PROJECTILES

1.A particle is projected with velocity u making an angle α with the horizontal. Find

- i.the time of flight
- ii.the horizontal range.
- iii.the maximum horizontal range.
- iv.the directions of projection for a given horizontal range.
- v.the greatest height attained.

2.If the maximum horizontal range for a particle is R, show that the greatest height attained is $\frac{1}{4}R$.

3.A particle is projected with a velocity 'v' so that its range on a horizontal plane is twice the greatest height attained. Show that the range is $\frac{2v^2}{5g}$.

4.Prove that if the time of flight of a bullet over a horizontal range R is T, the inclination of the direction of projection to the horizontal is $\tan^{-1}\left(\frac{gT^2}{2R}\right)$.

5.If r be the horizontal range of a projectile and h, its greatest height, prove that the initial velocity is

$$\left[2g\left(h+\frac{r^2}{16h}\right)\right]^{\frac{1}{2}}.$$

6.A projectile has initially a velocity whose horizontal and vertical components are u and v respectively. If R is the range and h, the greatest height attained, show that $\frac{4h}{R}=\frac{v}{u}$ and $\frac{8h}{g}=\left(\frac{R}{u}\right)^2$.Also prove that the maximum horizontal range is $2h+\frac{R^2}{8h}$.

7.If R be the range of a projectile on a horizontal plane and h its maximum height for a given angle of projection, show that the maximum horizontal range with the same velocity of projection is $2h+\frac{R^2}{8h}$.

8.The maximum height of a projectile is h and angle of elevation is α.Find out the difference of time when it is at a height of $h\sin^2\alpha$.

9.Obtain the equation of the path of a projectile and show that it may be written in the form $\frac{yR}{x(R-x)}=\tan\alpha$, where R is the horizontal range and α, the angle of projection. Deduce that the greatest height attained by the projectile is $\frac{1}{4}R\tan\alpha$ and that it occurs half way. Also show that the angle of projection is given by $\tan^{-1}\frac{yR}{x(R-x)}$.

10.A particle is projected so as just to clear a wall of height b, at a horizontal distance a, and to have a range c from the point of projection. Show that the initial velocity is given by $\frac{2u^2}{g}=\frac{a^2(c-a)^2+b^2c^2}{ab(c-a)}$.

11.A particle is projected under gravity with velocity $\sqrt{2gh}$ from a point at a height h above a level plane. Show that the angle of projection α for the maximum range on the plane is given by $\tan^2\alpha = \frac{a}{a+h}$ and that the maximum range is $2\sqrt{a(a+h)}$.

12.A gun is fired from the sea level. It is then taken to a height h above the sea level and fired, making the same angle α with the horizontal show that its range is increased by the fraction $\frac{1}{2}\left[\left(1+\frac{2gh}{v^2\sin^2\alpha}\right)^{\frac{1}{2}}-1\right]$ of itself. v being the velocity of projection.

13.A particle is projected from a point at a height $3h$ above a horizontal plane, the direction of projection making an angle α with the horizontal. Show that if the greatest height above the point of projection is h, the horizontal distance travelled before striking the plane is $6h\cot\alpha$.

14.Two shots are projected from a gun at the top of a hill with the same velocity u at angles of projection α and β respectively. If the shots strike the horizontal ground through the foot of the hill at the same point, show that the height h of the hill above the plane is given by $h = \frac{2u^2(1+\tan\alpha\tan\beta)}{g(\tan\alpha+\tan\beta)^2}$.

15.Two particles P and Q are projected with the same velocity simultaneously from a point O on the ground. Their angles of projection are α and β respectively where $\alpha+\beta=\frac{\pi}{2}$ $(\alpha<45)$. Two particles move in the same vertical plane through O. Taking the horizontal and vertical axes through O in that plane as Ox and Oy, find the coordinates of the position of P and Q in a time t after projection. Hence, show that the line joining the two points make an angle of 135 with Ox. Show that the distance between P and Q is $\frac{\sqrt{2}u}{g}\sin\alpha(\cos\alpha-\sin\alpha)$ when the particle P is at the maximum point on its path.

