## *COPLANAR FORCES*

1.The resultant of two forces $P$ and $2P$ is perpendicular to the force $P$. Find the angle between the two forces and the magnitude of the resultant.

2.The angle between two forces is 120. The larger of them is $80N$. If their resultant is perpendicular to the smaller, find the smaller force.

3.The sum of two forces is $24N$. Their resultant is at right angles to the smaller of the two forces, which is $12N$. Find the magnitudes of the two forces and the angle between them.

4.Two forces are such that one is twice the other. Their resultant is equal to the larger of the two forces. Find the angle between the two forces.

5.The resultant of two forces $P$ and $Q$ is $\sqrt{3}Q$. It makes an angle of 30 with the direction of the force $P$. Show that $P$ is either equal to $Q$ or equal to twice $Q$.

6.Two forces $P$ and $Q$ are acting at an angle such that their resultant $R = P$. If $P$ is doubled show that the new resultant is at right angles to $Q$.

7.When two forces $P$ and $Q$ are acting inclined at an angle $\theta$ to each other, their resultant is $5\sqrt{P^2+Q^2}$. When the two forces are inclined at $\left(90^\circ - \theta\right)$, the resultant is $3\sqrt{P^2+Q^2}$. Show that $\tan\theta = \frac{1}{3}$.

8.When two forces are inclined at an angle $2\alpha$, their resultant is equal to twice the resultant obtained when they are inclined of $2\beta$. Show that $\cos\alpha = 2\cos\beta$.

9.The magnitude of the resultant of two forces $P$ and $Q$ is $P$. The resultant of two forces $2P$ and $Q$ acting in the same directions is also $P$. Find the magnitude of $Q$. Find also, the angle which the direction of $Q$ makes with the direction of $P$.

10.The resultant of two forces acting inclined at an angle $\theta$ to each other is $R$. If $P$ and $Q$ are interchanged, show that $R$ turns by an angle $2\tan^{-1}\left(\frac{P-Q}{P+Q}\right)\tan\frac{\theta}{2}$.

11.Two forces $P$ and $Q$ inclined at an angle of $120^0$ have a resultant $R$. When they are inclined at an angle of $60^0$, the resultant becomes $n$ times as great as before. Prove that

$$P = \frac{R}{2\sqrt{2}}\left(\sqrt{3n^2-1}+\sqrt{3-n^2}\right) \text{ and } Q = \frac{R}{2\sqrt{2}}\left(\sqrt{3n^2-1}-\sqrt{3-n^2}\right).$$

12.The greatest resultant which two forces can have is $P$ and the least is $Q$. Show that if they act at an angle $\theta$ the resultant is $\left(P^2\cos^2\frac{\theta}{2}+Q^2\sin^2\frac{\theta}{2}\right)^{\frac{1}{2}}$.

13.If the greatest possible resultant of two forces $P$ and $Q$ is $n$ times the least, show that the angle $\alpha$ between $P$ and $Q$ when their resultant is twice the square root of their product is given by $\tan^2\frac{\alpha}{2} = \frac{1}{n^2-2}$.

14. The resultant of the forces $P$ and $Q$ is R. If $Q$ is doubled, $R$ is doubled and if $Q$ is reversed $R$ is again doubled. Show that $P:Q:R = \sqrt{2}:\sqrt{3}:\sqrt{2}$.

15.The resultant of two forces is the same when their directions are inclined at an angle $\phi$ and when they are inclined at an angle $\left(\frac{\pi}{4}-\phi\right)$. Prove that $\tan\phi=\sqrt{2}-1$.

16.Two unlike parallel forces *P* and *Q (P>Q)* act at *A* and *B* respectively. If *P* and *Q* are both increased by *x*, show that the resultant will move through a distance $\frac{x.AB}{P-Q}$.

17.*P* and *Q* are two like parallel forces, if *P* be moved parallel to it self through a distance *x*, show that their resultant moves through a distance $\frac{px}{P+Q}$.

18.Two like parallel forces *P* and *Q(P>Q)* act on a rigid body at *A* and *B* respectively. If *P* and *Q* be interchanged in position, show that the point of application of the resultant will be displaced through a distance *x* where $x=\frac{P-Q}{P+Q}AB.$

19.Two unlike parallel forces *P* and *Q(P>Q), x(m)* apart act at two points of a rigid body. Show that, if the direction of *P* be reversed, the resultant displaced through the distance $\frac{2PQ}{P^2-Q^2}x(m)$.

20.Three like parallel forces *2P+Q, 4P-2Q* and *8(N)* act at the vertices of a triangle. Find *P* and *Q* if the resultant of the three forces passes through the centroid of the triangle.

21.Three like parallel forces *P, Q, R* act respectively at the three vertices *A, B, C* of a $\Delta$ *ABC*. Prove that if the resultant of *P, Q, R* passes through the circumcentre of $\Delta$ *ABC* then $\frac{P}{\sin 2A}=\frac{Q}{\sin 2B}=\frac{R}{\sin 2C}$.

22.Three like parallelforces *P, Q, R* act respectively at the vertices A,B,C of a $\Delta$ *ABC*. Prove that if the resultant of *P, Q, R* passes through the orthocentre of $\Delta$ ABC, show that $\frac{P}{\tan A}=\frac{Q}{\tan B}=\frac{R}{\tan C}$.

Hence deduce that $P(b^2+c^2-a^2)=Q(c^2+a^2-b^2)=R(a^2+b^2-c^2)$ here *a, b* and *c* have the usual notations for a triangle.

23.A coplanar system of forces $5N, 4N, 4\sqrt{2}N, 5\sqrt{3}N, 4N, 7N, 10N$ and $6\sqrt{2}N$ act on a particle. The first force is horizontal. Other forces are making angles of $30°, 45°, 90°, 150°, 180°, 240°$, and $315°$ respectively with the direction of the first force. Find the algebraic sum of the horizontal and vertical resolved parts of each force.

24.In the equilateral triangle *ABC*, the mid points of the sides *BC* and *AC* are *D* and *E* respectively. A coplanar system of forces $\sqrt{3}N, 2\sqrt{3}$ and 3N act along the sides *DC, DE* and *DA* respectively. Find the algebraic sum of the components of each force in the direction of *BC* and in the direction perpendicular to it.

25.The magnitudes of a system of coplanar forces acting at a point are $2, 2, 3, 2\sqrt{3}, 5$ and 7. The first force is horizontal and the other forces are making angles 45, 60, 90, 135 and $240°$ respectively with the horizontal. Find the algebraic sum of the horizontal and the vertical components of each force in the system.

