BINOMIAL THEOREM

1. Expand the following binomials.

1. $(x-3)^5$ 2. $(3x+2y)^4$ 3. $(2x-y)^5$ 4. $(1-3a^2)^6$

5. $(x^2+x)^5$ 6. $(1-xy)^7$ 7. $\left(2-\frac{3x^2}{2}\right)^4$ 8. $\left(3a-\frac{2}{3}\right)^6$

9. $\left(1+\frac{x}{2}\right)^7$ 10. $\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$ 11. $(x^2+2y)^5$ 12. $\left(2x-\frac{3}{y}\right)^5$

2.. (1). Find the value of $\left(x+\sqrt{2}\right)^4+\left(x-\sqrt{2}\right)^4$

(2). Find the value of $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$

(3). Find the value of $\left(2-\sqrt{1-x}\right)^6+(2+\sqrt{1-x})^6$

(4). Find the value of $\left(\sqrt{x^2-a^2}+x\right)^5-\left(\sqrt{x^2-a^2}-x\right)^5$

3. Using the binomial theorem, expand $\left[(x+y)^5+(x-y)^5\right]$ and hence find the value $\left[(\sqrt{2}+1)^5+(\sqrt{2}-1)^5\right]$.

4 By using the binomial expansion, expand

$i)$ $(1+x+x^2)^3$ $ii)$ $(1-x+x^2)^4$

5)Find the 10th term of $\left(2x^2+\frac{1}{x}\right)^{12}$

6)Find the 6th term in the expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$.

7) Find the 5th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$.

8)Find the middle term in the expansion of $\left(3x-\frac{x^3}{6}\right)^7$

9)Find the term independent of x in the expansion of

$i)$ $\left(x^2+\frac{1}{x}\right)^9$ $ii)$ $\left(2x-\frac{1}{x}\right)^{10}$

10) *i.* Find the 12th term of $(2x-1)^{13}$

ii. Find the 28th term of $(5x+8y)^{30}$

iii. Find the 4th term of $\left(\frac{a}{3}+9b\right)^{10}$

iv. Find the 5th term of $\left(2a-\frac{b}{3}\right)^8$

v. Find the 7th term of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$

11)Prove that there is no term x^6 involving in the expansion of $\left(2x^2-\frac{3}{x}\right)^{11}$

12)Find the coefficient of x^5 in the expansion of the product $(1+2x)^5(1-x)^7$.

13)Using binomial theorem, find the values of $i)$ $(10.1)^5$ $ii)$ $(0.99)^{15}$

14)Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$.

15)If the coefficients of a^{r-1}, a^{r}, a^{r+1} in the binomial expansion $(1+a)^n$ are in arithmetic progression, prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

16)If P be the sum of odd terms and Q that of even terms in the expansion of $(x+a)^n$,prove that (i) $(P^2 - Q^2) = (x^2 - a^2)^n$

17) If the coefficients of 2nd,3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in arithmetic progression, show that $2n^2 - 9n + 7 = 0$.

18)The first three terms in the binomial expansion of $(x+y)^n$ are 1 , 56 and 1372 respectively. Find the values of x and y.

19)The coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:7:42. Find n.

20)Using binomial theorem , prove that when $6^n - 5n$ is divided by 25 always leaves a remainder 1, where n is a positive integer.

21) (i). Find the middle term of $\left(\frac{a}{x} + \frac{x}{a}\right)^{10}$

(ii).Find the middle term of $\left(1 - \frac{x^2}{2}\right)^{14}$

(iii).Find the coefficient of x^{18} in $\left(x^2 + \frac{3a}{x}\right)^{15}$

(iv).Find the coefficient of x^{18} in $\left(ax^4 - bx\right)^9$

(v).Find the coefficient of x^{32} and x^{-17} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$

(vi). Find the two middle terms of $\left(3a-\frac{a^3}{6}\right)^9$

(vii). Find the term independent of x in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

($viii$). Find the term independent of x in $\left(x-\frac{1}{x^2}\right)^{3n}$

22.(i).Find the greatest term in the expansion of $(1+4x)^8$ when x=2

(ii).Find the greatest term in the expansion of $(x+y)^{30}$ when $x=11, y=4$.