16.Two particles A and B are projected simultaneously from a point P on a horizontal plane, in directions making angles α and β with the horizontal. The particle A hits the top of a vertical post and B hits the foot of that post. The post subtends an angle θ at P. Show that $\tan\alpha-\tan\beta=\tan\theta$.

17.Show that the maximum horizontal range of a body projected with a velocity u, inclined at an angle θ to the horizontal from a point O is $R=\frac{u^2}{g}$.

A particle just touches the top of a pole of height $\frac{R}{4}$ at a horizontal distance of $\frac{R}{2}$ from O. Show that $\tan\theta=1$ or 3.

Show that the horizontal ranges for these two angles are R and $\frac{3R}{5}$.

18.A particle is projected under gravity with initial velocity u at an angle α to the horizontal from a point O such that it passes through a point P whose horizontal and vertical coordinates are x and y respectively with reference to the origin O. Show that $y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\alpha$.

If the direction of the path of the particle at P makes an angle β with the horizontal, deduce that $\tan\alpha + \tan\beta = \frac{2y}{x}$.

If there are two possible paths for the particle which will pass through P and if the angle between the two paths at P isa right angle, prove that $x^2 + 2y^2 - \frac{u^2}{g}y = 0$.

Q also clears a maximum height when it is at the same horizontal distance OA from O find $\alpha_1 + \alpha_2$.

19.The components of the initial velocity of a particle projected under gravity from a point O are u,v in the directions of horizontal and upward vertical axes ox, oy respectively.Show that when it has moved a horizontal distance x it attains a vertical height $y = \left(\frac{v}{u}\right)x - \left(\frac{g}{2u^2}\right)x^2$. The particle just clears a vertical wall of height $\frac{a}{2}$ at a horizontal distance a from O and has a range $4a$ on the horizontal plane through O.Determine u,v and show that the direction of projection makes an angle $\tan^{-1}\left(\frac{2}{3}\right)$ with the horizontal.

20. A particle P is projected under gravity from a point O with an initial velocity u making an acute angle with the upward vertical. After time T it is moving with a velocity v in a direction at right angles to its initial direction of motion. It is given that $OP = s$ at this instant.Show that

i. $v^2 = g^2T^2 - u^2$ ii. $s = \frac{1}{2}gT^2$

21.A particle is projected with velcity u at an angle θ to the horizontal. If the horizontal range of the particle is R, show that $R = \frac{u^2\sin 2\theta}{g}$ and obtain the equation $Rg\tan^2\theta - 2u^2\tan\theta + Rg = 0$. Hence show that the maximum horizontal range is $\frac{u^2}{g}$. Deduce if $R < \frac{u^2}{g}$, that there exists two angles of projection. If these angles are θ_1 and θ_2. Show that $\tan\theta_1 . \tan\theta_2 = 1$. Also prove that $\theta_1 + \theta_2 = \frac{\pi}{2}$.

22. A particle is projected with velocity u at an angle θ to the horizontal. After a time t of projection it passes through a point of height h. Show that $gt^2 - 2u\sin\theta t + 2h = 0$. Deduce that this motion is possible only if $\sin\theta > \frac{\sqrt{2gh}}{u}$. Show that there are two possible points on the trajectory which are at height h and the time taken by the particle to travel between these points is $\frac{2}{g}\sqrt{(u^2\sin^2\theta - 2gh)}$. Hence obtain the maximum height of the projectile.

23.A particle is projected from a point O with velocity u at an angle θ to the horizontal. After a time t the particle is at P. The angle of elevation of P from O is α .Show that the velocity of projection is $\frac{gt\cos\alpha}{2\sin(\theta-\alpha)}$. If the highest point on the path is P, Find the time from O to P and hence obtain $\tan\theta = 2\tan\alpha$.

24. A particle is projected from the point O with inclination θ to the horizontal. The particle passes the point P whose angle of elevation from O is α. If at the point P the particle moves making an angle β to the horizontal. Show that $\tan\theta = 2\tan\alpha - \tan\beta$.

