \sin\theta)$ is a variable point on the circle $x^2+y^2=1$ and *Q* is the other extremity of the diameter through *P*. *A* and *B* are the points with coordinates (1,0) and (0,1) respectively. If *AP* and *BQ* intersect at *U*. Show that the coordinates *U* satisfy the equations. $(x-1)\cos\frac{\theta}{2}+y\sin\frac{\theta}{2}=0$ and $(1+x-y)\cos\frac{\theta}{2}+(x+y-1)\sin\frac{\theta}{2}=0$. Deduce that u lies on a fixed circle S and obtain its equation. Show also that the point of intersection of *AQ* and *BP* also lies on *S*.

87. Show that, if the two circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ intersect orthogonally, then $2g_1g_2+2f_1f_2=c_1+c_2$. Find the values of λ and μ So that the equation $\lambda(y-7x)y+\mu y(4y-4x+3)+(4y-4x+3)(y-7x)=0$ represents a circle. Hence show that this equation with those values of λ and μ, represents the circumcircle *S* of the triangle *OAB* whose sides *OA, AB* and *BO* are given by the equations $y=0$, $4y-4x+3=0$ and $y-7x=0$ respectively. Obtain the equation of the circle which intersects *S* orthogonally at *O* and *A*.

88. Show that if the two circles given by $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ intersect orthogonally then $2g_1g_2+2f_1f_2=c_1+c_2$. A circle *S* with centre on the *X*-axis intersets orthogonally the circle S' given by $x^2+y^2-8x-6y+21=0$ and touches the circle S'' given by $x^2+y^2+4x+6y+9=0$. Show that there are two such circles of *S*, one touching the circle S'' externally and the other touching the circle S'' internally. Find the equations of these two circles.

89.Show that the equation of the chord of contact of the tangents drawn from an external point (x_0, y_0) to the circle $x^2+y^2+2gx+2fy+c=0$ is $xx_0+yy_0+g(x+x_0)+f(y+y_0)+c=0$. $x^2+y^2+2x+6y+1=0$ and $4x+3y-5=0$ are the equations of a given circle and a given straight line respectively. Show that the line does not cut the circle. A variable straight line intersets the given circle at two distinct points *P* and *Q*, and the tangents to the circle at *P* and *Q* meet on the given straight line. Show that this variable lines passes through a fixed point and find the corrdinates of this point.

90. Find a condition that the circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ may touch and prove that if they touch the point of contact lies on each of the lines.
$2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0$ and $(f_1-f_2)x-(g_1-g_2)y+f_1g_2-f_2g_1=0$. Show that the circles $x^2+y^2-2x+4y=0$ and $x^2+y^2-10x+20=0$ touch each other externally and find the coordinates of *A* the point of contact of the two circles. *P* is a point such that the lenght of the tangent from *P* to the first circle is *k* (a constant) times that of the tangent from *P* to the second. Prove that if $k^2\neq 1$ the locus of *P* is a circle through *A* and find its equation in terms of *k*.

91. If the two circles $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$. Intersect orthogonally, show that $2gg'+2ff'=c+c'$ Let *P* and *Q* be the points on the circle $S\equiv x^2+y^2-a^2=0$ with the coordinates $(-a,0)$ and $(a\cos\theta, a\sin\theta)$ respectively. The chord *PQ* is extended to a point *R* so that *PQ=QR*. Find the coordinates of *R* and show that as θ varies, *R* lies on a circle S'. Obtain the equation S' .A third circle S'' which touches the *Y*-axis, intersects both circles *S* and S' orthogonally. Show that there are two such circles S'' and obtain their equations.

92. Let $S_1 \equiv x^2 + y^2 - 2 = 0$ and $S_2 \equiv x^2 + y^2 + 3x + 3y - 8 = 0$. Show that S_1=0 and S_2=0 touch internally and find the coordinates of the point of contact *P*. A stratight line drawn through the point *P* cuts S_1=0 and S_2=0 again at the points *Q* and *R* respectively. Show that the mid-point of *QR* lies on the circle $x^2 + y^2 + \frac{3x}{2} + \frac{3y}{2} - 5 = 0$.

93. Show that the two circles with equations $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 - 4r^2 = 0$ never touch each other externally, but touch each otherr internally if $g^2 + f^2 = r^2$. Find the coordinininates of the point of contact in the latter case. Show that there are two circles, which pass through the origin and the point $(a,0)$ where $0 < a < 1$ and touch the circle whose equation is $x^2 + y^2 - 4 = 0$. Find the coordinates of the points of contact. Find also the equation of the circle having these points as ends of a diameter.

