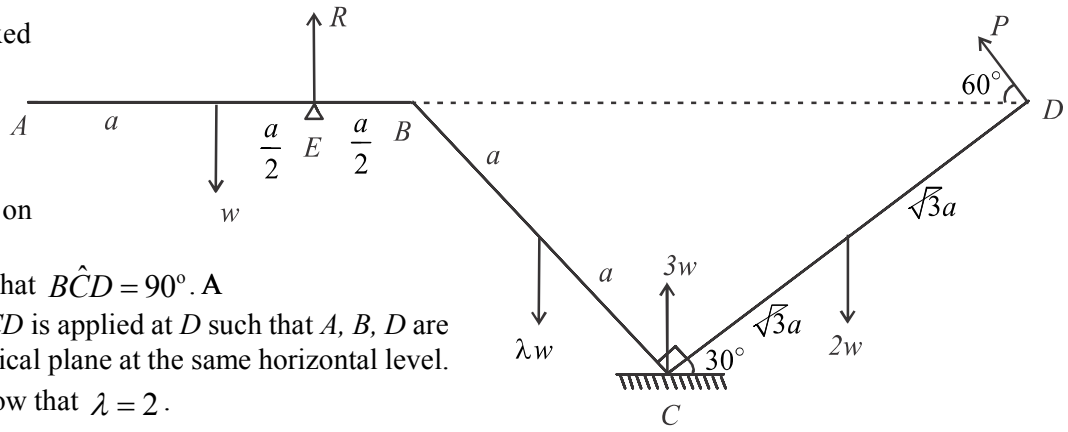
JOINTED RODS

1.Four equal uniform rods *AB, BC, CD* and *DA* are hinged freely at their ends to form a framework in the shape of a square. The weight of the rods *AB* and *AD* are each *w* and the weight of the rods *BC* and *CD* are each 2*w*. A light rod is connected to the mid-points of the rods *AD* and *CD*. The framework is suspended from the joint *A*. If the system is in equilibrium in the vertical plane, find the reaction at the joints *B* and *C*. Also find the tension of the light rod.

2.Three uniform rods, $AB = 2a, BC = 2a$ and $CD = 2\sqrt{3}a$ are smoothly joined at the ends *B* and *C*. The weights of the rods *AB, BC* and *CD* are *w*, λw and 2*w* respectively. The end *A* is smoothly hinged to a fixed point. *AB* is kept horizontally by a smooth fixed peg at the point *E* on the rod *AB* such that $AE = \frac{3a}{2}$. Jont *C* is placed on the horizontal ground such that $B\hat{C}D = 90^{\circ}$. A force '*P*' perpendicular to *CD* is applied at *D* such that *A, B, D* are kept at equilibrium in a vertical plane at the same horizontal level. The reaction at *C* is 3*W*. Show that $\lambda = 2$.



Further, show that the reaction at the peg *E* can be given by $R = w$. Also, show that the horizontal and vertical components of the force exerted on *AB* by *CB* at *B* are $\frac{\sqrt{3}w}{4}$ and $\frac{w}{4}$ respectively.

3. *AB* and *BC* are two uniform rods of equal length and weights 2*W* and *W* respectively. The rods are smoothly jointed at *B* and hung by a point *A*. *A* and *C* points are connected by an inelastic string that brings the system to equililbrium, the angle $A\hat{B}C = \frac{\pi}{2}$.

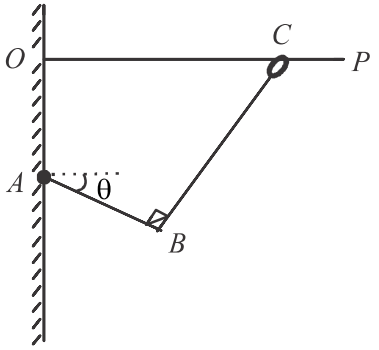
Show that

i.the rod *AB* is inclined an angle $\tan^{-1}\frac{1}{4}$ to the vertical,

ii.the tension in the string is $\frac{2w}{17}\sqrt{34}$ and

iii.the reaction at the joint *B* on *BC* is $\frac{w}{17}\sqrt{85}$, makes an angle $\tan^{-1}2$ with *BC*.