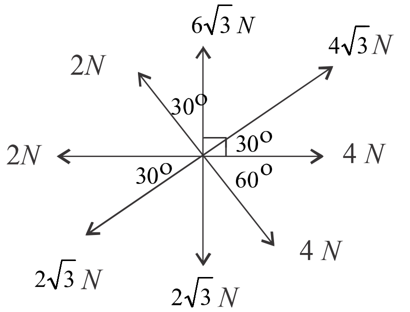
26. Forces $4P, 6P, 5\sqrt{2}P$ and $\sqrt{2}P$ act on a point *O* at the centre of a square *ABCD*. The first two forces act along the diameters *OA* and *OB* respectively and the other two forces act along directions perpendicular to *AB* and *BC*

respectively. Find the algebraic sum of the components of each force in the directions of *AB* and *AD*.

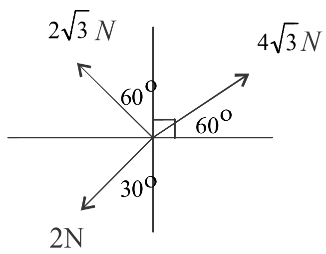
27.*ABCDEF* is a regular hexagon. A coplanar system of forces $,3\sqrt{3},2,4\sqrt{3},3$ and $2\sqrt{3}$ Newton act along the directions *AB, AC, AD, DE* and *AF* respectively. Find the algebraic sum of the components of each force in the system in the direction of AB and in a direction perpendicular to it.

28.Forces $2N,\sqrt{3}N,5N,5\sqrt{3}N$ and $2N$ act from one angular point of a regular hexagon towards the other five angular points. Find the algebraic sum of the components of each force in the direction of the first force and in a direction perpendicular to it.

29. Find the resultant force in magnitude and direction of the following system of forces.




***a***

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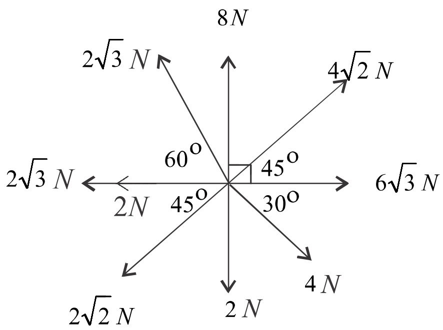


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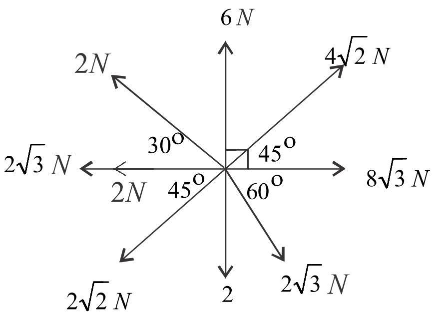


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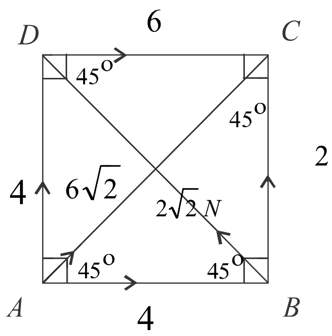


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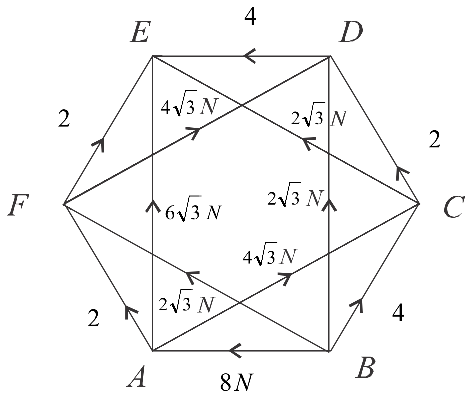



***f***

30.In the rectangle *ABCD*, $AB = 8m$ and $BC = 3m$ . The mid point of *AB* is *O*. Forces 2, 3, 10, 9, 5 and 10 Newton acting on a body act along *AD, BC, OC, OA, OD* and *BO* respectively. Find the algebraic sum of the moments of the system of forces.

*i*.about *A*      *ii*.about *O*

31.On a ruler $AB = 200\ cm$ long, forces 3*N*, 5N, 2*N* and 6*N* act perpendicular to the rod at distances 40 *cm*, 60 *cm*, 150 *cm* and 180 *cm* from A respectively.

*i.* Find the algebraic sum of the moments of the system of forces about an axis through *A*.

*ii.* When an additional force *P* is applied at *B* perpendicular to the rod, the moment of the system about the axis becomes zero. Find the value of *P*.

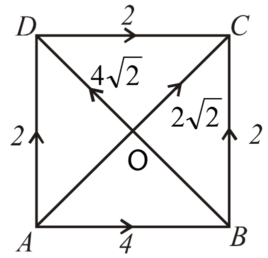
32.The side of an equilateral triangle *ABC* is 2*m*, *P, Q* and *R* are the mid points of *BC, CA* and *AB* respectively. Force $5,4\sqrt{3},2\sqrt{3},2\sqrt{3}$ and 8 Newton acting on a body act along the sides *BC, RC, AP, BQ* and *AQ* respectively. Find the algebraic sum of the moments of the system of forces

*i*.about *A*      *ii*.about *B*

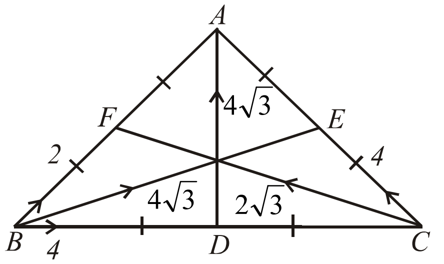
33.*ABC* is an equilateral triangle in which a side is *5(cm)* long. The mid points of *AB* and *AC* are *D* and *E* respectively. Forces of magnitudes $2\sqrt{3}$, 5, 9, 3 and $\sqrt{3}(N)$ act on a particle along the sides *BC, BE, BD, CD* and *EA* respectively. Find the algebraic sum of the moments about the points *A* and *B*.

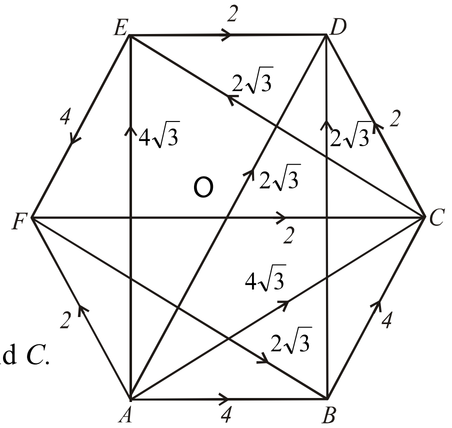
34.*ABCDEF* is a regular hexagon in which a side is *2(cm)*. Forces of magnitudes *1, 2, 3, 4, 6* and *5(N)* act on a particle along the sides *AB, BC, DC, DE, FE* and *FA* respectively. Find the algebraic sum of the moments about the point *A*.

35. A system of forces acts on a rigid body along the sides of square *ABCD* of side $2a$ as follows. Find the moments of the system of forces about *O* and *A*.




