FRICTION

Problems, solvable subject to F= μ R

1.A uniform rod of length $2a$ has one end on rough horizontal ground and is supported by a smooth horizontal rail perpendicular to the rod at a height h above the gorund. When the rod is on the point of slipping it makes an angle θ with the horizontal. Prove that the coefficient of friction between the rod and the ground is $\dfrac{aCos\theta \cdot Sin^2\theta}{h - aCos^2\theta Sin\theta}$.

2.A uniform ladder of weight w rests on rough horizontal ground against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the ladder is inclined at an angle α to the vertical. Prove that, if the ladder is on the point of slipping and μ is the coefficient of friction between it and the ground, then $\tan\alpha = 2\mu$.

3.A heavy uniform sphere of radius a has a light inextensible string attached to a point on its surface. The other end of the string is fixed to a point on a rough vertical wall. The sphere rests in equilibrium touching the wall at a point distant h below the fixed point. If the point of the sphere in contact with the wall is about to slip downwards and the coefficient of friction between the sphere and the wall is μ, find the inclination of the string to the vertical. If $\mu = \dfrac{h}{2a}$ and the weight of the sphere is w, show that the tension in the string is $\dfrac{w}{2\mu}\left(1+\mu^2\right)^{\frac{1}{2}}$

4.A uniform circular disc, of radius a and weight w, stands in a vertical plane on a rough horizontal floor with a point of its circumference in contact with an equally rough vertical wall and its plane at right angles to the wall. A particle of equal weight w is attached to a point P of the disc at the same height as the center O and between O and the wall. Show that the disc is on the point of slipping when $OP = \dfrac{2a\mu(1+\mu)}{1+\mu^2}$, where μ is the coefficient of friction.

5. A plank, whose mass can be neglected, rests with one end on rough ground and part of its length overreaching a rough vertical wall of height h, on which the plank rests in a vertical plane at right angles to the wall. The plank is inclined at θ to the horizontal and its coefficient of friction with the ground and with the wall is μ in each case. Show that, if $\mu < \tan\theta < \dfrac{1}{\mu}$, the distance x measured from the ground through which a man may walk along the plank before it slips, is given by $x = \dfrac{\mu h}{(1+\mu^2)Sin^2\theta \cdot Cos\theta}$

6.A rough circular cylinder of radius a is fixed with its axis horizontal and on it rests symmetrically a rod of length $2l$ and weight W, at right angles to the axis. A particle of weight w is attached to one end of the rod, which is allowed slowly to rotate without slipping untill it is in equilibrium. If it now makes an angle θ with the horizontal and is on the point of slipping, show that the angle of friction between rod and cylinder is θ and that $w = \dfrac{Wa\theta}{l - a\theta}$

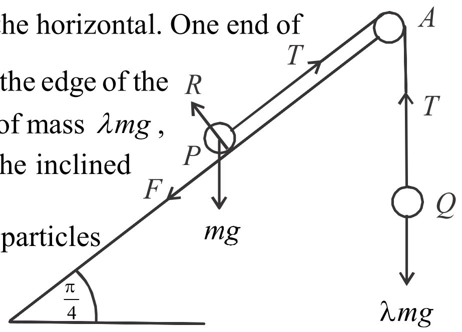
7.A solid hemisphere of weight W and radius a has its centre of gravity on the axis, at a distance x from the centre. It rests in equlibrium with its curved surface incontact with a rough plane inclined at an angle α to the horizontal. Prove that its plane base is inclined to the horizontal at an angle θ given by $xSin\theta = aSin\alpha$. If ϕ is the inclination of the base to the horizontal when a particle of weight W is attached to the centre of the base. The hemisphere again resting in equilibrium on the inclined plane, prove that $Sin\phi = 2Sin\theta$.

8.A uniform ladder of length $2a$, weight w rests in equilibrium inclined at an angle $\tan^{-1} 2$ to the horizontal, with end A touching the rough ground and end B on a smooth vertical wall, the coefficient of friction between the ground and the ladder is 1/3. How far, a child weight $2w$ can climb along the ladder without disturbing the equilibrium of the ladder? Also, find the least horizontal force which must be exerted on the ladder at A for the child be able to climb to the top of the ladder.

