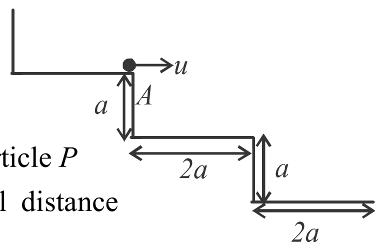
PROJECTILES -2

1.A straight narrow smooth tube *OA*, of length *l,* with both ends open, is fixed with the upper end *O* at a height $h(>l)$ above the horizontal ground, making an angle $\frac{\pi}{3}$ with the downward vertical. A particle, gently placed inside the tube at *O* slides down along the tube. Next, the particle leaves the tube at the end *A* and strikes the ground at a point *B* at a horizontal distance $\sqrt{3}l$ from *O*. Show that (*i*) the speed of the particle at *A* is $\sqrt{gl}$, and (*ii*) $h=\frac{3l}{2}$.

2.A particle is projected under gravity with speed *u* in a direction making an angle $\frac{\pi}{4}$ with the horizontal, from a point *O* on a horizontal ground towards a vertical wall of height *a* which is ata horizontal distance 2*a* from *O*. Show that if $u>2\sqrt{ga}$, then the particle will go over the wall.

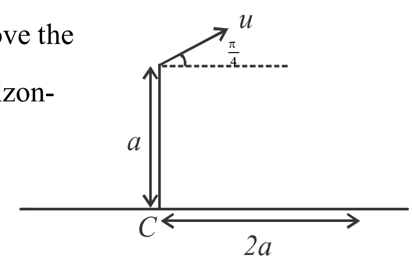
3.A particle *P*, projected horizontally with velocity *u* given by $u=\frac{3}{2}\sqrt{ga}$ from a point *A* at the edge of a step of a fixed stairway perpendicular to that edge, moves under gravity. Each step is of height *a* and length 2*a* (see the figure). Show that the particle *P* will not hit the first step below *A*, and it will hit the second step below *A* at a horizontal distance 3*a* from *A*.



4.A particle is projected under gravity from a point *O* with a speed *u* at an angle $\frac{\pi}{3}$ to the horizontal. Let *h* be the vertical distance of the particle above the level of *O* when it has travelled a distance *k* horizontally. Show that

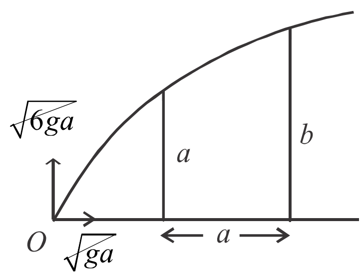
$$\sqrt{3}k=h+\frac{2gk^2}{u^2}.$$

5.The base of a vertical tower of height *a* is at the centre *C* of a circular pond of radius 2*a*, on horizontal ground. A small stone is projected from the top of the tower with speed *u* at an angle $\frac{\pi}{4}$ above the horizontal. (See the figure.) The stone moves freely under gravity and hits the horizontal plane through *C* at a distance *R* from *C*. Show that *R* is given by the equation $gR^2-u^2R-u^2a=0$



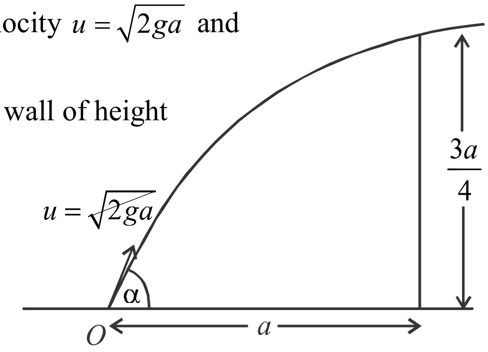
Find *R* in terms of *u, a* and *g*, and deduce that if $u^2>\frac{4}{3}ga$, then the stone will not fall into the pond.