25. A particle P is projected at t=0 from a point O at an angle θ to the horizontal. If the maximum height of the particle from O is a, show that $a = \dfrac{u^2\sin^2\theta}{2g}$. Two points A and B are at a height b above the level of O. If the direction of motion of the particle at A and B is inclined at an angle α to the horizontal. Show that $\tan^2\alpha = \left(\dfrac{a-b}{a}\right)\tan^2\theta$. Also deduce that when the particle is at the maximum height it moves horizontally.

26. A particle is projected from a point O with velocity u inclined at an angle α to the horizontal if the horizontal range on the plane through O is R. Show that $R = \dfrac{u^2}{g}\sin 2\alpha$. Also obtain that the maximum range is $\dfrac{u^2}{g}$. Show also that if $R < \dfrac{u^2}{g}$, there are two angles of projection α_1, α_2 $(\alpha_1 < \alpha_2)$. If the times of flight for these projectiles are T_1 and T_2 respectively. deduce that $T_1T_2 = \dfrac{2R}{g}$ and when $\alpha = 30^0$. $T_2 - T_1 = \dfrac{u}{g}\left(\sqrt{3}-1\right)$.

27. A particle is projected from a point O with velocity u inclined at an angle α to the horizontal. If the particle cannot rise a vertical height greater than $\dfrac{u^2}{8g}$, show that the maximum horizontal range is $\dfrac{\sqrt{3}u^2}{2g}$.

28. A particle projected from a point O inclined at an angle α to the horizontal, Just passes through two vertical walls of height h. The distance to the nearer wall from point O is a. The distance between the walls is h.

Show that $\tan\alpha = \dfrac{h(2a+h)}{a(a+h)}$ and $u^2 = \dfrac{g\left[a^2(a+h)^2 + h^2(2a+h)^2\right]}{2ah(a+h)}$

31. From a point O on the ground a ball is projected vertically upward with velocity v at time $t=0$. At the same instant a boy of height h projects another ball from a point a distance a away O, with velocity w making an angle α to the horizontal. If balls collide at $t=t$, show that $w(a\sin\alpha + h\cos\alpha) = av$.

If $w = \dfrac{v}{2}$ and $h = a\sqrt{3}$, show that $\alpha = 30^0$. Find the time for the collision of the balls.

29. A balloon is rising upward with uniform velocity u. As the ballon passes a point O, a man in the ballon throws a ball with velocity v relative to the balloon making an angle α to the horizontal. If the range on the horizontal plane through O is R, show that $gR = v(2u\cos\alpha + v\sin 2\alpha)$.

Prove that if $u=v$, the maximum R occurs when $\alpha = \dfrac{\pi}{6}$. If $u=v$ and $\alpha = \dfrac{\pi}{6}$.
Find the distance between the ballon and the ball when the ball hits on the horizontal plane through O.

30. A war-ship is steaming ahead with constant velocity v. A short gun is mounted on the war-ship so as to point straight backwards and is set at an angle of elevation θ rads. If $v\sqrt{3}$ is the velocity of projection relative to the

gun, show that the range R is given by $R=\frac{2\sqrt{3}v^2}{g}\left(\sqrt{3}\cos\theta-1\right)\sin\theta$. Show that R is a maximum when $\theta=\frac{\pi}{6}$ and find the maximum range.

31.A warship is steaming ahead with constant velocity v. A short gun is mounted on the warship. So as to point straight backwards and is set at an angle of elevation α. If the velocity of the bullet relative to the gun is $u(>v)$, show that the range of the bullet is $2u\sin\alpha\frac{(u\cos\alpha-v)}{g}$. Show also that the range is maximum when the angle of projection is $\cos^{-1}\left[\frac{v+\sqrt{v^2+8u^2}}{4u}\right]$

32.A particle is projected from a point O with velocity v at an angle α to the horizontal. After horizontal motion of the particle, at a point P on its trajectory it moves making an angle β with the horizontal. If OP makes an angle β to the horizontal show that time taken to reach point P is $\frac{4v\sin\alpha}{3g}$ and $3\tan\beta=\tan\alpha$.