9.Two equal uniform rods AB, BC each of length $2a$ are rigidly jointed at B so that $A\hat{B}C = \frac{\pi}{2}$. The rods are in limiting equilibrium in contact with a fixed rough cylinder of radius a and axis horizontal such that AB and BC are horizontal and vertical tangents to the cylinder. If the coefficient of friction at each contact is μ, show that $\mu^2 + 2\mu - 1 = 0$. Show further that $\mu = \sqrt{2} - 1$.

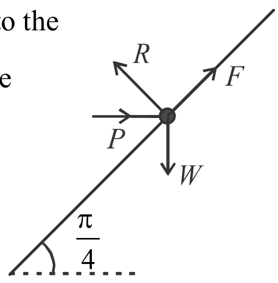
10. A uniform rod AB of length $2a$, rests with end A touching a rough vertical wall and against a fixed smooth peg P. AB makes an acute angle θ with upward vertical. If the distant between A and P is x, and the coefficient of friction between the rod and the wall is μ, when the end A is just about to slip downwards, prove that $(x-a)\tan^2\theta - \mu a \tan\theta + x = 0$.

11.A particle P of mass m is placed on a rough plane inclined at an angle $\frac{\pi}{4}$ to the horizontal. One end of a light inextensible string which passes over a fixed small smooth pulley fixed to the edge of the inclined plane at A, is attached to the particle P and the other end to a particle Q of mass λmg, as shown in the figure. The coefficient of friction between the particle P and the inclined plane is $\frac{1}{2}$. The line PA is a line of greatest slope of the inclined plane and the particles P and Q stay in equilibrium with the string taut.



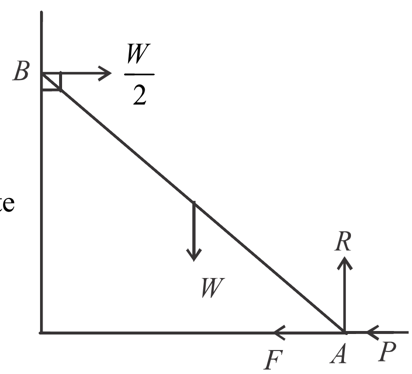
Show that $\frac{1}{2\sqrt{2}} \le \lambda \le \frac{3}{2\sqrt{2}}$. (The relevant forces are marked in the figure)

12.A small bead of weight W is threaded to a fixed rough straight wire inclined at an angle $\frac{\pi}{4}$ to the horizontal. The bead is kept in equilibrium by a horizontal force of magnitude P as shown in the figure. The coefficient of friction between the bead and the wire is $\frac{1}{2}$.



Obtain equations sufficient to determine the frictional force F and the normal reaction R on the bead, in terms of P and W. It is given that $\frac{F}{R} = \frac{W-P}{W+P}$. Show that $\frac{W}{3} \le P \le 3W$.

13.A uniform rod AB of length $2a$ and weight W has its end A placed on a rough horizontal floor and the end B against a smooth vertical wall. The rod is kept in equilibrium in a vertical plane perpendicular to the wall by a horizontal force of magnitude P applied at the end A towards the wall. In the figure, F and R denote the frictional force and the normal reaction at A, respectively. If the reaction at B from the wall is $\frac{W}{2}$ as shown in the figure and the coefficient of friction between the rod and the floor is $\frac{1}{4}$, show that $\frac{W}{4} \le P \le \frac{3W}{4}$

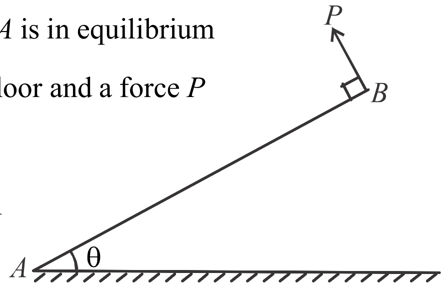


14.A particle of mass m is placed on a rough plane inclined at an angle α to the horizontal, where the coefficient of friction between the plane and the particle is $\mu(<\tan\alpha)$. The particle is held in equilibrium with a force P applied upwards to the particle along a line of greatest slope of the plane. Show that

$mg(\sin\alpha-\mu\cos\alpha)\le P\le mg(\sin\alpha+\mu\cos\alpha)$.