(iii).Find the greatest term in the expansion of $(2x+3y)^{28}$. When x=9, y=4.

(iv).Find the greatest term in the expansion of $(2a+b)^{14}$. When a=4, b=5.

(v).Find the greatest term in the expansion of $(3+2x)^{15}$. When $x=\frac{5}{2}$.

(vi).Find the greatest term in the expansion of $(1+x)^n$. When $x=\frac{2}{3}$, n=6.

(vii).Find the greatest term in the expansion of $(a+x)^n$, When

$$a=\frac{1}{2}, x=\frac{1}{3}, n=9$$

23. Find the mumericaly greatest coefficient in the following expansions.

(i). $(1+x)^{15}$ (ii). $(1+2x)^{35}$ (iii). $(2x+3y)^{11}$

(iv). $(3-2x)^{24}$ (v). $(2+3x)^{16}$ (vi). $(4+3x)^8$

(vii). $(3x+2y)^{20}$

24.It is given that the greatest coefficient of the expansion

$\left(x^{1/2}+x^{-3/4}\right)^{n}$ is in 9th term. Find n and coefficient of x^4.

25.If $x>0$, if the middle term of the expasion $(2+x)^{14}$ is the greatest, show that $\frac{7}{4}<x<\frac{16}{7}$.

26.If the greatest term of the expansion $\left(2+\frac{3}{8}x\right)^{10}$ is T_4, show that

$2<x<\frac{64}{21}$.

27. Prove the following.

(i). ${}^{n}C_{r-1}+{}^{n}C_{r}={}^{n+1}C_{r}$ (ii). $r.{}^{n}C_{r}=n.\ {}^{n-1}C_{r-1}$

(iii). $\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}}=\frac{n-r+1}{r}$ (iv). ${}^{n}C_{r}={}^{n}C_{n-r}$

(v). $\frac{{}^{n}C_{r}}{r+1}=\frac{{}^{n+1}C_{r+1}}{n+1}$

28. Prove that $\frac{{}^{n}C_{r}}{r+1}=\frac{{}^{n+1}C_{r+1}}{n+1}$.

Hence, by repeated application of this result show that

$$\frac{{}^{n}C_{r}}{(r+1)(r+2)(r+3)}=\frac{{}^{n+3}C_{r+3}}{(n+1)(n+2)(n+3)} .$$

29. (a). If $(1+x)^{n}=\sum_{r=0}^{n}{}^{n}C_{r}x^{r}$ show that

${}^{n}C_{0}+{}^{n}C_{1}+{}^{n}C_{2}+..........+{}^{n}C_{n}=2^{n}$

(*b*). Show that $r.{}^{n}C_r = n.{}^{n-1}C_{r-1}$. Hence show that

$${}^{n}C_1 + 2.{}^{n}C_2 + 3.{}^{n}C_3 + ... + n.{}^{n}C_n = n.2^{n-1}$$

30. Write down the r^{th} term T_r of the series

$2C_1 + 3C_2 + 4C_3 + ... + (n+1)C_n$ where $C_r = {}^{n}C_r$ Show that

$T_r = r.{}^{n}C_r + {}^{n}C_r$.

Hence show that

$2C_1 + 3C_2 + 4C_3 + ... + (n+1)C_n = 2^{n-1}(n+2) - 1$. Deduce that

$C_0 + 2C_1 + 3C_2 + 4C_3 + ... + (n+1)C_n = 2^{n-1}(n+2)$.

31. Using the expansion of $(1-x)^n$ show that $\sum_{r=0}^{n}(-1)^r \; {}^{n}C_r = 0$.

Consider the series $3\;{}^{n}C_0 - 8\;{}^{n}C_1 + 13\;{}^{n}C_2 - 18\;{}^{n}C_3 + ...$ up to $(n+1)$ term. Show that the general term T_r can be written as $T_r = (-1)^r(3+5r)\;{}^{n}C_r$ for $r = 0,1,2,3,......n$. Hence show that the sum of the above seris is *0*.