36.A system of forces acts on a rigid body as given in the equilateral triangle *ABC* of side $2a$. Find the moments of the system of forces about *B* and *E*




37.A system of forces act on a rigid body along the sides of regular hexagon *ABCDEF* of side $2a$ as follows.
Find the algebraic sum of moments of the system of forces about *O,A* and *C*.

38.*ABCD* is a rectangle with *AB=6, BC=4. O* is the mid-point of *AB*. Forces of magnitudes *3, 2, 5, 8, 10* and *9* act on a body along the sides *AB, BC, OC, OA, OD* and *BO* respectively.
Find the algebraic sum of the moments of this system of forces about the points *A* and *B*.

39.In the rectangle *ABCD*, $AB = 8\,m$ and $BC = 2\sqrt{3}\ m$. Forces $3\sqrt{3}, 9, 5\sqrt{3}$ and $x$ Newton acting on a body act along the sides *BA, BC, DC* and *DA* respectively. The resultant of this system is a force of 4 Newton making an angle of $30°$ with *AB*. Find the distance from *A* to the point where the resultant cuts *AB* and the value of $x$.

40.In the rectangle *ABCD*, $AB = 4m$ and $BC = 3m$. Forces 12, $x$, 18, 4 and 6 Newtons acting on a body act along the sides *AB, AC, BC, DC* and *DA* respectively. The resultant of this system is a force of 25*N* parallel to *AC*. Find the distance from *A* to the point where the resultant cuts *A* and the value of $x$.

41.In the trapezium *ABCD*, $A\hat{B}C = 90$, $AB = 16m$, $DC = 11m$ *and* $BC = 12m$. Forces x, 10, 13, 3 and 7 acting on a body lie along the sides *AB,CA, AD, BC* and *DC* respectively. The resultant of this system is a force of 15*N* parallel to *CA*. Find the distance from *A* to the point where the resultant cuts *AB* and the value of $x$.

42.In the rectangle *ABCD*, $AB = 12m$ and $BC = 5m$. The coplanar system of forces of 8*N*, 20*N*, 2*N*, 6*N*, 13*N* and 13*N* act along the sides *AB, BC, DC, DA, BD* and *AC* respectively. Find the magnitude, direction and the line of action of the resultant.

43.The side of an equilateral triangle *ABC* is $2a$. *D, E* and *F* are the mid points of the sides *BC, CA* and *AB* respectively. Forces 8*N*, 4*N*, $8\sqrt{3}N, 4\sqrt{3}N$ and 4*N* acting on a body lie along *BC, CD, AD, BE* and *EF* respectively. Find the magnitude, direction and the line of action of the resultant.

44.*ABC* is an equilateal triangle of side $2m$. *P, Q* and *R* are the mid points of the sides *AB, BC* and *CA* respectively. Forces $4\sqrt{3}, 3\sqrt{3}, 2\sqrt{3}, 3, 2$ and 6 Newton acting on a body act along *AB, BC, AC, AQ, BR* and *CP*. Find the magnitude, direction and the line of action of the resultant.

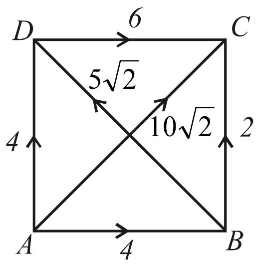
45.The side of a regular hexagon *ABCDEF* is $2m$. Forces of $1N$, $2N$, $3N$, $10N$, $5N$, and $6N$ act along *AB, CB, DC, DE, EF* and *FA* respectively. Find the magnitude, direction and the line of action of the resultant.

46. A system of forces acts along the sides of a square of side *2(m)* as follows.
Find the magnitude and direction of the resultant. Also find the point where the resultant cuts the side *AB.*

47.*ABCD* is a rectangle with *AB=4, BC=3 A* system of forces acts as shown in the figure. Find the magnitude and direction of the resultant. Also find the point where the resultant cuts the side *AB.*

48.*ABCDEF* is a regular hexagon of side *2(m).* A system of forces acts along the sides of hexgon as shown in the figure.Find magnitude and direction of the resultant. Also find the distance from *A* to the point where the resultant cuts the side *AB.*

49.A system of forces acts along the sides of an equilateral triangle *ABC* of side 2*(m)* Find the magnitude and direction of the resultant. Also find the line of action of the resultant.

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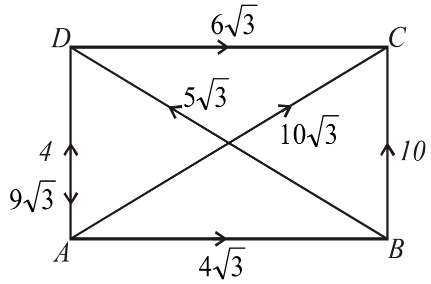


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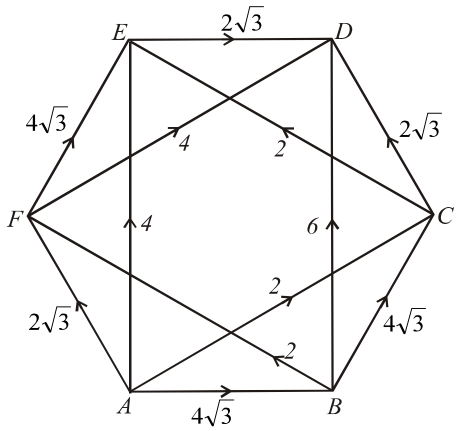


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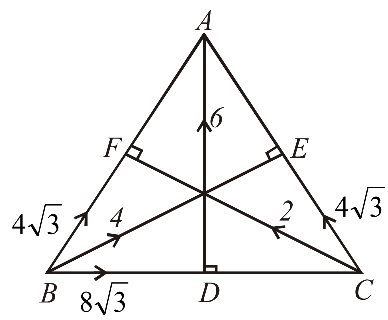



**49.**

50. *ABCD* is a rectangle in which *AB=6, BC=4*. The mid-point of *AB* is *E*. Forces of magnitudes *3, 6, 12* and *10* act on a body along the sides *AB, BC, CD* and *DA* respectively. The forces $x$, $y$ and $z$ act along the sides *DC, CE* and *ED* of the triangle *CDE*. If these forces together bring the equilibrum. Find the value of $x$, $y$ and $z$.

51 *ABC* is an equilateral triangle. The mid-point of the sides *BC, CA* and *AB* are *D,E* and *F* respectively. Forces of magnitudes $4\sqrt{3}$, $2\sqrt{3}$, $\sqrt{3}$, $x$, $y$ and $z$ act on a body along the sides *DA,EB,CF,BC,AC* and *AB* respectively. If these forces together bring the equilibrium. Find the values of $x$,$y$ and $z$.Also find the resultant of the forces $x$,$y$ and $z$.