15.A rod AB of weight w which divides its center of gravity in the ratio 1:3 from A is in equilibrium inclined at an angle θ to the horizontal by keeping end A on a rough horizontal floor and a force P applied perpendicular to the rod at B as shown in the figure.

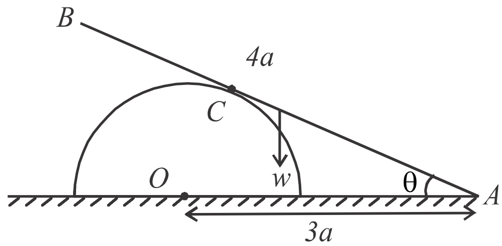
The rod and the force P is in the same vertical plane. If the coefficient of friction between the rod and the floor is μ.



Show that $\mu\ge\dfrac{\cos\theta\sin\theta}{4-\cos^2\theta}$.

16.The end A of a rough uniform rod AB of weight w and length $4a$ is in contact with a smooth horizontal plane. A rough right semi circular cylinder is fixed such that the plane surface coincide with the horizontal plane as shown in the figure. The vertical plane through the rod is perpendicular to the axis of the cylinder and that plane meet the axis of the cylinder at the point O. The rod is in equilibrium such that, $OA=3a$ and $O\hat{A}B=\theta$. Find the magnitude and the direction of the resultant reaction at the point C, which the rod and the cylinder is in contact with each other. Also find the magnitude of the reaction at A. Hence or otherwise, show that $\cos\theta\ge\dfrac{2}{3}$.

If the coefficient of friction at C is μ, show that, $\mu\ge\tan\theta$.



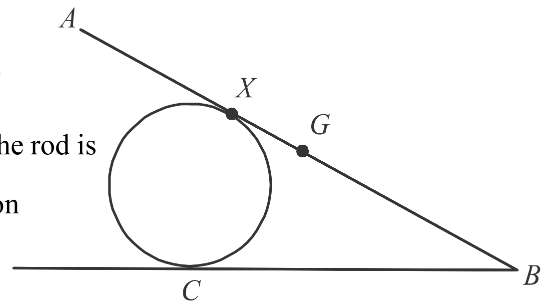
17.A uniform rod AB of length $2a$ and weight W rests in equilibrium with one end A against a smooth vertical wall and the other end B on a rough horizontal floor. The vertical plane containing the rod is perpendicular to the plane of the wall. The inclination of the rod to the vertical is θ and the coefficient of friction between the rod and the floor is μ, show that $\theta\le\tan^{-1}(2\mu)$ .A gradually increasing horizontal force P is applied at a point C of the rod ; where $BC=\dfrac{2}{3}a$ .This force acts away from the wall and in the vertical plane through AB. Find in terms of P, W and θ the force of friction at B and normal reaction at A, whilst equilibrium.

Given that $\mu=\dfrac{3}{4}.\theta=45^\circ$, show that equilibrium is broken by the rod slipping when P exceeds $\dfrac{3W}{8}$.

18. State the necessary condition for a system consisting of only three coplanar forces which are not parallel to each other to be in equilibrium.

A sphere of weight W resting on a rough inclined plane of inclination α is kept in equilibrium by a horizontal string attached to the highest point of the sphere. show that $\lambda\ge\dfrac{\alpha}{2}$; where λ is the angle of friction between the plane and the sphere.

19. In the diagram AB is a uniform rod of length $4a$ and weight W and G is the centre of mass of the rod. The rod is resting against a fixed vertical circular plane containing AB. The rod inclined at an angle $\frac{\pi}{4}$ to the horizontal. The point of contact at X of the rod with roller is at a distance a from the end A and the contact at X of the rod is smooth. Show that in order that equilibrium be maintained in this position the coefficient of friction μ at B must not be less than $\frac{1}{2}$.



