MATHEMATICAL INDUCTION

1)using the principle of mathematical induction prove the following, for all $n \in z^{+}$.

i) $1.2 + 2.3 + 3.4 + \ldots\ldots\ldots + n(n+1) = \frac{1}{3} n(n+1)(n+2)$.

ii) $1.3 + 3.5 + 5.7 + \ldots\ldots\ldots + (2n-1)(2n+1) = \frac{1}{3} n(4n^2 + 6n - 1)$

iii) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

iv) $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots\ldots\ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$.

2)using the principle of mathematical induction, prove that

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\ldots\left(1-\frac{1}{n+1}\right) = \frac{1}{n+1}$$. for all $n \in z^{+}$.

3)Using principleof mathematical induction prove that $(10^{2n-1} + 1)$ is divisible by 11 for all $n \in z^{+}$.

4)Using principle of mathametical induction prove that $(7^{n} - 3^{n})$ is divisible by 4 for all $n \in z^{+}$.

5)Using principle of mathametical induction prove that $(2.7^{n} + 3.5^{n} - 5)$ is divisible by 24 for all $n \in z^{+}$.

6)Using principle of mathametical induction prove that $(n^{2} + n)$ is even for all $n \in z^{+}$.

7)Using principle of mathametical induction prove that $(1+x)^n \geq (1+nx)$ for all $n \in z^+$, where $x >$ -1.

8)Using principle of mathametical induction prove that $n < 2^n$ for all $n \in z^+$.

9)Using principle of mathametical induction prove that

$$(1^2 + 2^2 + 3^2 + ... + n^2) > \frac{n^3}{3} \text{ for all } n \in z^+.$$

10)Using principle of mathametical induction prove that $(2n+7) < (n+3)^2$, for all $n \in z^+$.

11)Using principle of mathametical induction prove that

$$\sin^2 \alpha + \sin^2 (\alpha + \beta) + \sin^2 (\alpha + 2\beta) + ... + \sin^2 (\alpha + (n-1)\beta)$$
$$= \frac{n}{2} - \frac{\sin(n\beta)}{2 \sin \beta} \cos(2\alpha + (n-1)\beta)$$

12)Using principle of mathematical induction prove that

$$\frac{d^n}{dx^n}(e^{ax} \sin bx) = (a \sec \theta)^n . e^{ax} \sin(bx + n\theta) \text{ for all } n \in z^+.$$

where $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.

13)If $a_1 = \cos\theta,\ a_2 = \cos 2\theta$ and $a_n = \cos(n\theta)$, show by principle of mathematical induction that $a_n = 2a_{n-1}\cos\theta - a_{n-2}$, for all $n \in z^+$, $n > 2$ and $\theta \in \Re$, .

Show that $\frac{d^r}{dx^r}(xe^x) = (x+r)\, e^x$ for any positive integer *r*.

If $y = x^2 e^x$ prove that $\frac{dy}{dx} = 2xe^x + y$. Deduce that

$\frac{d^r y}{dx^r} - \frac{d^{r-1} y}{dx^{r-1}} = 2(x+r-1)e^x$. Hence, Show that

$\dfrac{d^n y}{dx^n} = n(2x+n-1)e^x + y,$ for any positive integer *n*.

14)Using principle of mathematical induction prove that

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, for all $n \in z^+$, $\theta \in \Re$, .

15)using the principle of induction show that the sum of *n* terms of the series

$8+88+888+......$ is $\dfrac{8}{9}\left[\dfrac{10}{9}(10^n-1)-n\right]$.

16)A number sequence is given by $u_1, u_2, ..., u_n, ...$ $n \geq 1$ is an integer.

Given that $u_1 = 1, u_{n+1} = 3u_n + 2$ Show by induction for all $n \in z^+$ that

$u_n = 2.3^{n-1} - 1$.

17) If $f(n) = 9.3^{2n} - 8n - 9$ where n is an integer show that

$f(n+1) = 9f(n) + 64(n+1)$. Hence use mathematical induction to show that $f(n)$ is always divisible by 64.