## EQUILIBRIUM OF A RIGID BODY

***Using triangle of forces solve the following***

52.A uniform sphere of radius $a$ and weight $w$ is suspended from a point $O$ on a smooth wall by means a light inextensible string of length $2a$. Find the tension in the string and reaction by the wall.

53.A uniform rod $AB$ of length $2a$ and weight $w$ is hinged to a vertical wall at $A$.The end $B$ is joined to a point $C$ on the wall a height $2a$ above $A$ by a light inextensible string so that the rod makes an angle $\theta$ with downward vertical. Find the reaction of the hinge and tension in the string.

54.A uniform rod $AB$ of length $2a$ and weight $w$ is resting, one end on a rough horizontal ground and other end against a smooth vertical wall. If the rod makes an angle $60^0$ to the horizontal, find the reactions of the points of contacts.

***Using Lami's theorem solve the following.***

55.A uniform sphere of radius $a$, weight $w$ is in equillbrium on a smooth inclined plane inclination $\alpha$ to the horzontal by means a light inextensible string of length $b$ attached to a point on the sphere and to a point on the plane. Find the tension in the string and normal reaction by the plane.

56.A uniform lamina $ABCD$ in the form of a square of side $2a$ has weight $w$ . One end of a light inextensible string is attached to the mid point of $AD$ and its other end is fastened to a point $O$ on a smooth wall. The lamina is in equilibrium in a vertical plane perpendicular to the wall with $AC$ horizontal and $A$ in contact with the wall. Find the reaction at $A$ and tension in the string.

57.A ring of weight $w$ which can slide freely on a fixed smooth vertical circular wire is supported by a light inextensible string attached to the highest point of the circular wire when the ring is in equilibrium if the string subtends an angle $\theta$ at the centre of the circle, find the tension in the string and reaction of the wire on the ring.

58.Two beads of weights $w_1$ and $w_2$ slide on a smooth circular wire in a vertical plane, being connected by a light thread which subtends an angle $2\theta$ at the centre of the circle. When the beads are in equilibrium on the upper half of the wire. Prove that the inclination $\phi$ of the string to the horizontal is given by $\tan\phi = \dfrac{w_1 \sim w_2}{w_1 + w_2}\tan\theta$ .

***Using "cot" theorem solve the follwing.***

59.A heavy bar $AB$ whose centre of gravity is at a point $G$ such that $AG:GB = a:b$ is supported by a string attached to the end $A$. The other end $B$ is pulled away from the vertical by a horizontal force until the inclination of the string to the horizontal is $\varphi$. Prove that the inclination $\theta$ of the bar to the horizantal is given by $(a+b)\tan\theta = a\tan\varphi$ .

60.A uniform rod, of length $a$, hangs against a smooth vertical wall being supported by means of a string, of length $l$, tied to one end of the rod, the other end of the string being attached to a point in the wall. Show that the rod can rest inclined to the wall at an angle $\theta$ given by $Cos^2\theta = \dfrac{l^2 - a^2}{3a^2}$ . What are the limits of the ratio of $a:l$ in order that equilibrium may be possible?

61.A rod $AB$ of length l has its centre of gravity at $G$. Such that $AG = \frac{l}{4}$. This rod is in equlibrium wth its ends in contact with two smooth planes each inclined at an angle $\alpha$ to the horizontal. The inclination of the rod to the horizontal is $\beta$ and lies in a vertical plane which is perpendicular to the line of intersection of the two planes. Show that $2\tan\alpha\tan\beta = 1$.

62.A rod which centre of gravity divides it in the ratio $a{:}b$ rests in side a smooth sphere with the inclination $\theta$ to the horizontal. The angle subtended by the rod at the centre of the sphere is $2\alpha$. Show that $\tan\theta = \frac{b-a}{b+a}\tan\alpha$.

63.A solid cone with semi-vertical angle $\alpha$ and height $h$, resting against a smooth vertical wall with its base, by means a string attached to the vertex of the cone and to a point in the wall. Show that the maximum possible length of the string is $h\sqrt{1+\frac{16}{9}\tan^2\alpha}$

***Solve the following problems by resolving forces and taking moments.***

64.A uniform ladder $5m$ long and weight $w$ rests, one end against a smooth vertical wall and the other end is in contact with the ground by means a light inextensible string connected to the ladder and bottom of the wall. The string is normal to the ladder and the end of the ladder on the ground is $3m$ away from the foot of the wall. Find the tension in the string and reactions on the ladder when a man of weight $2w$ has climbed the upper end of the ladder.

65.A uniform rod of weight $w$ rests in contact with a smooth horizontal plane and a smooth inclined plane by means a light inextensible string attached to the lower end of the rod and to the point of intersection of the plane and inclined plane. If the inclination of the inclined plane to the horizontal is twice that of the rod. Find the tension in the string and reaction on the rod.

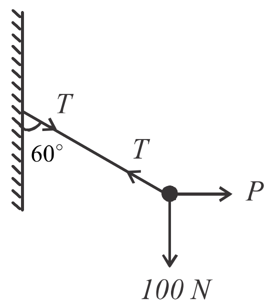
66.*i.*A right circular cylinder of radius $a$ is kept on a smooth horizontal plane so that its axis is horizontal. It is prevented from rolling by a string of length $3a$ connected to the centre of the cylinder and a point $A$ on the plane. A uniform rod $AB$ of length $4a$ and weight $w$ is hinged at $A$ and is kept in contact with the cylinder so that it is perpendicular to the axis. Find the tension in the string.

*ii*. Two equal spheres of radius a and weight $w$ rest in side a smooth hemispherical vessel of radius $4a$. Find the reactions between the spheres and the vessel and spheres.

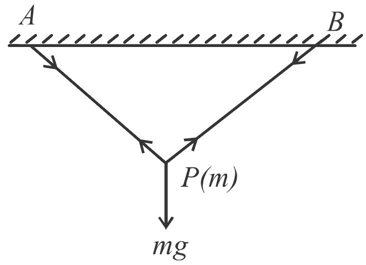
67.A uniform heavy rod of length $2a$ rests, inside a smooth hemispherical bowl of radius $a$ with one end is in contact with the bowl and the other end is projecting beyond the rim of the bowl. The rim of the bowl is horizontal and the rod is inclined at an angle $\theta$ to the horizontal. Show that $2\cos 2\theta = \cos\theta$

***Solve the following problems***

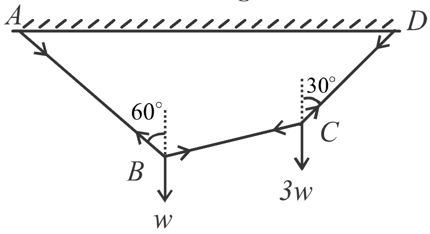
68.A particle of mass 10($kg$) is suspended by a light inextensible string whose other end is joined to a point on a wall and by a horizontal force $P$ acting on the particle. When the particle is in equilibrium, find the tension in the string and the force $P$.