A monkey of weight W stands at X and then starts walking slowly up the rod. Show that if $\mu = \frac{11}{13}$, the rod is on the point of slipping when it has moved a distance $\frac{a}{2}$.

20. A uniform solid hemisphere is placed on a rough inclined plane inclination α to the horizontal with its base in contact with the plane. where $\alpha < \tan^{-1}\frac{8}{5}$. The solid hemisphere is kept in equilibrium on the plane by a force acting at the end of the perpendicular radius to plane on the surface of the hemisphere along the line of greatest slope of the plane. Find the minimum force required to prevent the hemisphere from slipping down the plane. Now if this force is gradually increased. Show that the equilibrium is broken either by slipping or rolling according as $\mu \gtrless 1 - \frac{5}{8}\tan\alpha$ where μ is the coefficient of friction. Also find the friction force when it topples.

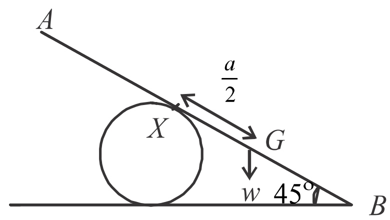
21. Two rough inclined planes each inclined to the horizontal at an angle of 45° intersect along horizontal line. A cylinder of radius a and weight w is in equlibrium between the planes so that the axis of the cylinder is parallel to the line of intersection of the inclined planes. Now a couple is applied on the cylinder in a plane perpendicular to the axis of the cylinder. The angle of friction between the planes and the cylinder is 30°. Show that if the moment of the couple is greater than $\frac{\sqrt{6}wa}{4}$, the cylinder rotates.

22. A rough uniform cylinder of radius r and weight w is placed in contact with a rough horizontal plane and equally rough vertical wall along two generators of the cylinder. An increasing force P is applied on the cylinder vertically upwards in mid cross section at a distance $2r$ from the wall.
Show that as P increases , the cylinder doesnot roll along the wall. Show that when the equilibrium is broken, the value of P is given by $P = \frac{\mu(1+\mu)w}{2\mu^2 + \mu + 1}$. where μ is the coefficient of friction at the points of contact.

23. A right circular cylinder of base radius a height h and weight $2w$ is placed on an inclined plane of inclination α to the horizontal so that the base of the cylinder in contact with the inclined plane. The coefficient of friction at the contact is μ.
Find the conditions required to break the equilibrum rolling and slipping. What is the condition required to slip before rolling.

24.A uniform right circular solid cone of base radius a , height h and weight w is in equilibrirum, base of the cone in contact with the rough horizontal plane. A gradually increasing horizontal force P is applied at the vertex of the cone. Show that the equilibrirum is broken by slipping or rolling according as $\mu \gtrless \frac{a}{h}$ .where μ is the coefficient of friction

25.A cylinder is fixed to a rough horizontal plane. A uniform ladder AB of weight w and length $2a$ is in equilibrium against the cylinder at a point X with its end B on the horizontal ground. The contact at X is smooth If the coefficient of friction at B is μ , show that $\mu \geq \frac{1}{2}$.



A child of weight w begins to walk up along the ladder from X. If $\mu = \frac{11}{12}$,

show that the maximum distance he can walk is $\frac{a}{4}$.

26.Two rough planes each inclined at an angle 60^{o} with the horizontal intersect each other along a horizontal line. A uniform cylinder of weight w and radius a is placed between these two planes symmetrically. The axis of symme try of the cylinder is horizontal and at both points of contact of the cylinder with the planes the coefficient of friction is μ . A couple of moment G is applied to the cylinder so as to rotate it in a vetical plane. Find the maximum value of G just before the cylinder begins to rotate.

27.A uniform wire of weight $3w$ is bent to form an equilateral triangle ABC. The wire frame is in equilibrium in a vertical plane so that C is below AB and the mid-point of AB in contact with a rough peg. A gradually increasing horizontal force P is applied at C. If the wire frame is in equilibirium making AB an angle α to the horizontal show that $P = w \tan \alpha$. Show also that the minimum value of μ is $\frac{1}{\sqrt{3}}$. where μ is the coefficient of friction between the peg and the wire frame.