4. *AB* and *BC* are two rods of lengths 2*a,* 4*a* and weights *w*, 2*w* respectively. These rods are smoothly hinged at *B*. One end *A* of rod *AB* is smoothly hinged to a vertical wall as shown in the figure. A light small ring which can freely slide along a rough horizontal wire *OP* is attached to the end *C* of the rod *BC*. At the equilibrium, the rod *AB* makes an angle θ to the horizontal and $A\hat{B}C = \frac{\pi}{2}$. If $\tan\theta = \frac{3}{4}$, show that the magnitude of horizontal and vertical components of the reaction at joint *B* are $\frac{18w}{25}$ and $\frac{w}{25}$ respectively. If the coefficient of friction between the ring and the wire is μ, show that $\mu \geq \frac{18}{49}$.



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5.Four uniform smooth rods each of length $2b$ and weight w are smoothly jointed at A, B, C and D to form a rhombus $ABCD$. The system is kept in equilibrium in a vertical plane so that the rods AB and AD rest upon two smooth pegs at the same horizontal level. The distance between the pegs is $2a$. The rods AB and AD each makes an angle θ to the vertical. A weight of w hang at the lowest point C. Show that

i. The reaction on the peg is $\frac{5w}{2}\cos ec\theta$

ii., if the reaction at B makes an angle α to the vertical, $3\tan\alpha = 2\tan\theta$ and

iii. $5a = 12b\sin^3\theta$

6. A rhombus $ABCD$ is formed of four uniform rods AB, BC, CD and AD each of length $2a$ and weight w, smoothly jointed at their ends. The rhombus is suspended from A and a light inextensible string is connected to the points L and M on the rods AB and BC respectively. Here $AL = CM = \frac{a}{2}$.

The string LM and AC are vertical and the system is in equilibrium in a vertical plane with the vertex A is above C. Given that $B\hat{A}D = B\hat{C}D = 60^o$.

i.Find the reaction at C and show that its inclination to the horizontal is $\tan^{-1}(2\sqrt{3})$.

ii.Show that the tension of the string LM is $\frac{8w}{3}$.

iii.Find the magnitude and the direction of the reaction at B.

7.Three uniform rods of AB, BC and CD each of length $2a$ and weight W are jointed freely at B and C. The rod BC is horizontal and the end A and D are kept on a smooth horizontal plane. Two light, inextensible strings of same length are connected to the midpoints of rods AB and CD and the other ends are connected to the midpoint of rod BC. The strings are kept taut and $ABCD$ is in equilibrium in a vertical plane. $A\hat{B}C = 120^o$, Show that the tensions in the strings is $2W$. Find the magnitude of the reaction at the joint B and show that the reaction makes an angle $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ with the horizontal.

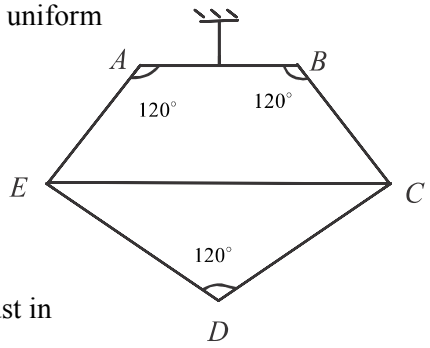
8. Four uniform smooth rods AB, BC, CD and DA each of length $2a$ and weight w are smoothly jointed at A, B, C and D so as to form a rhombus. The rods AB and AD are placed on two smooth pegs on the same horizontal level at a distance $2l$ apart such that the joint A is above C. A weight $2w$ is hung at C. The rhombus is kept in equilibrum symmetrically in a vertical plane with $B\hat{A}D = 120^\circ$

i.Find the reaction on a rod by a peg.

ii.Show that the reaction at the joint B is $\frac{\sqrt{43}w}{2}$ and find the inclination of the reaction to the horizontal.

iii.Also show that $l = \sqrt{3}a$.

9.The figure represents a frame in the form of a pentagon *ABCDE* formed of five uniform rods of weight *w* per unit length jointed at their ends. $AE = BC = 2a$, and $ED = CD = 2b$. The angles at vertices *A, B* and *D* are 120° each. The frame is suspended from the mid-point of *AB* and is in equilibrum with the symmetrical shape maintained by a light rod *CE* of length $2b\sqrt{3}$ connecting the joints *C* and *E*. Show that the reaction at the joint *D* is of magnitude $2b\sqrt{3}w$ and find the thrust in the light rod *CE*.