69. A particle $P$ of mass $m$ is suspended by means of two light inextensible strings $PA$ and $PB$. The string $PA$ makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with horizontal and $A\hat{P}B = 90^\circ$ see the figure. Find the tensions in the strings.




70. A string $ABCD$ joined to two points $A$ and $D$ at the same horizontal level, bears two weights $w$ and $3w$ at $B$ and $C$. $AB$ and $CD$ incline to vertical by angles $60^\circ$ and $30^\circ$ respectively. Show that $BC$ part is horizontal. Also find the tensions of the strings.




71. A particle of weight $w$ is to be placed in equilibrium on a smooth inclined plane of inclination $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. Find the force required to keep the particle in equilibrium

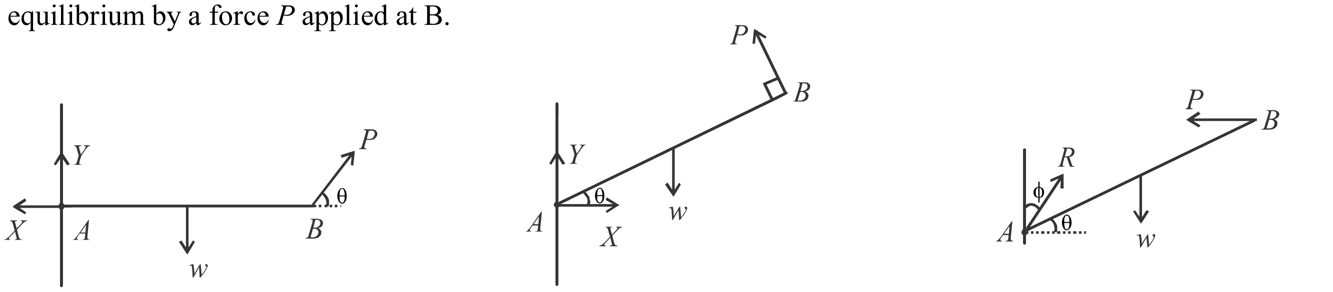
*i.* If the force is parallel to the plane.
*ii.* If the force is horizontal.

Also find the normal reaction between the particle and the plane.

72. The centre of gravity of a rod $AB$ of length $(a+b)$ and weight $w$ is at a distance $a$ from $A$. The rod is kept in equilibrium on two pegs on the same horizontal level at a distance $c$ apart. The lengths of the parts of the rod projecting beyond the pegs are equal. Show that the reactions on the rod by the pegs are

$\left[\frac{b-a+c}{2c}\right]w$ and $\left[\frac{a-b+c}{2c}\right]w$

73. The following diagrams show a uniform rod $AB$ of weight w is smoothly hinged to a wall at $A$ and kept in equilibrium by a force $P$ applied at B.



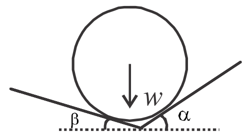

*i.* Show that $X\tan\theta = \frac{w}{2}$ *ii.* Show that $Y = \frac{w}{2}(1+\sin^2\theta)$ *iii.* Show that $\tan\theta.\tan\phi = \frac{1}{2}$

74. A uniform rod is hinged to a point smoothly. By applying a horizontal force whose magnitude is half of the weight of the rod at the other end, the rod is kept in equilibrium. Find the inclination of the rod to the vertical.

75. A uniform rod $AB$ of weight $w$ is smoothly hinged at $A$ to a fixed point. By applying a horizontal force $P$ ar the lower end $B$, the rod is kept in equilibrium with inclination $\theta$ to the horizontal. Find the force $P$ and the reaction at $A$.

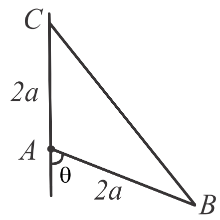
76. A rod whose centre of gravity divides it in the ratio $a : b$ rests its ends in contact with two smooth inclined planes of inclination $\alpha$ and $\beta$ to the horizontal. Find the inclination of the rod to the horizontal and the reactions on the planes.

77.A uniform sphere of weight $w$ is in equilibrium between two smooth inclined planes as shown in the diagram. By marking the reactions on the sphere by the planes find them. Now a second equal sphere is placed on the plane of inclination $\alpha$, so that it is in equilibrium in contact with the first sphere. Find the new reactions.




78.A uniform rod $AB$ of weight $W$ and length $2a$, is hinged to a vertical wall at $A$ smoothly. A weight $w$ is suspended at $B$ and the rod is kept in equilibrium horizontally by a light inextensible string attached to $B$ and a point in the wall at a height $a$ above $A$. Show that tension in the string is $\frac{\sqrt{5}}{2}(W+2w)$. Find the total reaction at $A$.

79. The diagram shows a uniform rod $AB$ weight $w$ smoothly hinged to a wall at $A$ and kept in equilibrium by a light inextensible string attached to $B$ and to a point $C$ in the wall such that $AC = 2a$. Mark all the forces acting on the rod. Show that the reaction at the hinge meet the string at the mid - point $D$ of $BC$. and it is normal to the string $BC$. Also find this reaction and tension in the string.




80.A uniform rod $AB$ of weight w and length $2a$ is in equilibrium making an angle $\theta$ to the horizontal with end $A$ against a smooth wall and contact with a smooth peg $C$. Find

*i.*Reaction of the wall.

*ii.*Reaction of the peg.

*iii.*Positon of the peg .

81.A non - uniform rod $AB$ of length $\sqrt{3}a\,(m)$ is in equilibrium in side of a fixed smooth sphere of radius $a(m)$. If the rod is inclined $30^\circ$ to the horizontal find the reactions at the ends of the rod, and the position of centre of gravity of the rod.

82.A uniform rod $AB$ is in equilibrium making an angle $\alpha$ to the horizontal, with upper end $A$ in contact with a smooth peg and lower end $B$ is jointed to a point $C$ on the same horizontal level of $A$, by a light inextensible string. If the inclination of the string to the horizontal is $\beta$ show that $\tan\beta = 2\tan\alpha + \cot\alpha$

Also show that $AC = \dfrac{AB\sec\alpha}{1+2\tan^2\alpha}$.

83.If a body is in equilibrium under the action of three forces, then show that the lines of action of the forces should meet in a point. A uniform rod $AB$ of length $2a$ and weight $w$, is in equilibrium suspending from $A$ by a string and applying a horizontal force at a point $C$ on the rod a distance $b$ from $B$ with end $B$ is below $A$. If the inclinations of the rod and the string to the vertical are $\theta$ and $\alpha$ respectively. Show that $(2a-b)\tan\alpha = a\tan\theta$.

When $\theta = \frac{\pi}{4}$ and $b = 0$, show that the magnitude of horizontal force is $\frac{w}{2}$ and tension in the string is $\frac{\sqrt{5}}{2}w$.