28.A uniform rod of weight w and length $4a$ is in equilibrium in a fixed rough sphere of radius $3a$, making an angle α to the horizontal in the vertical plane through the centre of the sphere. Show that $\tan\alpha = \frac{9\mu}{5-4\mu^2}$, where μ is the coefficient of friction at the points of contact If $\alpha = 45^{o}$. Show that the coefficient of friction $\mu = \frac{\sqrt{161}-9}{8}$.

29.A hollow hemisphere of radius a and weight $2w$ is attached an end of a light inextensible string of length $2a$ to a point on its rim. The other end of the string is joined to a point o in a rough wall and the hemisphere is in equilib rium a point on its curved surface in contact with the wall. when the hemisphere is in limiting equilibrium to move down the plane containing the edge makes an angle 60^{o} to the horizontal .

Show that $\mu \geq \frac{\sqrt{3}\left[2+\sqrt{15}-2\sqrt{5}\right]}{4-\sqrt{3}}$, where μ is the coefficient of friction between wall and hemisphere.

30.The plane base of a smooth hemisphere of radius a is fixed to a rough horizontal plane. A uniform rod AB of length $2a$ and weight w is in equlibirium a point C on the rod in contact with the smooth curved surface of the hemisphere and the end A in contact with the rough horizontal plane. The rod is in limiting equilibrium making an angle α with the plane. Show that $\sin\alpha.(\alpha+\lambda)=\sin\lambda$. where λ is angle of friction. If $\mu=\frac{1}{2}$, find the value of α. Also obtain the total reaction at the points of contact.

31.A uniform circular disc of centre O weight W and radius a is in contact with a rough horizontal ground and a rough vertical wall. A particle of weight w is fixed at a point P on the circumference of the disc which is above the point O at a horizontal distance $\frac{a}{2}$ from the wall. The angle of friction at both contacts is λ, and the disc is in limiting equilibrium in a vertical plane perpendicular to the wall. Show that $\cos 2\lambda-\sin 2\lambda=\frac{W}{w+W}$.

If $w=W\left(\sqrt{2}-1\right)$ obtain the value of λ.

32.A rough cylinder of radius a is fixed so that its axis is horizontal. A uniform rod AB of weight w_1 and length $4a$ is placed symmetrically on this cylinder. When another particle of weight w_2 is jently attached to the end B, the rod turns over the cylinder without slipping and settles in equilibrium inclined to the horizontal. If the angle of friction at the contact is λ, Show that $\lambda=\frac{2w_2}{w_1+w_2}$.

If $w_1=w$ and $w_2=\frac{\pi w}{12-\pi}$, show that coefficient of friction is $\frac{1}{\sqrt{3}}$.

33.A rough cylinder of weight $3w$ and radius a is placed on a rough horizontal plane. A uniform plank AB of weight $2w$ and length $4a$ is in equilibrium end A in contact with the plane and a point C on the plank so that $AC=3a$ in contact with the cylinder, in the verical plane through the centre of the cylinder perpendicular to the axis. If the coefficient of friction at the points of contact is μ, Show hat $\mu\geq\frac{8}{21}$.

34.A uniform cube of side $2a$ and weight w is placed on a rough horizontal plane. A uniform rod AB of length $4a$ and weight $2w$ which is hinged at A to a point on the plane is in contact with the smooth edge of the cube at a point C on the rod such that $AC=3a$. The system is in equilibrium so that the rod lies in the vertical plane through the centre of gravity of the cube perpendicular to the edge.

If the coefficient of friction between the plane and the cube is μ show that $\mu\geq\frac{8\sqrt{5}}{47}$.