32. If $x + y = 1$ prove that $\sum_{r=0}^{n} r\;{}^{n}C_r x^r.y^{n-r} = nx$

33. Let $S = 1.2C_1 + 2.3C_2 + 3.4C_3 + ... + n(n+1)C_n$. Where $C_r = {}^{n}C_r$.

Show that $S = 2n + \sum_{r=2}^{n} r(r+1)C_r$. Show further that

$\sum_{r=2}^{n} r(r+1)C_r = n(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + 2n\sum_{r=2}^{n} {}^{n-1}C_{r-1}$. Hence deduce that $S = n(n+3)2^{n-2}$.

34. Prove that $\dfrac{{}^{n}C_r}{r+1} = \dfrac{{}^{n+1}C_{r+1}}{n+1}$. Hence, if $(1+x)^n = \sum_{r=0}^{n} {}^{n}C_r x^r$ show

that $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$ Where $C_r = {}^n C_r$.

35. Write down the general term of the series

$\frac{{}^nC_1}{2} - \frac{2\left({}^nC_2\right)}{3} + \frac{3\left({}^nC_3\right)}{4} + ... + (-1)^{n+1}\frac{n\left({}^nC_n\right)}{n+1}$. Show that the sum of n terms of the series is $\frac{1}{n+1}$.

36. (i). Find the sum of ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9$.

(ii). Find the sum of n terms of the series

$$\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ...$$

(iii). Show that $\sum_{K=0}^{10} {}^{20}C_K = 2^{19} + \frac{1}{2}\,{}^{20}C_{10}$

37. (i). Find the sum of all coefficients in the binomial expansion of $\left(x^2 + x - 3\right)^{319}$.

(ii). If the sum of the coefficients in the expansion of $\left(a^2x^2 - 2ax + 1\right)^{51}$ vanishes, then find the value of a.

38. (i). If $\left(1 + x - 2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{40}x^{40}$ then show that $a_1 + a_3 + a_5 + ... + a_{39} = -2^{19}$.

(ii). If the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924x^6$ then show that $n = 12$.

(iii). If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is α and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are β and γ show that $\beta + \gamma = \alpha$

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39.(i).If the coefficients of three consective terms in the expansion of $(1+x)^n$ are 165, 330 and 462 respectively, then find the value of n.

(ii).If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$ then prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

(iii).Show that $\sum_{r=1}^{n}\frac{r.^nC_r}{^nC_{r-1}}=\frac{n(n+1)}{2}$.

(iv).Find the sum $\sum_{r=1}^{n} r^2\frac{^nC_r}{^nC_{r-1}}$.

40. Show that,

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots\dots\dots + C_{n-1}x^{n-1} + C_nx^n$$

Where $C_r = ^n C_r$.

(a). Prove that $C_0 + C_1 + C_2 + C_3 + \dots\dots\dots + C_n = 2^n$

(b). When n is odd integer. Prove that

$$C_0 + C_2 + C_4 + \dots\dots\dots + C_{n-1} = C_1 + C_3 + C_5 + \dots\dots\dots + C_n$$

(c). When n is even integer

$$C_0 + C_2 + C_4 + \dots\dots + C_n = C_1 + C_3 + C_5 + \dots\dots + C_{n-1}$$

Hence deduce that,

$$C_0 + C_2 + C_4 + \dots\dots = C_1 + C_3 + C_5 + \dots\dots = 2^{n-1}$$

41.Write down the expansions of $(1+x)^n$ and $(x+1)^n$ in usual notaion.

By multiplying above expansions and considering x^n coefficients on both sides, show that,

$$C_0^2 + C_1^2 + C_2^2 + \dots\dots + C_n^2 = \frac{(2n)!}{(n!)^2}$$

42.Let $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_rx^r + ... + C_nx^n$

where ${}^nC_r = C_r$

(a).By differentiating both sides with respect to x and substituting x=1,

show that, $C_1 + 2C_2 + 3C_3 + ... + nC_n = n.2^{n-1}$

(b).By integrating both sides with respect to x and substituting x=1, show that,

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + ... + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{(n+1)}$$

43. State the expansion of $(1+x)^n$ for positive integral n. Show that

$C_0 + C_1 + C_2 + ... + C_n = 2^n$ and

$C_1 + 2C_2 + 3C_3 + ...+ nC_n = n.2^{n-1}$

Hence show that

$C_0 + 2C_1 + +3C_2 + ...+ (n+1)C_n = (n+2).2^{n-1}$.