84.A non - uniform rod $AB$ rests with its end $A$ on a smooth inclined plane of inclination $\alpha$ to the horizontal and end $B$ against a smooth vertical wall. making an angle $\beta$ to the horizontal. If $G$ is the centre of gravity of the rod,

show that $\dfrac{AG}{GB} = \sin\alpha\sin\beta\sec(\alpha+\beta)$.

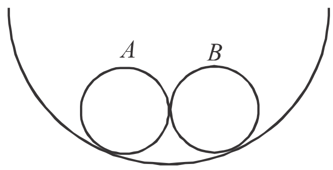
85.A uniform rod of length 2$a$ rests, partly in a fixed smooth bowl of radius $r$ and rim horizontal, with one end in contact with the inner surface of the bowl and a point on the rod in contact with the rim of the bowl. A part of the rod projecting beyond the rim of the bowl. If the inclination of the rod to the horizontal is $\theta$, show that $2r\cos 2\theta = a\cos\theta$.

86.A uniform rod $AB$ of length 3$r$ is placed in a fixed smooth bowl of radius $r$, and brim horizontal, such that end $A$ is in contact with the inner surface of the bowl. When the rod is in equilibrium. find the inclination of the rod to the horizontal.

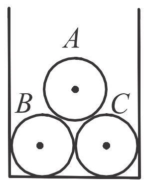
87.A smooth hemispherical bowl of radius $a$, is fixed so that its brim is horizontal. A uniform rod $ACB$ is in equilibrium such that end $A$ in contact with the inner surface of the bowl, end $B$ is projecting beyoned the brim and point $C$ is in contact with the bowl. If the rod makes an angle of $30^\circ$ with the horizontal, show that the length of the rod is $\frac{4\sqrt{3}a}{3}$. Also find the reactions on the rod.

88.A smooth hemispherical bowl of radius $a$ is fixed with its brim is horizontal. A uniform rod of weight $w$ is in equilibrium so that. one end in contact with the bowl and the other end away the brim of the bowl. If the length of the portion of the rod in the bowl is $\sqrt{3}a$. Show that the inclination of the rod to the horizontal is $\frac{\pi}{6}$. Also find the reactions on the rod.

89.Two equal smooth spheres $A$ and $B$ each of weight $w$ and radius $a$ are placed in a smooth hemisphere of radius 3$a$ as shown in the figure. Find the reactions between $A$ and $B$. Also find the reactions between hemisphere and the spheres.




90.There equal smooth spheres each of weight $w$ and radius $a$ are placed in a smooth cylinder of radius $b$ as shown in the diagram. Find




*i*. reaction between $A$ and $B$

*ii*. reaction between $B$ and the base of the cylinder

*iii*.write an equation for the reaction between $B$ and the wall of the cylinder.

91.Two smooth spheres each of weight $w$ and radius $b$ are placed inside of a cylinder of radius $a(<2b)$ and axis vertical. Show that the reactions between spheres and the wall of the cylinder are $\frac{(a-b)w}{\sqrt{2ab-a^2}}$.

92.A smooth hemispherical bowl of radius $a$ and weight $w$ is kept on a smooth horizontal table. A rod of weight $w$ and length 2$l$ rests partly in the bowl with one end of the rod in contact with the bowl and a point on the rod in contact with the brim of the bowl. When the hemisphere and the rod are in equilibrium the inclination of the brim of the bowl to the horizontal is $\frac{\pi}{6}$. If the inclination of the rod to the horizontal is $\theta\left(<\frac{\pi}{2}\right)$, show by geometrically or otherwise that

*i*. $\theta = \frac{1}{2}\left[\cos^{-1}\left(\frac{1}{4}\right) - \frac{\pi}{6}\right]$

*ii*. $l = \frac{a}{2\cos\theta}$.

93.The ends of a uniform rod of length *a* are *A* and *B*. The rod is kept in equilibrium in a vertical plane with its end *A* in contact with a smooth vertical wall and by means a string of length *l* attached to the end *B* and to a point in the wall vertically above *A*. When the system is in equilibrium obtain an expression for the angle at which the rod is inclined to the vertical. Show that in order to keep the equilibrium in this manner, $2a \geq l \geq a$.

94.A smooth bowl of radius *r* is fixed, so that its rim is horizontal. A rod *AB* whose centre of gravity divides it in the ratio *a:b,* rests inside the bowl so that end *B* is above end *A*. If the inclination of the rod to the horizontal is $\theta$. Show that $\cos\theta = \dfrac{b-a}{2\sqrt{r^2-ab}}$.

95.Two uniform smooth spheres of equal weights are placed in side a smooth hollow right circular cone, whose semi-vertical angle $\alpha$ and is fixed so that its vertex is down wards and its axis is inclined at an angle $\alpha$ to the verti cal. Each sphere touches the cone at only one point. If the common perpendicular to the spheres makes an angle $\beta$ with the vertical. Show that $2\cot\beta = \tan 2\alpha$

96.A hollow cylinder with radius R is fixed so that its axis is horizontal. Two cylinders of weight 2*w* and radius *r* are placed inside this cylinder symmetrically. Another cylinder of radius *r* and weight *w* is kept symmetrically on the two cylinders. Show that the equilibrium is possible only if $R \leq r\left(1+2\sqrt{19}\right)$

97. A smooth hemispherical bowl of radius *a* is fixed with its rim uppermost and horizontal. A uniform rod *AB* of weight 2*w* and length *l* rests with its one end inside the bowl and the rod is leaning on the rim. The end *B* is out side of the bowl and a mass *w* is attached there. Show that $l = 3a(2\cos\theta - \sec\theta)$. Find the maximum possible inclination of the rod to the horizontal. Where $\theta$ is the inclination of the rod to the horizontal.

98.A solid hemisphere of weight *w* and radius *r* rests in equilibrium on a smooth horizontal table with its curved surface in contact with the table. A light string of length *l* is connected to a point on the rim of the solid and to a point on the table, so that the plane face of the solid is inclined to the horizontal, if *r>l,* show that the tension in the string is $\dfrac{3w}{8}\dfrac{r-l}{\sqrt{2rl-l^2}}$.

99.Three cylinders *A, B* and *C*, each of weight *w* and radius *r* are placed inside a cylinder of radius *R* so that the cylinder *C* is symmetrically on *A* and *B*. The axes of all cylinders being horizontal. Show that for the equilibrium of the system $r \geq \dfrac{R}{2\sqrt{7}+1}$.

100.A rod has length 2*l* and weight *w.* A weight *w* is fastened to apoint on the rod which a distance *d* away from its mid-point. The rod with the weight *w* is in equilibrium in a smooth sphere of radius *R (R>l)* If the rod makes an angle $\theta$ to the horizontal show that $\tan\theta = \dfrac{wd}{(W+w)\sqrt{R^2-l^2}}$.