6.A particle is projected from a point *O* on a horizontal floor with a velocity whose horizontal and vertical components are $\sqrt{ga}$ and $\sqrt{6ga}$, respectively. The particle just clears two vertical walls of heights *a* and *b* which are at a horizontal distance *a* apart as shown in the figure. Show that the vertical component of the velocity of the particle when it passes the wall of height *a* is $2\sqrt{ga}$. Show further that $h=\frac{5a}{2}$.

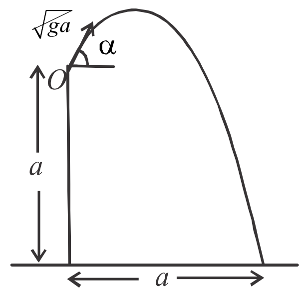


7.A particle is projected from a point O on a horizontal floor with initial velocity $u=\sqrt{2ga}$ and at an angle $\alpha\left(0<\alpha\dfrac{\pi}{2}\right)$ to the horizontal. The particle just clears a vertical wall of height $\dfrac{3a}{4}$ located at a horizontal distance a from O.

Show that $\sec^2\alpha-4\tan\alpha+3=0$. Hence, show that $\alpha=\tan^{-1}(2)$.



8.A particle is projected from a point O at a vertical distance a above a horizontal ground with initial velocity $\sqrt{ga}$ and at an angle $\alpha\left(0<a<\dfrac{\pi}{2}\right)$ to the horizontal, as shown in the figure. The particle strikes the ground at a horizontal distance a from O. Show that $\tan\alpha=1+\sqrt{2}$.

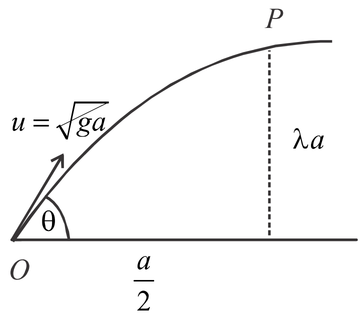


9.A particle is projected from a point on the horizontal ground, at an angle $\alpha\left(0<\alpha<\dfrac{\pi}{2}\right)$ to the horizontal, with initial speed $u=\sqrt{2gR}$. where R is the horizontal range of the projectile on the ground. Show that the angle between the two possible initial directions of projection is $\dfrac{\pi}{3}$

10. A particle is projected under gravity with speed $u=\sqrt{ag}$ in a direction making an angle $\theta(0^\circ<\theta<90^\circ)$ with the horizontal from a point O on a horizontal ground.

During its flight, if it passes the point P which is at a horizontal distance of $\dfrac{a}{2}$ and height λa from O,

Show that $\tan^2\theta-4\tan\theta+(8\lambda+1)=0$. Also deduce that $\lambda\le\dfrac{3}{8}$



11.A particle projected from a point O on a horizontal ground with a velocity $u=\sqrt{2ga}$ making an angle $\theta\left(0<\theta<\dfrac{\pi}{2}\right)$ to the horizontal, moves under gravity and hits a target at a point P. The horizontal and vertical distances of P measured from O are a and ka, respectively, where k is a constant. Show that

$\tan^2\theta-4\tan\theta+4k+1=0$ and **deduce** that $k\le\dfrac{3}{4}$. Now, let $k=\dfrac{11}{16}$. Show that the angle between the two possible directions of projection is $\tan^{-1}\left(\dfrac{4}{19}\right)$.

12.A particle is projected from a point O on a level ground with a velocity u at an angle of elevation 45°. The particle reaches a point P at a distance R on the level ground from O. Show that the equation of the path of the projectile, referred to horizontal and vertical axes through O is $y=x-\dfrac{gx^2}{u^2}$.

i. Show that $R=\dfrac{u^2}{g}$. If a point Q is on the path of the projectile such that $x=\dfrac{R}{4}$,

ii. what is the angle it makes with the horizontal ?

iii. find the magnitude and direction of the velocity of the particle at Q.

iv. find the ratio $OQ:QP$.