33.A particle is projected with velocity u and inclination θ to the horizontal. Relative to the horizontal and vertical axes through the point of projection show that the equation of the path is given by

$2u^2y=2u^2x\tan\theta-gx^2(1+\tan^2\theta)$. Hence find the horizontal range. If the horizontal range is $\frac{\sqrt{3}u^2}{2g}$, find the angles of projection. Show that the maximum height of the particle does not exceed the value $\frac{3u^2}{8g}$.

34.Two particles P, Q are projected simultaneously from a point A, with the same velocity u, and inclinations θ, $\frac{\pi}{2}-\theta$ $\left(\theta<\frac{\pi}{4}\right)$ with the upward vertical. Show that the velocity of P relative Q is a constant. Hence show that PQ always inclined to the horizontal with an angle $\frac{\pi}{4}$. If the particle Q reaches the same horizontal level as A, show that the distance between P and Q is $\frac{4a^2}{g}\sin\theta.\cos\left(\theta+\frac{\pi}{4}\right)$.

35.An insect is at the centre of the bottom of a cylinder of radius a and height $3a$. It can jump with velocity u upwards in any direction. Show it can come out of the cylinder only if $u^2\geq ga(3+\sqrt{10})$.

36.Two particles are projected from the same point at the same instant, and the same vertical plane with velocites u and v and angles of projection α and β to the horizontal respectively.

i.Show that at any instant of the motion of the particles the lines joining the positions of the particles are parallel.

ii.Show that the velocities of the particles become parallel after a time $\frac{uv\sin(\alpha-\beta)}{g(v\cos\beta-u\cos\alpha)}$.

37..A and B are two points on a horizontal ground at a distance a. A particle projected from A to B with velocity u with an angle of projection 2α $(<90^{\circ})$ and at the same time a second particle is projected from B to A with velocity v and angle of projection α. If the particles collide show that $v=2u\cos\alpha$. Hence show that the time for the collision is

$\frac{a}{u(1+2\cos 2\alpha)}$. Also if $v=2u\cos\alpha$,

*i.*Show that the projectiles have the same maximum heights

*ii.*Show that horizontal range of second particle is always greater than that of first particle.

38.A is a point on the trajectory of a projectile projected from a point *O*. $OA=2a$ and *OA* makes with the horizontal an angle α . The particle is projected from *O* making an angle θ with *OA* and, with velocity *v*. If the velocity of the particle at *A* is perpendicular to *OA* Show that

$$i.\ \sqrt{2}\sin\theta\cos(\theta+\alpha) = ga\cos^2\alpha . \quad ii.\ 2\tan\theta.\tan\alpha = 1 .$$

when the velocity of the particle is parallel to *OA* ,If the perpendicular

distance from the particle to *OA* is *a*, obtain a relation between $\tan\theta$ and $\tan\alpha$, Hence deduce that $\theta=\frac{\pi}{4}$.

39. A particle projected from a point *A* on a horizontal plane with velocity *u* at an angle 60^{o} to the horizontal, strikes at a point *B* of a vertical wall which is at a distance *a* from *A*, and after rebounding it returns again to *A*. The vertical height from the plane to *B* is *h*. The coefficient of restitution between the particle and the wall is *e*. Show that

$$i.\ \sqrt{3}\,eu^2 = 2ga(1+e) \qquad ii.\ (1+e)h = \sqrt{3}\,a .$$

40.A particle projected with velocity *v* making an angle 60^{o} to the horizontal from a point *A* on a smooth horizontal plane, strikes a smooth vertical wall and at the second collision on the horizontal plane it comes again to *A*. If for all collisions the coefficient of restitution is *e,* show that $\sqrt{3}\,ev^2=2ga$. where *a* is the distance from *A* to the wall.

41.A particle is projected from a point *A* with velocity *v* at an angle θ inclined to the horizontal. The horizontal line through *A* meets the trajectory at *B*. The particle moves at *C* and *D* making to the horizontal at an angle β .

*i.*Show the time to move from *C* to *D* is $\frac{2v}{g}\cos\alpha.\tan\beta$ *ii.* Show that $\frac{CD}{AB} = \frac{\tan\beta}{\tan\alpha}$.

iii. If the height to *C* and *D* from *AB* is *h*, Show that $h = \frac{v^2\cos^2\alpha}{2g}(\tan^2\alpha-\tan^2\beta)$.