101. A hollow circular smooth cylinder of radius *r* is fixed with its axis is horizontal. Three equal uniform cylinders each of radius *a* are placed in side this cylinder. So that one of them is on the other two. If all contacts are smooth and the cylinders are in equilibrium.Show that $1+\dfrac{2\sqrt{3}}{3} < \dfrac{r}{a} < 1+2\sqrt{7}$

102.A uniform solid hemisphere of weight $W$ rests with its curved surface on a smooth horizontal table. A weight $w$ is suspended from a point on the rim of the hemisphere.If the plane base of the rim is inclined to the horizontal at an angle $\theta$ , prove that $\tan\theta=\frac{8w}{3W}$

103.A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If $\theta,\varphi$ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\varphi=\frac{3}{8}+\tan\theta$ .

104. A unifrom solid sphere ,of weight $w$ and radius $a$ , is hung by a string of length $a$ from $a$ fixed point o. A uniform rod , also of weight $w$ and length $4a$ has one end freely attached to the same point o . If the rod rests touching the sphere, show that the inclinations of the string and the rod to the vertical are each equals to $\frac{\pi}{12}$

show also that the tension in the string is. $\frac{w\cos\left(\frac{\pi}{12}\right)}{Sin\frac{\pi}{3}}$ and find the reaction between the sphere and the rod.

105. A uniform smooth rod $AB$ of length $2a$ and weight $w$ can turn freely about its fixed end $A$. A small smooth ring $C$ of weight $2w$ can slide along the rod. The ring is Joined to a fixed point $D$ , in the same horizonatal level as the point $A$, by an inextensible string of length $\frac{a}{4}$ .The string and the rod are in same vertical plane and $AD$ = $\frac{a}{4}$ In the position of equilibrium , find the reaction between the rod and the ring and show that the rod makes an angle $\frac{\pi}{3}$ with the horizontal .Also , find the tension in the string and the reaction at the end $A$.

106. If three coplanar forces acting upon a rigid body keep it in equilibrium, show that they must either meet in a point or be parallel.

A unifrom, smooth hemispherical bowl of weight $w$ and radius $r$ rests on a smooth horizontal table, and partly inside it rests a unifrom rod of length $2l$ and of weight $w$. The inclination of the base of the hemisphere to the horizontal is $\frac{\pi}{6}$.*If* $\theta\ (<\frac{\pi}{2})$ is the inclination of the rod to the horizontal and $R$ is the reaction at the rim of the bowl, prove geometrically or otherwise that

(i) $\theta=\frac{1}{2}\left[\cos^{-1}\left(\frac{1}{4}\right)-\frac{\pi}{6}\right]$ (ii) $l=\frac{1}{2}r.\sec\theta$ (iii) $R=\frac{w}{(8+\sqrt{3}-\sqrt{15})^{\frac{1}{2}}}$

107.A uniform rod $AB$ of length $2a$ and weight $w$ is hung from a smooth peg $O$ by means of a string $AOQ$ of length $2l$, One end of which is fastened to the rod at $A$, while the other end is attached to a small light smooth ring $Q$ which slides on the rod. The rod is inclined at an angle $\theta$ to the horizontal. By considering the equilibrium of the ring $Q$ and of the rod $AB$ deduce the following results.

(i)The straight portions of the string are each inclined to the vertical at an angle $\theta$:

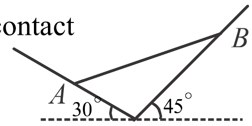
(ii)The angle $\theta$ is given by $a\cos^3\theta=l\sin\theta$

(iii)The tension in the string is $\frac{1}{2}w\sec\theta$ .

108.A smooth solid hemisphere of radius $3a$ is fixed so that its plane face is horizontal. A smooth sphere of radius $2a$ and weight $w$ is in equilibrium on the curved surface of the hemisphere by means of a light inextensible string of length $2a$, joined its one end to the highest point on the hemisphere and other end to the sphere. Find the tension in the string and reaction at the point of contact.

109. A uniform rod $AB$ of length $4a$ and weight $w$ is smoothly hinged at $A$ so that it can freely rotate in the vertical plane through the point $A$. The rod is in equilibrium in contact with a smooth peg $C$ at a distance $3a$ from $A$, inclined to the horizontal at $30^\circ$. Find the reaction at the hinge and the peg.

110.A uniform rod $AB$ of length $2a$, weight $w$ is in equilibrium between two inclined planes, in contact with its ends $A$ and $B$ as shown in the diagram. Show that the inclination of the rod to the vertical is $\tan^{-1}\left(\sqrt{3}+1\right)$. Also find the reactions at $A$ and $B$.




110.A uniform rod $AB$ of length $2a$, weight $w$ smoothly hinged to a vertical wall at $A$, has a light inextensible string joined to $B$ which passes over a smooth pully $C$ carring a weight $w$ at its other end. The rod stays in equilibrium making an angle $30^\circ$ with downward vertical. Find the tension in the string and the angle between rod and the string. Also find the horizontal and vertical components of the reaction at the hinge.

111. A uniform rod $AB$ of length $4a$, weight $W$ smoothly hinged at $A$, is in equilibrium in a vertical plane through the point $A$, by a light inextensible string whose one end joined to a light, smooth ring $C$ sliding along the rod, and passing over a smooth pulley $D$, carring a weight $w$ at its other end. The pulley $D$ is vertically above $A$ such that $AD = 5a$.

*i*.Show that the string is perpendicular to the rod.

*ii*.If the rod makes an angle $\theta$ with the vertical show that $\tan\theta = \dfrac{5w}{2W}$.

*iii*.Show that the minimum of $\dfrac{w}{W}$ is $\dfrac{3}{10}$.

*iv*.Find the horizontal and vertical components of the reaction at $A$.

112.one end of a light inextensible string, passing over a smooth peg $A$, is joined to a point $B$ on the spherical surface of a uniform sphere of weight $2w$ and other end is attached to end $C$ of a uniform rod $CD$ of weight $3w$. The sphere and the rod are in equilibrium with the mid - point $E$ of the rod in contact with the sphere. In the position of equilibrium the parts $AC$ and $AB$ of the string make angles $\alpha$ and $\beta$, respectively with the vertical. Show that $\sin\alpha = \dfrac{2\sin\beta}{3\cos(\alpha+\beta)}$ and $\cos(\alpha+\beta) = \dfrac{2}{3}$. Hence obtain $\alpha = \beta$. Show also that $\cos\alpha = \dfrac{\sqrt{5}}{\sqrt{6}}$. Also find the tension in the string and the reaction at the point of contact $E$.

113. $A$ and $B$ are two fixed points on a horizongal line at a distance $c$ apart. Two fine light strings $AC$ and $BC$ of lengths $b$ and $a$ respectively support a mass at $C$. Show that the tensions of the strings are in the ratio $b(a^2+c^2-b^2):a(b^2+c^2-a^2)$.