35.A uniform cube of weight $2w$ is on a rough horizontal plane.A gradually inceasing force is acted at the mid-point of the highest edge in the vertical plane through the centre of gravity of the cube making an angle α, so at to move the cube upwards direction. If the cube is about to roll about the lowest edge of the cube, when the equilibrium is broken show that the friction force and normal reaction are given by $\frac{w}{1+\tan\alpha}$ and $\frac{w[2\cos\alpha+\sin\alpha]}{\sin\alpha+\cos\alpha}$ respectively. If the cube breaks the equilibrium by rolling before slipping show that $2\tan\alpha > 1-2\mu$.

If $\alpha = 45^{\circ}$ and $\mu = \frac{1}{2}$, determine the way of breaking the equilibrium.

36.A uniform right circular solid cone of base radius a , height $2a\sqrt{3}$ and weight w is in equilibrium on an inclined plane of inclination α to the horizontal. The coefficient of fricton is $\frac{1}{\sqrt{3}}$. when the inclination of the plane is increased determine the way the equilibrium is broken.

47.Two equal uniform rodsAB and BC , each of weight W , are freely jointed together at B and rest in a vertical plane with ends A and C on a rough horizontal plane and with angle ABC equal to a right angle. A light inextensible string attached to the middle point of BC is pulled in the direction parallel toAC with a gradually increasing force P.If the system is in equilibrium while force P is less than $2W$, show that the ratios of the friction to the normal reaction, at A and C, are $\frac{2W-P}{4W-P}$ and $\frac{2W+3P}{4W+P}$ respectively.If equilibrium is broken while force P less than $2W$, show that it is broken by the extremity C slipping while A remains at rest and also that the coefficient of friction assumed the same for both rods, must be less than $\frac{4}{3}$.

48.Define the angle of friction.

A wedge of mass M is placed withone face in contact with a rough horizontal floor. on the other face inclined at an angle α to the horizontal a small object of mass m is placed. The object is just prevented from slipping down by a horizontal force and the wedge remains at rest. The angle of friction between the object and the wedge is $\lambda(<\alpha)$ If μ is the coefficient of friction between the wedge and the horizontal floor, show that $\mu \geq \frac{m}{M+m}\tan(\alpha-\lambda)$

Find the least value μ should take if the object is on the point of slipping up, while the wedge remains at rest. $(\alpha+\lambda<\frac{\pi}{2})$.

49. A Uniform rod AB of weight w and length 2a, is placed at an angle α to the horizontal, with A on a rough horizontal floor and B against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and the coefficient of friction at both A and B is $\mu(<1)$. show that, if the rod is in limiting equilibrium, then $\tan\alpha = \frac{1-\mu^2}{2\mu}$. If the inclination of the rod to the horizontal is $\theta(<\alpha)$, and a couple of moment Mis applied in the vertical plane through the rod , so that the rod is just prevented from slipping down, show that $(1+\mu^2)M = (1-2\mu\tan\theta-\mu^2)\ wa\cos\theta$

50.A uniform ladder of weight W and length $2a,$ rests with one end against a smooth vertical wall and the other on the ground which is rough, the coefficient of friction being μ . A man whose weight is four times that of the ladder can climb the whole length of the ladder, without the ladder slipping. If θ is the inclination o f the ladder to the down ward vertical at the smooth end, show that $\mu \geq \frac{9}{10}\tan\theta$, by considenring a general position of equilibrium of the system. Now if the foot of the ladder is at a distance $a\sqrt{2}$ from the wall and $\mu \leq \frac{1}{2}$, find the least value of the couple required in the vertical plane through the rod, so that, again, the man can climb the whole length of the ladder without the ladder slipping.

51. A heavy ring of radius *a,* mass *M* hangs in a vertical plane from a rough peg. An insect of mass m creeps up along the ring starting from the lowest point of the ring. Show that, the insect can reach the peg only if its mass is smaller than $\frac{M\sin\lambda}{(1-\sin\lambda)}$. Where λ is the angle of friction between the peg and the ring. If the mass of the insect is greater than $\frac{M\sin\lambda}{1-\sin\lambda}$, show that the distance the inscect can go along the ring, before slipping the ring is $a\left\{\lambda+\sin^{-1}\left(\frac{(m+M)\sin\lambda}{m}\right)\right\}$