42.A bird flies with uniform velocity *u* along a straight line inclined an angle β to the horizontal . As the bird passes a point *A*, a child at a point *O* on the ground, a height *h* vertically below *A*, throws a stone to a direction making an angle α to the horizontal. Where $\alpha > \beta$ .Show that whichever the velocity of the stone the stone does not hit the bird unless $\tan\alpha > \frac{\sqrt{2gh}}{u}\sec\beta + \tan\beta$.

43. A particle is projected under the gravity in a vertical plane with a velocity u at angle θ to the horizontal, at a point *C* which is at a height *k* from a point *O*.Consider a rectangular Cartisian system of coordinates by taking horizontal and vertrical lines through the point O in the plane of projection as *Ox* and *Oy* axes respectively.If at time t the particle is at the point (*x,y*), show that $y = k + x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$.

A particle *P* is projected under the gravity in the verticle plane at the point *A*(*0,h*), where *h* is positive ,with a velocity *v* at an angle α to the horizontal. At the same instant another particle *Q* is projected under the gravity in the vertical plane at the point $B\left(0,\frac{h}{2}\right)$ with a velocity *w* at an angle $\beta(>\alpha)$ to the horizontal. If the two particles *P* and *Q* meet at a point whose horizontal distance is *d*, show that $v\cos\alpha = w\cos\beta$ and $h = 2d(\tan\beta - \tan\alpha)$.

Show also, that the time for the two particles to meet is $\dfrac{h}{2(w\sin\beta - v\sin\alpha)}$

44.A particle is projected under gravity from a point *O* on the ground with speed *u* at an angle α to the horizontal, in a plane perpendicular to a wall at a distance *a* from *O*. If the particle is at a height *y* when it is at a horizontal distance *x* from *O,* show that $gx^2\sec\alpha = 2u^2(x\sin\alpha - y\cos\alpha)$.

If the particle just passes over the wall and falls on the ground at a point of distance *d* from the wall, show that the height of the wall is $\dfrac{ad}{a+d}\tan\alpha$.

Find the maximum height attained by the particle in terms of *a, d* and α.

45.A particle projected from a point *O* on a horizontal ground with a velocity $u = \sqrt{2ga}$ making an angle $\theta\left(0 < \theta < \dfrac{\pi}{2}\right)$ to the horizontal, moves under gravity and hits a target at a point *P*. The horizontal and vertical distance of *P* measured from *O* are *a* and *ka*, respectively, where *k* is a constant. Show that $\tan^2\theta - 4\tan\theta + 4k + 1 = 0$ and deduce that $k \le \dfrac{3}{4}$. Now, let $k = \dfrac{11}{16}$. Show that the angle between the two possible directions of projection is $\tan^{-1}\left(\dfrac{4}{19}\right)$.

46. A particle is projected at an angle α to the horizontal from a point at height *h* from horizontal ground .

The particle reaches the ground at a point of horizontal distance 2*h* from the point of projection.

find the speed of projection in terms of *g*, α and *h* Given that the direction of motion of particle with horizontal when it reaches the ground is β, show that $\tan\beta = 1 + \tan\alpha$.

47. A particle is projected from a point *O* on a horizontal plane with initial velocity *u* inclined an angle β to horizon tal. The particle clears two parallal walls each of height *h*. If *t* is the time taken by the particle to pass a wall show that $gt^2 - 2u\sin\beta.t + 2h = 0$. Hence or otherwise find the time taken by the particle to pass the walls. Also deduce the maximum height ascended by the particle, time taken to reach maximum height, time of flight and horizontal range.

47.A particle is projected under gravity with velocity $\sqrt{2ga}$ at an angle α to the horizontal from a point *O* which is at a height *h* from a horizontal ground along the vertical plane through *O*.

*i.*Taking the horizontal and the vertical axes through *O* as the *x* and *y* axes, if point $P \equiv (x, y)$ be any point on the path of the projection show that $y = x\tan\alpha - \dfrac{x^2}{4a}\sec^2\alpha$.

