Combined Maths

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Combined Mathematics II

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Test Paper 07

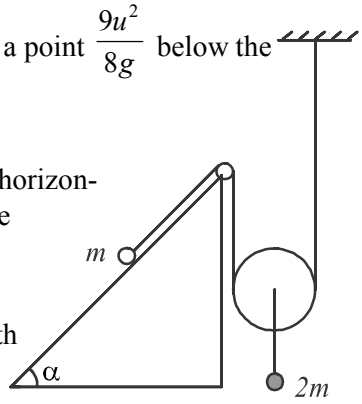
∗ Answer **all** *Q*uestions in Part **A** and **five** *Q*uestions Only in Part **B**.

PART A

1.Two equal smooth spheres A and B moving with velocities λu and $\frac{u}{\lambda}$ in opposite directions, collide directly. The coefficient of restitution between the spheres is e and $\lambda > 0$. If after the collision the sphere A gets reversed, show that $\lambda^2 < \frac{1+e}{1-e}$

2.A point A, at a height h above a horizontal floor, a particle is projected with velocity u making an angle 60° with horizontal. where $h > \frac{9u^2}{8g}$. Find the velocity of the particle when it passes through a point $\frac{9u^2}{8g}$ below the horizontal level through the point A.

3.A particle of mass m can move on a fixed smooth inclined plane of inclination α to the horizontal , as shown in the diagram, when the system is released from rest find acceleration of the particles.



4.A motor car of mass 1000 (kg) moves on a level road with uniform velocity $u\,(ms^{-1})$ with its engine working at H (kw). When the motor car moves up along an inclined plane of inclination α to the horizontal, its maximum velocity is $v\,(ms^{-1})$. If the resistance to the car in both occasions remains the same, show that $H = \frac{guv\sin\alpha}{u-v}$

5.One end of a light elastic string of natural length $2l$ and modulus of elasticity $2mg$ is joined to a fixed point O and a particle P of mass m is joined to the other end. The particle P moves on a horizontal circle below O with uniform angular velocity ω. If the string makes an angle θ with the vertical, show that $\cos\theta = \frac{g - l\omega^2}{2l}$.

6.A uniform rod AB of weight $2w$ and length $4a$ is in limiting equilibrium in contact with a rough peg P , such that $AP = 3a$ with its end A is joined to a point C by a light inextensible string and a weight w is suspended from B, as shown in the diagram.The string and the rod lie in

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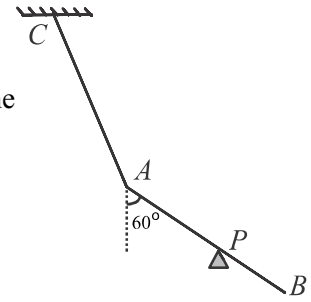
the same vertical plane and they make angles 30° and 60° respectively with the vertical. Find the tension in the string and show that the coefficient of friction between the peg and the rod is $\frac{\sqrt{3}}{4}$.



7.The events *A* and *B* are two exhaustive and mutually exclusive events in event space *S*. If $P(A)=\frac{2}{5}$, find $P(B)$. *C* is another third event in the space *S* such that $P(C)=\frac{1}{2}$ and *A* and *C* are independent events. A' and C' denote the complementory events of *A* and *C,* respectively.

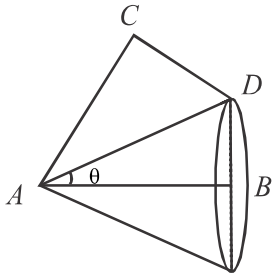
i.Find $P(A\cap C)$

ii.Find $P(A\cup C)$ and deduce that $P(A'\cup C')$.

8.First half of a set of observations has the mean 10, and in the second half , first $\frac{1}{6}^{th}$ of the observation has the mean 20, and last $\frac{1}{3}^{rd}$ of the observations has the mean 30. Find the mean of the whole observations.

9.Let $\underline{a}$ and $\underline{b}$ be two unit vectors, such that $\underline{a}+\alpha\underline{b}=\beta\underline{a}-\underline{b}$. Where α and β are two scalars. Show that the angle between the vectors $\underline{a}$ and $\underline{b}$ is $\cos^{-1}\left(\frac{\beta-\alpha}{2}\right)$.

10.A uniform hollow cone is suspended by a light inextensible string from a smooth peg *C* as shown in the diagram. The semi - vertical angle of the cone is θ and at the equilibrium its axis is horizontal. Show that $A\hat{C}D=2\cot^{-1}(3\tan\theta)$.