94. Obtain the equation of the chord of contact of tangents drawn to the circle $x^2 + y^2 = a^2$ from the external point (x_0, y_0). A circle through the points (1, 1) and (-1, 0) intersets the circle $S \equiv x^2 + y^2 - a^2 = 0$ at the distinct points *P* and *Q*. The tangents drawn at *P* and *Q* to the circle *S*=0 meet at *R*. Show that the point *R* lies on the line $(2a^2 - 3)x + (a^2 - 1) - a^2 = 0$.

95. Two circles are said to intersect orthogonally when the tangents at their points of intersection are at right angles. Find the condition for the two circles. $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ to intersect orthogonally. Prove that the equation $x^2 + y^2 + 4x + 2\lambda y - 6 = 0$ ....(*) where λ is the parameter, represents a system of circles passing through the points $\left(-2 + \sqrt{10}.0\right)$ and $\left(-2 - \sqrt{10},0\right)$. $S = 0$ is a circle belonging to the system represented by (*). Show that there exists a unique circle $S' = 0$ belonging to the same system which is orthogonal to $S = 0$. Find $S' = 0$ when $S \equiv x^2 + y^2 + 4x + 4y - 6$. Find also the general equation of the circle otrhogonal to both $S = 0$ and $S' = 0$.

96. *a.*Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and let $S' \equiv x^2 + y^2 + 2g'x + 2f'y + c' = 0$ $S = 0$ is a variable circle passing through a fixed point, and $S' = 0$ is a fixed circle . The circle $S = 0$ cuts the circle $S' = 0$ at the opposite ends of a diameter. Show that the centre of $S = 0$ lies on a fixed straight line.

b. A and *B* are the two distinct points (x_1, y_1) and (x_2, y_2) respectively. Find the equation of the circle having *AB* as a diameter.

CD is the diameter perpendicular to *AB*. Show that the coordinates of *C* and *D* take the form

$$\left[\frac{1}{2}(x_1 + x_2) + \lambda, \frac{1}{2}(y_1 + y_2) + \mu\right] \text{ and } \left[\frac{1}{2}(x_1 + x_2) - \lambda, \frac{1}{2}(y_1 + y_2) - \mu\right]$$

where λ and μ are to be dertermined.

97.The equations $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ represent two non intersecting circles. Let O_1 and O_2 be the centres of the two circles. A pair of common tangents can be drawn to the two circles from a point *T* lying between O_1 and O_2. Identify the point *T* and find its coordinates in terms of the coordinates of O_1 and O_2 and the radil of the two circles.

Identify also the point *T* on the extended line O_1O_2 through which a second pair of tangents can be drawn to the two circles and find its coordinates. Find the equations of the four common tangents to the two circles $x^2 + y^2 - 18x + 6y + 86 = 0$ and $x^2 + y^2 + 18x - 6y + 74 = 0$.

98. State the conditions for two circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ to touch each other internally or externally. Let $S \equiv x^2+y^2+2gx+2fy+c=0$ be a circle and $P_1(x_1,y_1)$ be a point which lies outside the circle $S=0$. Show that the length of a tangent from the point P_1 to the circle $S=0$ is given by $\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$. Find also, the equation of the circle that passes through the origin and intersects orthogonally each of the circles C_1 and C_2.

(100) Show that, if the circle $S' \equiv x^2+y^2+2g'x+2f'y+c'=0$ cuts the circle $S \equiv x^2+y^2+2gx+2fy+c=0$ at the ends of a diameter of the circle $S=0$, then $2g^2+2f^2-c=2gg'+2ff'-c'$

A variable circle cuts each of the circles $S_1 \equiv x^2+y^2-25=0$ and $S_2 \equiv x^2+y^2-2x-4y-11=0$ at the ends of a diameter. Show that the centre of the variable circle lies on the straight line $x+2y+2=0$.

(101) If the two circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ touch each other, show that the point of contact lies on the straight lines $2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0$ and $(f_1-f_2)x-(g_1-g_2)y+f_1g_2-f_2g_1=0$. If the circles $x^2+y^2+4x+2y+k=0$ and $x^2+y^2+4x-4y+4=0$ touch each other, find the possible values of k.

Determine whether the circles in each case touch internally or externally.

(102) Show that,for all values of g and f the circle $x^2+y^2+2gx+2fy-r^2=0$ bisects the circumference of the circle $x^2+y^2-r^2=0$. Show that two circles can be drawn through the point $(1,1)$ touching the straight line $y+5=0$ and bisecting the circumference of the circle $x^2+y^2-4=0$. Find the equations of these two circles.