114.A heavy body rests on a smooth plane inclined to the horizontal at an angle $\alpha$ being held by a string to which a force $P$ is applied. If the normal reaction of the plane is equal to the force $P$, show that the inclination of $P$ to the inclined plane is $90^\circ - 2\alpha$.

115.A bead is free to slide on a smooth vertical circle and is connected by a string equal in length to the radius of the circle , to the highest point of the circle ; find the tension of the string and the reaction of the circle.

116.A uniform rod $4a$ in length is placed with one end inside a smooth horizontal bowl whose rim is horizontal and whose radius is $a\sqrt{3}$. Show that a quarter of the rod will project over the edge of the bowl. Also prove that the shortest rod that will thus rest will be of length $2\sqrt{2}a$.

117.A uniform rod $ACB$ of length $4a$ rests with one end $A$ in contact with the inside smooth surface of a hemispherical bowl of radius $r$ and $C$ in contact with its horizontal rim. If $CB = a$, show that the inclination of the rod to the horizontal is $30^\circ$ and $r = a\sqrt{3}$.

118.A smooth hemispherical bowl of radius $r$ is held with its rim horizontal. A uniform rod rests with one end against the inner surface of the bowl. If the portion of the rod within the bowl is of length $c$, show that the total length of the rod is $\frac{4}{c}(c^2-2r^2)$.

119.A heavy uniform sphere rests touching two smooth inclined planes one of which is inclined at $60^\circ$ to the horizontal If the pressure on this plane is one - half of the weight of the sphere, prove that the inclination of the other plane to the horizontal is $30^\circ$.

120.A uniform rod rests on a fixed smooth sphere with its lower end pressing against a smooth vertical wall which touches the sphere. If $\theta$ is the angle which the rod makes with the vertical when in equilibrium , prove that $a = 2l\sin\frac{\theta}{2}\cos^3\frac{\theta}{2}$ where $l$ is the length of the rod and '$a$' the radius of the sphere.

121.A rod $AB$ of length $3a$ is placed inside a sphere in a vertical plane through its centre inclined at an angle $\alpha$ to the horizontal. The rod subtends an angle $\beta$ at the centre of the sphere. If the centre of gravity of the rod lies at a distance $a$ from $A$, show that $\tan\frac{\beta}{2} = 3\tan\alpha$.

122. A smooth uniform hemispherical bowl of radius $a$ is fixed rigidly with its axis horizontal. A uniform rod of weight $W$ and of length $2l$ is in equilibrium in a vertical plane through the centre of the bowl with one end touching a point inside the bowl and with the other end extending outside the edge. If the rod is inclined to the horizontal at an angle $\theta$, show that $l\cos\theta = 2a\cos 2\theta$. Show also that the reaction at the edge of the bowl is $\frac{Wl}{2a}$. Deduce that $3l^2 \geq 2a^2$ for equilibrium.

123.Two spheres each of weight $W$ and radius $a$ are joined by a string of length $2l$ at two points on the surface of each sphere. The two spheres are in equilibrium touching each other and with the string passing over a smooth peg. Show that the tension in the string is $\frac{W(a+l)}{\sqrt{l^2+2al}}$ and that the reaction between the sphere is $\frac{aW}{\sqrt{l^2+2al}}$.

124.The extreme diameter of a uniform semicircular plate is $AB$. This plate is suspended from $A$. Find the angle made by $AB$ with the vertical. (The centre of gravity of a semicircular plate of radius $r$ lies on its axis at a distance of $\frac{4r}{3\pi}$ from the centre.)

125.When a uniform hollow hemisphere is in equilibrium when suspended from a point on its edge, show that the angle made by the plane containing the edge with the vertical is $\tan^{-1}\frac{1}{2}$. (The centre of gravity of a hollow hemisphere lies on its axis at a distance of $\frac{r}{2}$ from the centre.)

126.A uniform thin rod $AB$ of length $2a$ and weight $W$ is hinged at $A$. The rod is held in equilibrium in a vertical plane by means of a light rod joined to a point $C$ on the rod $AB$ and to a point $D$ vertically below $A$. If $AC = \frac{a}{2}$ and $AD = \frac{2}{3}a$, find the thrust in the rod $DC$ and the reaction at $A$.

127.The end $A$ of a uniform rod $AB$ of weight $W$ and of length $a$ is hinged freely to a vertical wall. The end $B$ of the rod is connected to point $C$ on the wall by means of a light inextensible string of length $a$. $C$ is at a distance $b$ vertically above $A$. Show that the reaction of the hinge at $A$ is $\frac{W\sqrt{a^2+2b^2}}{2b}$. Find also the tension in the string.

128.A non-uniform rod $AB$ is in equilibrium in a vertical plane with its lower end $A$ on a smooth plane inclined at an angle $\alpha$ to the horizontal and with its upper end $B$ against a smooth vertical wall . If it is inclined at an angle $\beta$ to the horizontal , show that the centre of gravity of the rod divides it in the ratio $\sin\alpha\sin\beta : \cos(\alpha+\beta)$.

129.A uniform rod $4a$ in length is placed with one end inside a smooth horizon      tal bowl whose rim is horizontal and whose radius is $a\sqrt{3}$ . Show that a  quarter of the rod will project over the edge of the bowl.

130.A uniform rod *ACB* of length $4a$ rests with one end *A* in contact with the inside smooth surface of a hemispherical bowl of radius *r* and *C* in contact with its horizontal rim. If $CB = a$, show that the inclination of the rod to the horizontal is $30^{\circ}$ and $r = a\sqrt{3}$ .

131.A smooth hemispherical bowl of radius *r* is held with its rim horizontal . *A* uniform rod rests with one end against the inner surface of the bowl. If the portion of the rod within the bowl is of length *c*,show that the total length of the rod is $\frac{4}{c}(c^2 - 2r^2)$ .

132.A body of weight *W* is lying on an inclined plane.The body is kept in equilibrium by a horizontal force *P*. If the body can also be held in  equilibrium by a force *Q* acting along the plane, then prove that $\frac{1}{Q^2} = \frac{1}{P^2} + \frac{1}{W^2}$ .

133.A bead of weight *W* is free to slide on a smooth vertical circular wire and is connected by a string, equal in length to the radius of the circle, to the highest point of the circle, find the tension of the string and the reaction of the circle.

134.A ring of weight *W* which can slide freely on a smoorh vertical circle is suspended by a string attached to the heighest point. If the thread subtends an angle $\theta$ at the centre, find the tension in the thread and the reaction of the circleon the ring.

135.A uniform plane lamina in the form of rhombus, one of whose angles is $120^{\circ}$ is supported by two forces applied at the centre in the directions of the diagonals so that one side of the rhombus is horizontal, show that if *P* and *Q* are the forces and P is greater, then $P^2 = 3Q^2$ .