10. A rhombus *ABCD* is formed by joining freely, four uniform rods *AB, BC, CD* and *DA* each of length 2*a* and weight *W* ,at the points *A, B, C* and *D*. The system rests on the surface of a smooth cylinder of radius *r* which is fixed with its axis horizontal. The rods *AB* and *AD* rest in the vertical plane perpendicular to the axis of the cylinder. If the rods *AB* and *AD* make an angle $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ with vertical.

i.Find reactions on *AB, AD* which exerted by the cylinder.

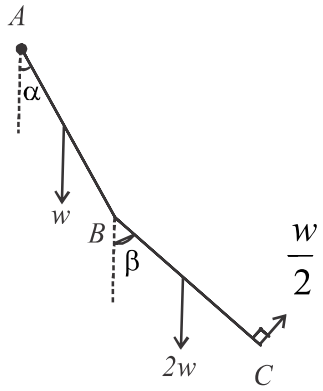
ii.Find magnitude and directions of reactions at joint *A* and *C*.

iii.Deduce the horizontal and vertical components of reaction either at *B* or *D*.

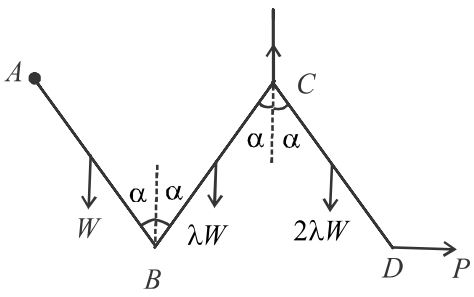
iv.Show that $r = \frac{a}{5}$.

11. Two uniform rods *OA* and *AB*, each of length 2*a* and weights *aw* and *bw* respectively, are freely jointed at *A* and suspended from *O*. The system is kept in equilibrium by a horizontal force *P* at *B*. If the rods *OA* and *AB* make the angles α and β to the vertical respectively, show that $\frac{\tan\alpha}{\tan\beta} = \frac{b}{a+2b}$.

12. Two uniform rods *AB* and *BC*, each of length 2*a* are jointed smoothly at *B*. The rod *AB* is of weight *W* and the rod *BC* is of weight 2*W*. The end *A* is hinged smoothly to a fixed point. This system is kept in equilibrium in a vertical plane with rods *AB* and *BC* making angles α and β, respectively, with the downward vertical by a force $\frac{W}{2}$ applied at *C* in the direction perpendicular to *BC* shown in the figure. Show that $\beta = \frac{\pi}{6}$ and find the horizontal and the vertical components of the reaction at the joint *B* on the rod *BC* exerted from the rod *AB*. Also, show that $\tan\alpha = \frac{\sqrt{3}}{9}$.



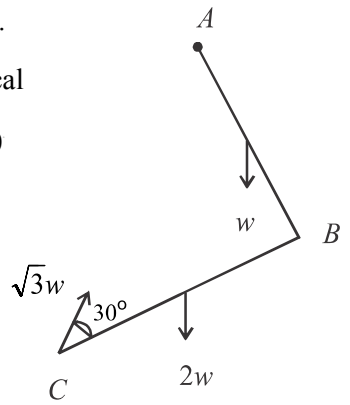
13. Three uniform rods *AB, BC* and *CD* each of length 2*a* are smoothly joined at the ends *B* and *C*. The weights of the rods *AB, BC* and *CD* are *W*, λW and $2\lambda W$, respectively. The end *A* is smoothly hinged to a fixed point. The rods are kept in equilibrium in a vertical plane by a light inextensible string attached to the joint *C* and to a fixed point vertically



above C and by a horizontal force P applied to the end D such that A and C are at the same horizontal level and each of the rods making an angle α with the vertical, as shown in the figure. Show that $\lambda = \frac{1}{3}$.Show also that the horizontal and vertical components of the force exerted on AB by CB at B are $\frac{W}{3}\tan\alpha$ and $\frac{W}{6}$, respectively.

14. Four uniform rods each of weight W are hinged smoothly to form a rhombus $ABCD$. The system is suspended freely from D. The vertex A and the midpoint of CD are joined by a light rod such that $A\hat{B}C = \frac{\pi}{3}$. Find the reaction of joint C and the stress of the light rod.