18) If $u_r = \dfrac{1}{r(r+1)(r+2)}$ show by using mathematical induction that

$\sum_{r=1}^{n} u_r = \dfrac{n(n+3)}{4(n+1)(n+2)}$ Hence, deduce that

a. $\sum_{r=1}^{n} u_r$ is convergent

b. for all positive integral values of *n*. $\dfrac{1}{6} \leq \dfrac{n(n+3)}{4(n+1)(n+2)} \leq \dfrac{1}{4}$

19). (i). Show by using the principal of mathematical induction that

$$1^2.n + 2^2(n-1) + 3^2(n-2) + ... + n^2.1 = \frac{n}{12}(n+1)^2(n+2).$$

(ii). Show by using principal of mathematical induction that

$\sum_{r=1}^{n} r(r+2) = \frac{n}{6}(n+1)(2n+7)$ Hence find the sum of the series

$3ln2 + 4ln2^2 + ... + (n+2)ln2^n$

20). Show by using the principel of mathematical induction that,

i. $1.1! + 2.2! + 3.3! + ... + n.n! = (n+1)! - 1$

ii. $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{(2n+1)}$

iii. $2.1! + 5.2! + 10.3! + ... + (n^2+1).n! = n(n+1)!$

21). Use the principle of mathematical induction to show that for any

positive integer n, $\sum_{r=1}^{n} r.2^{r-1} = 1 + (n-1)2^n$.

22). When n is a positve integer, if $f(n) = 3^{2n} + 7$ show that

$f(n+1) - f(n)$ isdivisible by 8. Hence or otherwise deduce that $f(n)$ is divisible by 8

23). Show by using the principle of mathematical induction that

$$\sum_{r=1}^{n} \frac{2r+3}{r(r+1)3^r} = 1 - \frac{1}{(n+1)3^n} .$$

24).Let S_n be the sum of the first n terms of the series

$\frac{3}{1.2}.\frac{1}{2} + \frac{4}{2.3}.\frac{1}{2^2} + \frac{5}{3.4}.\frac{1}{2^3} + ...$ using the principle of mathematical

induction or otherwise show that $S_n = 1 - \frac{1}{(n+1)2^n}$.

25). If n is a positive integer prove by mathematical induction that the remainder when $4.6^n + 5^{n+1}$ is divided by 20 is 9.

26)Use the principle of mathematical induction to show that

$\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a positive integer for all positive integers.

27).Use the principle of mathematical induction to prove that

$$3 + 33 + 333 + ... + \underbrace{33.....3}_{n-times} = \frac{1}{27}\left(10^{n+1} - 9n - 10\right)$$

28). Prove by the principle of mathematical induction that

$$\cos\alpha\cos 2\alpha\cos 4\alpha..........\cos\left(2^{n-1}\alpha\right)=\frac{\sin(2^n\alpha)}{2^n\sin\alpha}.$$

29). Show by using the principle of mathematical induction that

$1.n+2.(n-1)+3.(n-2)+.....+n.1=\frac{n}{6}(n+1)(n+2)$ for all positive intergers.

30).Let $A_{n+1}=(1-\alpha)(1-A_n)+A_n$ for $n=1,2,3$ and $A_1=\beta$ where α and β are real numbers. Prove by the principle of mathematical induction, that for every positive integer *n*. $A_n=1-(1-\beta)\alpha^{n-1}$. Find $\sum_{r=1}^{n} A_r$.

31). Prove by using the principles of mathematical induction that $n!\geq 2^{n-1}$ for every positive integer n. Deduce that $\sum_{k=1}^{n}\frac{1}{k!}\leq 2-\frac{1}{2^{n-1}}$. Hence show that $e\leq 3$ where e is the base of natural logarthms.

32). Let *P* be an integer. By using the principle of mathematical induction prove that $P^{n+1}+(P+1)^{2n-1}$ is divisible by P^2+P+1, for all positive integral *n*.

33). Prove, by using the principle of Mathematical Induction, that

$$\sum_{r=1}^{n} r(r+1).....(r+k) = \frac{n(n+1)...(n+k+1)}{k+2}$$ for $n \in Z^+$, k being a fixed non-negative integer. Find constants *A, B, C* and *D* such that

$x^3 \equiv Ax(x+1)(x+2) + Bx(x+1) + Cx + D$ for $x \in IR$.

Hence, find $\sum_{r=1}^{n} r^3$. Show that the series $\sum_{r=1}^{\infty} r^3$ is not convergent.

34). Using the Principle of Mathematical Induction, prove that

$$\sum_{r=1}^{n} (n+2r) = n(2n+1)$$ for any positive integral n.