44.Write down in usual notation the expansions of $(1-x)^n$ and $(1-x)^{n-1}$

show that

${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^n\,{}^nC_n = 0$ and

${}^{n-1}C_0 - {}^{n-1}C_1 + {}^{n-1}C_2 - {}^{n-1}C_3 + ... + (-1)^{n-1}\,{}^{n-1}C_{n-1} = 0$

Hence show that,

$a\,{}^nC_0 - {}^nC_1(a-1) + {}^nC_2(a-2) - .... + (-1)^n\,{}^nC_n(a-n) = 0$

45. Let $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{2n}x^{2n}$ Show

that $a_2 = \dfrac{n(n+1)}{2}$ and $a_3 = -\dfrac{n}{6}(n-1)(n+4)$.

Show also that

(i). $a_1 + a_2 + a_3 + ... + a_{2n} = 0$.

(ii). $a_0 - a_1 + a_2 - a_3 + ... + a_{2n} = 3^n$.

46.(i).If n is a positive integer and when ${}^nC_r = \dfrac{n!}{r!(n-r)!}$,prove that

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r.$$

(ii).When n is a positive integer prove by mathematical induction that

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_1 x^2 + ... + {}^nC_n x^n.$$

(iii).By using $(1+x)^{2n} = (1+x)^n(1+x)^n$ and using binomial expansion show that

$$\left({}^nC_0\right)^2 + \left({}^nC_1\right)^2 + ... + \left({}^nC_r\right)^2 + ... + \left({}^nC_n\right)^2 = {}^{2n}C_n.$$

47. Given that $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$

Prove that,

(i). $a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$

(ii). $a_0 - a_1 + a_2 - a_3 + ... + a_{2n} = 1$

(iii). $a_1 + 2a_2 + 3a_3 + ... + 2na_{2n} = n.3^n$

48. (a).The coefficients of the expansion $(1+x)^n$ are $C_0, C_1, C_2........C_n$ Find

(i). $C_1C_n + C_2C_{n-1} + C_3C_{n-2} + ... + C_nC_1$

(ii). ${}^4C_0\,{}^4C_4 - {}^4C_1\,{}^4C_3 + {}^4C_2\,{}^4C_2 - {}^4C_3\,{}^4C_1 + {}^4C_4\,{}^4C_0$

(b). When n is a positive integer. If

$(1+x)^n = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ and

$$(1+x)^{n+1} = b_0 + b_1x + b_2x^2 + ... + b_{n+1}x^{n+1}.$$

Prove that $b_r = a_r + a_{r-1}$

49.If n is a positive integer prove that

$(1+x)^n = 1 + C_1x + C_2x^2 + ... + C_rx^r + ... + C_nx^n$. where

$${}^nC_r = C_r = \frac{n!}{r!(n-r)!}.$$

If $(3-2x)^{20} = a_0 - a_1x + a_2x^2 - .... + a_{20}x^{20}$

(i). Show that $a_0 + a_2 + a_4 + ... + a_{20} = \frac{1}{2}\left(5^{20}+1\right)$

(ii). Show that the greatest coefficient is a_8 .

50. (i).If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

where $C_r = {}^nC_r = \dfrac{n!}{r!(n-r)!}$.

(ii).Prove that $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = \dfrac{(2n)!}{(n!)^2}$.

(iii).Find the term independent of x in the expansion

$$\left(x^2 - \frac{1}{x}\right)^6\left(x + \frac{1}{x}\right)^4.$$

51. (i).Find the greatest term of the expansion $\left(\dfrac{1}{5} + \dfrac{5x}{2}\right)^{11}$ when x=2.

(ii). $a_0, a_1, a_2, ...a_{2n}$ are constants and n is a positive integer.