(103) Show that if the circles given by $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$ intersect orthogonally. then $2gg'+2ff'=c+c'$ .The centre of a circle C lies on the straight line $3x-y-5=0$. Also, the circle C' given by $x^2+y^2+4x+6y+9=0$ intersects the circle C orthogonally. Show that the equation of C can be written in the form $x^2+y^2+10y+21+\lambda(x+3y+11)=0$, where $\lambda \in \Re$.

Show further that, for different values of λ, these circles C have a common chord and verify that the centre of C' lies on this common chord.Also,find the length of this common chord.

(104) Find the coordinates of the centre and the radius of the circle S whose equation is given by $x^2+y^2-2x-2y+1=0$, and sketch the circle S in the xy -plane.

Let P be the point on the circle S, furtherest from the origin Write down the coordinates of the point P and show that the equation of the tangent line to the circle S at the point P is given by $x+y=2+\sqrt{2}$.

A circle S' which touches the line l, also touches the circle S externally at a point distinct from P. Let (h,k) be the coordinates of the centre of the circle S'. By considering the positions of O and the centre of S' with respect to the line l, show that $h+k<2+\sqrt{2}$

Show further that the coordinates of the centre of S' satisfy the equation $h^2-2hk+k^2+4\sqrt{2}(h+k)=8(\sqrt{2}+1)$.

(105) Let l_1 and l_2 be the straight lines given by $2x+y=5$ and $x+2y=4$ respectively. Show that the acute angle between l_1 and l_2 is $\tan^{-1}\left(\frac{3}{4}\right)$, and find the equation of the bisector of this angle.

Let A be the point of intersection of l_1 and l_2 ,and let $R=\{(x,y): x+2y\le 4 \ and\ 2x+y\ge 5\}$. Find the coordinates of the point A and shade the region R in the xy-plane.

Show that the equation of the circle S of radius $\sqrt{5}$ which lies in the region R and which touches both lines l_1 and l_2 is $x^2+y^2-14x+8y+60=0$.

Using the usual formula for the chord of contact, show that the equation of the chord of contact of the tangents drawn from the point A to the circle S is $x-y=10$.

Find the equation of the circle passing through the point A and the points of contact of S with l_1 and l_2.

(106)Let the equations of two circles be $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$. If these cirles intersect orthogonally, show that $2gg'+2ff'=c+c'$.

Show that the circle C , with the equation $x^2+y^2-8x-6y+16=0$ touches the x- axis.

Two circles, C_1 of radius r and C_2 of radius $R(>r)$, with common centre at the origin O,touch the circle C at the points A and B , respectively.Find the values of r and R ,and the coordinates of A and B.

Let S be a circle which intersects both the circles C and C_1 orthogonally and which touches the y- axis. Find the two possible equations for S.

The common tangent drawn to the two circles C and C_2 at the point B , meets the x- axis at P and the y-axis at Q. Show that the equation of the common tangent is $4x+3y=40$, and that the equation of the circle with the line segment PQ as a diameter is $3(x^2+y^2)-30x-40y=0$.

107.let l_1 and l_2 be the straight lines given by $2y-x=0$ and $y-2x+3=0$ respectively. Show that the equation of the acute angle bisector between the lines l_1 and l_2 is $y-x+1=0$. Let this bisector be l'. Let A be the point of intersection of the lines l_1 and l_2. Find the coordinates of A.

Find parametric coordinates of any point on l'. A circle S has radius $\sqrt{5}$ with its centre lying on l' and touching both lines l_1 and l_2. Show that there are two such circles for S and their equations are given by $x^2+y^2-14x-12y+80=0$ and $x^2+y^2+6x+8y+20=0$. Identify the circle S which lies in the region R given by $R=\{(x,y): 2x-y\le 3 \text{ and } 2y\le x\}$. Show that the equation of the chord of contactof the tangents drawn from the point A to this circle S is $x+y+6=0$. Show also that the equation of the circle passing through the origin and points of contact S with l_1 and l_2 is $3x^2+3y^2+8x+14y=0$.

108.Sketch the circles $S_1\equiv x^2+y^2=4$ and $S_2\equiv x^2+y^2-10x+24=0$. A circle S_3 touches the circles S_1 and S_2 externally. Show that the centre of S_3 lies on the curve $24x^2-y^2-120x+144=0$.

109.Show that the circle given by $S\equiv x^2+y^2-2x-4y+1=0$ touches x- axis. Find the equation of the chord of contact l of the tangents drawn to S from $A(-3,0)$. If the points of intersection of S and l be B and C. Find the equation S_1 of the circumcircle of the triangle ABC. show also that the centre of the circle which is orthoganal to both circles S and S_1 moves on the line $2x+y-2=0$.

110.A circle touches y- axis at the point (0, 2), and it makes a chord on the x- axis is of length 3. show that there exist two circles and their equations are $x^2+y^2+5x-4y+4=0$ and $x^2+y^2-5x-4y+4=0$.