15. Two rods AB, BC of weights W, $2W$ and each of length $2a$ are smoothly jointed at B. The end A of rod AB is smoothly hinged at a point. Angles made by AB, BC with vertical are α, β respectively. The system is kept in equilibrium by a force $\sqrt{3}w$ at an angle 30 with CB as shown in the figure. Find α, β and the reaction at joint B.



16. Three uniform rods each of weight w are smoothly jointed at P, Q, R so as to form an isosceles triangle.

Here $PQ = 8a$, $Q\hat{P}R = \frac{\pi}{6}$, $P\hat{Q}R = \frac{2\pi}{3}$.The joint P is smoothly hinged to a wall and the rod PQ is kept in a horizontal position by means a smooth peg in the same level of P at a distance $6a$ from P.

i.Find the reaction by the peg.

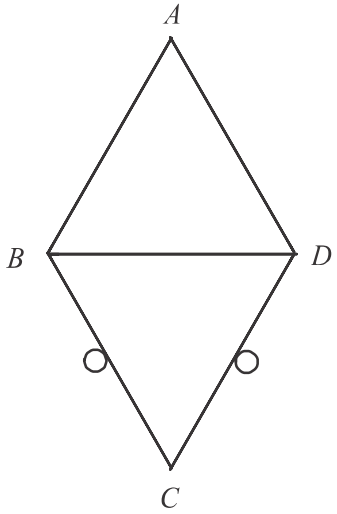
ii.Show that the reaction at the point Q is $\frac{\sqrt{19}w}{2}$ and find the reaction at R.

iii.Now instead of the peg, by applying a force F at R perpendicular to PR, the system is kept in equilibrium as before. Find the value of F. Show that the reaction at the joint Q makes an angle $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ to the horizontal.

17. Four uniform rods each of weight W are smoothly jointed at A, B, C, D and suspended from A. The framework is kept in equilibrium in a vertical plane by joining the mid-points of AB and AD by a light rod and suspending loads each of weight W at B and D. If $B\hat{A}D = 2\alpha$, $B\hat{C}D = 2\theta$ and $AB = AD = 2a$, find the reaction at C and show that the thrust in the light rod each is $w\left[5\tan\alpha + \tan\theta\right]$.

18. Two uniform rods AB, AD each of length $4a$ and weight $2w$ and another two uniform rods BC, CD each of length $3a$ and weight W are smoothly jointed at A, B, C and D and framework is suspended from A. such that $A\hat{B}C = A\hat{D}C = 90^\circ$, by joining mid-points P and Q of BC and CD by two light rods AP and AQ. If the framework is in equilibrium in a vertical plane find the reaction at the joint C. Also find the stresses of the rods AP and AQ.

19.Four rods *AB, BC, CD, DA* each of length $2a$ and weight w are smoothly jointed at ends *A, B, C* and *D*. The system is at equilibrium in a vertical plane by a light rod of length $2a$, joining the points *B* and *C* with the midpoints of *BC* and *CD* touching two pegs in the same horizontal level as shown in the figure. Find the reactions of the pegs on rods *BC* and *CD* and also find the reactions at joints *A* and *C*. Also, show that the thrust of the light rod is $\frac{2w}{\sqrt{3}}$.



20. *ABCDEF* is a framework formed with six uniform rods each of weight w and length a, smoothly jointing at their ends. The framework is suspended from *A*, and the regular hexagonal shape is maintained by a light rod joining the points *L* and *M* on *BC* and *FE* respectively, such that $BL = FM = b$.
A uniform circular lamina of weight w and radius $\frac{a}{\sqrt{3}}$ is placed symmetrically on the rods *DC* and *DE* such that its plane is on the same vertical plane of the framework *ABCDEF*.

i. Show that the reaction of *D* is $\frac{17w}{6\sqrt{3}}$ horizontally.

ii. Show that the thrust of the light rod is $\frac{34w}{3\sqrt{3}}$.

iii. Show that $b = \frac{7a}{34}$.

21.Three rods *AB*, *BC*, *CD* of weights $\frac{w}{2}$, w, $\frac{w}{2}$ and length a, $2a$, a are smoothly jointed at *B* and *C*. The rods are in equilibrium symmetrically on a fixed smooth sphere of radius $2a$, with *A*, *D* and mid - point of *BC* in contact with the sphere. Show that the reaction between sphere and *BC* is $\frac{91w}{50}$. Find the horizontal and vertical components of reaction at *B* and *C* and show that the distance to the point where these reactions meet from *BC* is $\frac{41a}{12}$.