If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$

show that $a_2 = \frac{n}{2}(n+1)$ and $a_3 = -\frac{n}{6}(n-1)(n+4)$

Also prove that

(a). $a_1 + a_2 + ... + a_{2n} = 0$.

(b). $a_0 - a_1 + a_2 - a_3 .... + a_{2n} = 3^n$.

52. Let $(1+kx)^{10} = a_0 + a_1 x + a_2 x^2 + ... + a_{10} x^{10}$, $x \in \Re$ where

$a_2 = \frac{20}{9}$ and k is a positive constant. Find the value of k.

Show that $a_1 + a_3 + a_5 + a_9 = \frac{11^{10} - 7^{10}}{2.9^{10}}$ Deduce the value of

$a_0 + a_2 + a_4 + a_6 + a_8 + a_{10}$.

53. Write down in the usual notation, the binomial expansion of $(a+b)^n$, where a and b are real and n is a positive integer .Using appropriate values for a, b and n write down the corresponding formulae for $(1-x)^4, (x+1)^4$ and $(1-x^2)^4$. Hence show in the usual notation that

(i). $\left({}^4C_0\right)^2 - \left({}^4C_1\right)^2 + \left({}^4C_2\right)^2 - \left({}^4C_3\right)^2 + \left({}^4C_4\right)^2 = 6$

(ii). ${}^4C_0\,{}^4C_1 - {}^4C_1\,{}^4C_2 + {}^4C_2\,{}^4C_3 - {}^4C_3\,{}^4C_4 = 0$

54. For all $x \in IR$ let in the usual notation

$(1+x)^n = {}^nC_0 + {}^nC_1 x + ... + {}^nC_r x^r + ... + {}^nC_n x^n$ where n is a positive integer. By considering the product of $(1+x)^{n-1}$ and $(1+x)$ show that

${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for $r = 1, 2, ...n-1$.

Deduce that ${}^nC_0 - {}^nC_1 + {}^nC_2 - .... + (-1)^{n-1}\,{}^nC_{n-1} + (-1)^n\,{}^nC_n = 0$

verify the above result by an alternative method. If n is an even integer, deduce that ${}^nC_0 + {}^nC_2 + {}^nC_4 + ... + {}^nC_n = 2^{n-1}$.

55.By using the principle of mathematical induction, prove that

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r \text{ where } n \text{ is a positive integer, and } {}^nC_r = \frac{n!}{(n-r)!r!}$$

Deduce that $(p+q)^n - p^n - q^n$ is divisible by pq, where p, q and n are positive integers.

56.(a).Three consecutive coefficients in the expansion of $(1+x)^n$ where n is a positive integer are 45, 120 and 210. Find the value of n.

(b).Is it possible for three consective coefficients in the expansion of $(1+x)^n$, where n is a positive integer to be in geometric progression ? Justify your answer.

57.State the binomial theorem for a positive integral index.

Let a, b and d be integers such that $a=b+d$. Show that $a^n - b^{n-1}(b+nd)$ is divisible by d^2 for positive integral n.
If U is the n^{th} term of an arithmetic progression whose first term is a and the common difference is d, prove that $a^n - (a-d)^{n-1}U$ is divisible by d^2. Deduce that $7^{60} - 3^{64}$ is divisible by 16.

58.State the binomial therom for a positive integral index. By choosing appropriate values for x and y in $3(x+y)^n$, show that 3^{2n+1} can be expressed as $7k + 3(2^n)$ where k and n are positive integers. Hence, show that $3^{2n+1} + 2^{n+2}$ is divisible by 7 for positive integral n.

59.(a).If $n>1$, then show that $(1+x)^n - nx - 1$ is divisible by x^2 .

(b).By substituting suitable values for x and y in the expansion $5(2x+y)^n$,show that 5^{2n+1} can be expressed in the form of $5.3^n + 11k$. Where n and k are integers. Hence, show that for all positive integers of n, $3^{n+3} - 5^{2n+1}$ is divisible by 11.

60.(i).Write down the expansion of $(a+b)^n$. Where n is a positive integer.