111.Find the values of k and m so that the equation $(y+mx)^2 = ky(3x+4y-9)$ represent a circle. Obtain the equations of corresponding circles.

112.A circle with centre lying on the line $2x-2y+9=0$ intersect the circle $x^2+y^2=4$ orthogonally. Show that this circle passes through two fixed points and find the coordinates of these points.

113.A circle passing through the points (1,2) and (2,1) touches the x-axis. show that there exist two circles. Find the equations of these circles.The common chord of these circles cuts x and y axes at A and B.

Find the area of the triangle OAB The common tangents drawn to this circles meet each other at the point C. Find C. Also find the equations of these tangents.

114.The line $l \equiv x-2y-5=0$ intersects the circle $S \equiv x^2+y^2-4x+8y+10=0$ at A and B. Two tangents are drawn to S at the points A and B. Find the coordinates of point of intersection M of this tangents. Find the circle passing through A, B, M. Show that the circumference of this circle is bisected by the circle $S=0$ Find the circle passing through A and B orthogonal to $S=0$.

115.a)Obtain the conditions under which

i) two circles intersect each other orthogonally,

ii) a circle bisects the circumference of another circle.

b) A circle $S=0$ passes through the points of intersection of the circle $S_1 \equiv x^2+y^2+2x+2y-1=0$ and the line $l \equiv x+2y=0$.

i) If the circle $S=0$ and the circle $S_2 \equiv x^2+y^2+10y-9=0$ intersect each other orthogonally, find the circle $S=0$.

ii) If the circle $S=0$ bisects the circumference of the circle $S_3 \equiv x^2+y^2+2x+2y-7=0$, find the circle $S=0$.

116.Find the coordinates of the point A at which the two circles $S_1 \equiv x^2+y^2-2x+4y=0$ and $S_2 \equiv x^2+y^2-10x+20=0$ touch each other externally. If P is a point such that the length of a tangent from P to $S_1=0$ is half of the length of the tangent from P to $S_2=0$, show that the locus of the point P is a circle through the point A. Suppose that a circle $S_3=0$ intersects the circle $S_2=0$ at the points A and $B(4, 2)$.

i . Show that the locus of the centre of the circle $S_3=0$ is a straight line.

ii. Find the coordinates of the centre C of the circle $S_3=0$ when CA is perpendicular to CB.

117.Let $m \in \Re$. Show that the point $P \equiv (0,1)$ does not lie on the straight line l given by $y=mx$.
Show that the coordinates of any point on the striaght line through P perpendicular to l can be written in the form $(-mt, t+1)$, where t is a parameter.
Hence, show that the coordinates of the point Q, the foot of the perpendicular drawn from P to l are given by

$\left(\frac{m}{1+m^2}, \frac{m^2}{1+m^2}\right)$.

Show that, as m varies, the point Q lies on the circle S given by $x^2 + y^2 - y = 0$, and sketch the locus of Q in the xy-plane. Also, show that the point $R \equiv \left(\frac{\sqrt{3}}{4}, \frac{1}{4}\right)$ lies on S.

Find the equation of the circle S' whose centre lies on the x-axis, and touches S externally at the point R.
Write down the equation of the circle having the same centre as that of S' and touching S internally.

118.Let $A \equiv (-2,-3)$ and $B \equiv (4,5)$. Find the equations of the lines l_1 and l_2 through the point A such that the acute angle made by each of the lines l_1 and l_2 with the line AB is $\frac{\pi}{4}$.

The points P and Q are taken on l_1 and l_2 respectively such that $APBQ$ is a square.
Find the equation of PQ, and the coordinates of P and Q. Also, find the equation of the circle S through the points A, P, B and Q.
Let $\lambda > 1$. Show that the point $R \equiv (4\lambda, 5\lambda)$ lies outside the circle S.
Find the equation of the chord of contact of the tangents drawn from the point R to the circle S.
As $\lambda (> 1)$ varies, show that these chords of contact pass through a fixed point.

119. i.The equation of the diagonal AC of a rhombus $ABCD$ is $3x - y = 3$ and $B \equiv (3, 1)$. Also, the equation of CD is $x + ky = 4$, where k is a real constant. Find the value of k and the equation of BC.

ii.Sketch the circles, C_1 and C_2 given by the equations $x^2 + y^2 = 4$ and $(x-1)^2 + y^2 = 1$ respectively, indicating clearly their point of contact.
A circle C_3 touches C_1 internally and C_2 externally. Show that the centre of C_3 lies on the curve $8x^2 + 9y^2 - 8x - 16 = 0$.