22.Four equal uniform rods *PQ, QR, RS* and *SP* each of weight w, are smoothly jointed to form a rhombus *PQRS*. It is suspended from the point *P* and kept in equilibrium in a vertical plane with the angle *QRS* equal to $2\tan^{-1}2$ by a light inextensible string connected to the midpoints of the rods *PS* and *RS*.

i. Find the reactions at the joints *R* and *S*. *ii*. Show that the tension of the string is $4w$.

23. A rhombus is formed by using four equal uniform rods *AB, BC, CD, DA* each of weight w and length $2a$.
The rhombus is suspended from *A* and rhombic form is maintained by using a string *AC*, and it is in equilibrium so that $B\hat{A}D = 60^{\circ}$.

(*i*) Find the magnitade and direction of the reaction at the joint *B*.

(*ii*) Show that the depth from A to the point of intersection of the lines of the reactions at the joints *A* and *B* on the rod *AB* is $\sqrt{3}a$.

(*iii*) Show that the tension in the string *AC* is $2w$.

24. A frame *ABCDE* in the shape of a regular pentagon has been made with five identical uniform rods, each of length $2a$ and weight *w,* freely jointed at their ends. The frame has been kept in a vertical plane with the side *CD* of the pentagon in contact with a horizontal plane. The shape of the pentagon is maintained by a string joining the mid points of the rods *BC* and *DE*. Show that the reaction at the joint *A* is $\frac{w}{2}\cot\frac{\pi}{5}$. Find also the tension in the string.

25.The weights of three uniform rods *AB, BC* and *CD*, each of length $4a$ are λW, *W* and λW respectively. The rods are smoothly jointed at *B* and *C*. The ends *A* and *D* are freely hinged to two fixed points in the same horizontal level, a distance $8a$ apart from each other, such that the system is in equilibrium in a vertical plane with *BC* lying below *AD*.

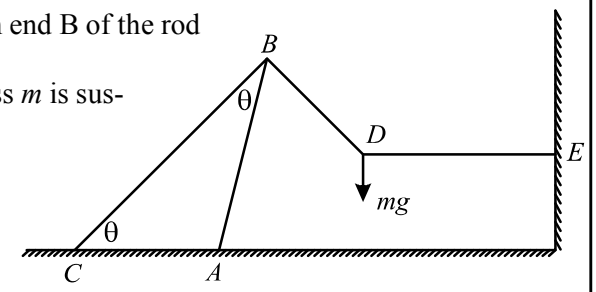
*i.*Find the horizontal and vertical components of the reaction at *A*.

ii.Find the horizontal and vertical components of the reaction on the rod *BC* at the point *B*.

iii the lines of action of the reactions on the rod *BC* at *B* and *C* meet at a distance $\frac{\sqrt{3}}{2}$ a vertically below *BC*, find the value of λ .

26. Four equal rods each of weight *w* and length $2a$ are smoothly jointed at their ends to form a rhombus *ABCD*. The rods *BC* and *CD* are placed on two smooth pegs at a distance $2c$ apart on the same horizontal level. The mid points of *AB* and *AD* are joined to the vertex C by means of two equal light rods. If $B\hat{A}D = \frac{2\pi}{3}$, show that the stress of light rods is $\frac{w}{3a}(3\sqrt{3}a - 4c)$. Find the reaction of the hinge *A*.

27.*AB* is a fine uniform rod of mass *M* with its end *A* is smoothly hinged to the ground. One end of a light inextensible string is tied to a point *E* in a wall. The other end passes over the rough end B of the rod and tied to a point *C* on the ground such that *AB=AC*. A particle of mass *m* is suspended from *D* such that *DE* is horizontal. $A\hat{B}D = 90^0$ and $A\hat{B}C = \theta$. The parts *BC, AB, BD, DE* are in the same vertical plane. $\left(0 < \theta < 45^0\right)$



(i). Calculate the tensions in *DE, DB* parts of the string.

(ii). Show that the tension in the string *BC* is $\frac{2m + M\cos^2 2\theta}{2\sin\theta.\cos^2\theta}.g$.

(iii). If $\theta = 30^0$, find the magnitude and direction of the reaction at *A*.