Expand $(x+2)^5$ and hence express $(x+2)^5 - (x-2)^5$ as a polynomial in x. Find the exact value of $2.1^5 + 1.9^5$

(ii).By substituting a suitable value for x in $\left(1+\frac{x}{5}\right)^6$ find to 4 significant figures correctly the value of $(1.01)^6$.

61.(i).Show that $6^n - 1$ is divisible by 5, for positive integral n.

(ii).Show that for positive integral n, $10^n - 9n - 1$ is divisible by 9.

(iii).By substituting x=2 and x=4 in $(1+x)^n$ Show that the expressions $3^n - 2n - 1$ and $5^n - 4n - 1$ are divisible by 4. Hence deduce that $5^n + 3^n - 6n - 2$ is divisible by 4.

62.Let $l = \left(4\sqrt{6} - 9\right)^{2n+1}$ and $m = \left(4\sqrt{6} + 9\right)^{2n+1}$. By expanding ,show that $m - l$ is an integer I. Also show that $0 < l < 1$. Deduce that the integral part and fractional part of m are I and l. Prove that $lm = 15^{2n+1}$.

63.Let $n \in Z^+$. The coefficient of x^{n-2} in the expansion of $\left(2+\frac{3}{x}\right)(1+x)^n$ is 120. Find the value of n.

64.Let $x \in \Re$ and in the usual notation, $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$, where $n \in Z^+$.

Show that $\sum_{r=0}^{n} {}^nC_r = 2^n$. By writing ${}^nC_r$ using factorials, show that

$\frac{{}^nC_r}{(r+1)} = \frac{{}^{n+1}C_{r+1}}{(n+1)}$ and deduce that $\frac{{}^nC_r}{(r+1)(r+2)} = \frac{{}^{n+2}C_{r+2}}{(n+1)(n+2)}$ for positive integral r with $0 \le r \le n$. **Hence** , show that

$$\sum_{r=0}^{n} \frac{{}^nC_r}{(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}(2^{n+2} - n - 3) .$$

65.Let $f(x) = \left\{x + \sqrt{x^2 - 1}\right\}^6 + \left\{x - \sqrt{x^2 - 1}\right\}^6$.

Using binomial expansion,express $f(x)$ as a polynomial in *x*.

Hence, find the value of $\frac{1}{\left(2 - \sqrt{3}\right)^6} + \frac{1}{\left(2 + \sqrt{3}\right)^6}$.

66.If the coefficient of *x* and the coefficient of x^2 in the binomial expansion of $(1+px)^{12}$, where *p* is a non-zero constant, are - *q* and 11*q* respectively, find the values of *p* and *q*.

67.Using the binomial expansion for a positive integral index, show that $\left(1+\sqrt{3}\right)^6 + \left(1-\sqrt{3}\right)^6 = 416$. **Hence**, find the integer part of $\left(1+\sqrt{3}\right)^6$.

68.Let $n \in Z^+$ and $n \ge 5$. The coefficient of x^{n-10} in the binomial expansion of $\left(3x + \frac{2}{x}\right)^n$ is less than 100. Find the value of *n*.

69.Let $a \in IR$. The term independent of x in the binomial expansion of

$\left(x+\frac{a}{x^3}\right)^{2n}$ is $\frac{969}{2}$. Find the value of a.

70.Let $n \in Z^+$. State, in the usual notation, the binomial expansion for

$(1+x)^n$. Show, in the usual notation, that

$\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{n-r}{r+1}$ for $r = 0,1,2,.....,n-1$.

The coefficients of x^r, x^{r+1} and x^{r+2} taken in that order, in the binomial expansion of $(1+x)^n$ are in the ratios 1 : 2 : 3. In this case, show that $n=14$ and $r=4$.

71.Write down the binomial expansion of $(5+2x)^{14}$ in ascending powers of x.

Let T_r be the term containing x^r in the above expansion for $r = 0,1,2,....,14$.

Show that $\frac{T_{r+1}}{T_r} = \frac{2(14-r)}{5(r+1)}x$ for $x \neq 0$.

Hence, find the value of r which gives the largest term of the above expansion, when $x = \frac{4}{3}$.