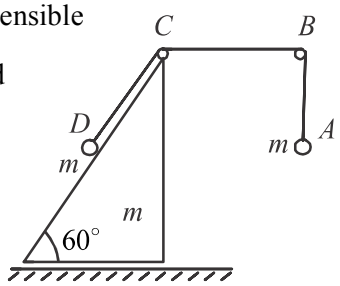


PART B

11.A lift ascending upward with uniform acceleration 2*g*, has the velocity *u* at time $t=0$. A man in the lift releases a particle *A* under gravity at the time $t=T$. At the moment the particle *A* reaches to its highest point, another second particle *B* is released under gravity by the man. On the same diagram sketch velocity time graphs of lift and the particle *A* and *B*. Hence , show that when the particle *B* comes to instantaneous rest, the velocity *A* is $3u+6gt$.

12.A smooth wedge of mass *m* is placed on a smooth horizontal table as shown in the diagram. A particle *D* of mass *m* placed on its face inclination 60° to the horizontal, is joined to one end of a light inextensible string which passes over a smooth fixed pulley *C* at the top of the face and another fixed pulley *B* on the same horizontal level as *C*, and to the other end of string is attached a particle *A* of mass *m.*

The system is released from rest. In the following motion, of the system find, the tension in the string. Also find the acceleration of the wedge and the particle *A*.



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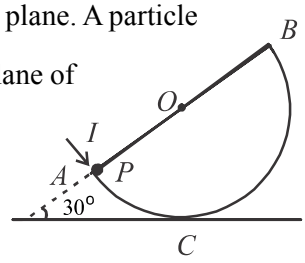
13.A smooth hemisphere of centre *O* and radius *a* is fixed to a point *C* on a fixed horizontal plane. A particle *P* of mass *m* placed at *A* is given an impulse *I*, perpendicular to *AB*, so as to move in *ABC* plane of in side surface of the hemisphere. see the diagram.



i. Find the initial velocity of the particle *P*.

ii. As the particle *P* moves between *C* and *B*, when *OP* makes an angle α with down ward vertical. show that the velocity of the particle *P* is $\left[\frac{I^2}{m^2} - ga(1-2\cos\alpha)\right]^{\frac{1}{2}}$ and the reaction exerting on the particle by the hemisphere is $\frac{m}{a}\left[\frac{I^2}{m^2} - ga(1-3\cos\alpha)\right]$.

iii.If the particle moves up to the point *B*, show that $I \geq \frac{m}{2}\sqrt{10ga}$.

14.Let *O, A, B* be three non - collinear points. The position vectors of the points *A* and *B* relative to the point *O* are $\underline{a}$ and $\underline{b}$ respectively. *D* is a point on *AB* such that $BD = 2DA$. Show that the position vector of *D* relative to *O* is $\frac{2\underline{a}+\underline{b}}{3}$.

Let $\overrightarrow{BC} = k\underline{a}$ $(k>1)$. If the points *O, D* and *C* are collinear. find the value of *k* and the ratio *OD* : *DC*. Express $\overrightarrow{AC}$ interms of $\underline{a}$ and $\underline{b}$. Also if the line through *O* parallel to *AC* meets *AB* at *E*, show that $6DE = AB$.

15.*ABCDEF* is a regular hexagen of length of a side is *a*(*m*). The forces 3, 2, 2, 3, 2, and 2 (Newtons)act along the sides $\overrightarrow{AB}, \overrightarrow{BC}$, $\overrightarrow{DC}, \overrightarrow{DE}, \overrightarrow{FE}$ and $\overrightarrow{FA}$, respectively.

i. Find the magnitude and direction of the resultant force of the system of forces.

ii. When a force *R* acting at *A* and a couple *G* are added to the system, the system is in equilibrium. Find the magnitude and direction of *G*.

iii.Now the force acting along *AB* is reversed. Explain giving reasons whether the new system of forces reduces to a couple or a single force.

16.A frame work is made with seven equal light rods jointing smoothly at their ends as shown in the adjacent diagram. The framework is hinged smoothly at *A*. Loads 10(*N*), 10(*N*) and 20(*N*) are applied at *B, D* and *C* and by applying a force *P* at *E*, making an angle θ with *CE*, the rods *BD* and *AE* are kept horizontally. If the horizontal component of *P* is $20\sqrt{3}(N)$, find the magnitude of *P*.

By using Bow's notation, draw a stress diagram to the joints *B, C, D* and *E*. Hence find the stresses in all rods stating

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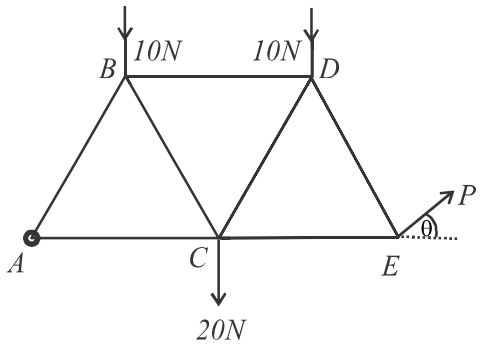
whether tension or thrust.



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