*ii.*Show that the maximum range that the particle strikes the horizontal ground is $2\sqrt{a(a+h)}$.

*iii.*In the case of the maximum range, show that the angle of projection is given by $\tan^2\alpha = \dfrac{a}{a+h}$.

48.A particle is projected from a point *O* on a level ground with a velocity *u* at an angle of elevation $45°$. The particle reaches a point *P* at a distance *R* on the level ground from *O*. Show that the equation of the path of the projectile, referred to horizontal and vertical axes through *O* is $y = x - \dfrac{gx^2}{u^2}$.

i. Show that $R = \dfrac{u^2}{g}$. If a point *Q* is on the path of the projectile such that $x = \dfrac{R}{4}$, *ii*. what is the angle it makes with the horizontal ?

iii. find the magnitude and direction of the velocity of the particle at Q.

iv. find the ratio $OQ : QP$.

49.A tennis player passes a ball above a net of height h, to other player. He projects the ball from a point, distance d from the net with velocity v at an angle α to the horizontal. Prove that $\tan^2\alpha - \dfrac{2v^2}{gd^2}\tan\alpha + 1 + \dfrac{2v^2h}{gd^2} < 0$.

Hence deduce that, if v is greater than a specific minimum value of u, α should take a value in between α_1 and α_2 $(> \alpha_1)$ α_1, α_2 and u are to be determined.

50.A ship sails with uniform velocity u in a straight path. A gun is fixed to the ship making an angle θ with the horizontal such that it aims the direction of motion of the ship. At a certain instant a bullet is released from the gun with a velocity $\sqrt{3}u$ so that it hits a castle of height h which is at a horizontal distance d from the gun. Assuming that the gun is in the level of sea, if the bullet hits castle, show that

$gd^2 + 2u^2(1+\sqrt{3}\cos\theta)^2 h - 2u^2\sqrt{3}d\sin\theta(1+\sqrt{3}\cos\theta) = 0$.

51.A gunner fires a shell of mass m with velocity u at an angle θ to the horizontal targeting a point which is at a distance R on the horizontal plane through the point of projection. But as soon as it is projected he recognized that the target is at a horizontal distance $\dfrac{R}{4}$ closer than his targeted point. Then he uses a technique to prevent the possible mitake as another magnetic shell of same mass is fired directly upwards to hit directly with the first shell, when the first shell is at its maximum height to coalesce two shells and by this to reduce the horizontal velocity and decreases the horizontal range of the combined shell. Show that the time taken for the first shell to reach the maximum height T is given by $T = \dfrac{u\sin\theta}{g}$ and the horizontal range R if the first shell travels freely is given by,

$R = \dfrac{2u^2\sin\theta\cos\theta}{g}$.

If the signal for projection of the second magnetic shell could be sent, at the same instant when the first shell is fired, show that the initial velocity of the second shell is also $u\sin\theta$. Subsequently show that the horizontal velocity of the combined shell reduces by half of the initial horizontal velocity and the horizontal range reduces by $\dfrac{R}{4}$. Where g is acceleration due to gravity.

52. Point O is at a height d vertically above a horizontal ground. A particle is projected from O with speed $\sqrt{2gd}$ making an angle $\theta\left(0 < \theta < \dfrac{\pi}{2}\right)$ above the horizontal under gravity and hits a target at a point P on the horizontal ground. The horizontal distance of P measured from O is kd. Where $k(>0)$ is a real constant.

Show that $k^2\tan^2\theta - 4k\tan\theta + k^2 - 4 = 0$ and deduce that $0 < k \le 2\sqrt{2}$. If $k = 2\sqrt{2}$, show that the angle of projection is $\tan^{-1}\dfrac{1}{\sqrt{2}}$.

53. A particle is projected from a point A on the ground with velocity u inclined θ to the horizontal. The particle just clears a point at a distance a, horizontally from A and height h from the ground. Show that $h = a\tan\theta - \dfrac{ga^2}{2u^2}\sec^2\theta$.

If the particle hits the ground at a distance $3a$ away from A, show that $3ga\tan^2\theta - 2u^2\tan\theta + 3ga = 0$, and **deduce** that $u^2 \ge 3ga$.