Combined Maths

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General Certificate of Education (Adv, Level) Examintaion, November 2024

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Combined Mathematics I

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Test Paper 06

* Answer **all** *Q*uestions in Part **A** and **five** *Q*uestions Only in Part **B**.

PART A

(1). Find the set of all possible real values of *x* which satisfies the inequality $\dfrac{x^2 - x + 1}{2 - x} \geq 1$.

(2). The first three terms of the expansion $\left(x + \dfrac{k}{x^2}\right)^{15}$ are x^{15}, $30x^{12}$ and Ax^9 respectively. Find the values of *A* and *k*.

(3). Show that $\dfrac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y} = \tan(x + y)$. Hence deduce that $\tan\dfrac{7\pi}{24} = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2$.

(4). Find all real solutions of $4 - x^2 = |2x - 1|$.

(5). If $x = e^{-t}(\cos t + \sin t)$. Show that $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 2x = 0$.

(6). Using a suitable substitution show that $\displaystyle\int_e^{e^2} \frac{1}{x\sqrt{\ln x}}\, dx = 2\sqrt{2} - 2$.

(7). $3 - i$ is a root of the equation $z^2 - (a + bi)z + 4(1 + 3i) = 0$. Find the values of *a* and *b*.

(8). The parametric equation of the curve *c* is given by $x = 4\cos t, y = 3\sin t$. Show that the equation of the tangent at $t = p$ is $3x\cos p + 4y\sin p = 12$. Also, this tangent cuts the *x* and *y* axes at *P* and *Q*.

Show that the area of the triangle *OPQ* is $\dfrac{12}{\sin 2p}$.

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(9). Let $g(x) = ax^2 + bx + c$. When $g(x)$ is divided by $(x-1),(x+1)$ and $(x-2)$, the remainders are 1, 25 and 1 respectively. Show that $g(x)$ is a perfect square .

(10). There are three coplanar parallel lines. If any p points are taken on each of the lines, show that maximum number of triangles with vertices on these points is $p^2(4p-3)$.

PART B

11. Let $f(x) = x^2 - px + q$, $x \in \Re$

(i). The roots of $f(x) = 0$ are α and β. Find the quadratic equation with the roots $\alpha^3 - p\alpha^2$, $\beta^3 - p\beta^2$.

(ii). Find interms of p and q, the maximum value of the function $\dfrac{1}{f(x)}$.

(iii). If the roots of $f(x) = 0$ are real and coincident, then show that the roots of the equation (i) above are also real and coincident.

12. (i). For $n\,(>1)$ positve integers, by using the principle of mathematical induction, show that

$2^{3n} - 7n - 1$ is divisible by 49.

(ii). Show that $n(n+1) = (n-1)(n-2) + 4(n-1) + 2$. Hence for positive integers $n \geq 3$ deduce that $\dfrac{n(n+1)}{(n-1)\,!} = \dfrac{1}{(n-3)\,!} + \dfrac{4}{(n-2)\,!} + \dfrac{2}{(n-1)\,!}$. Using the above results, show that the sum of infinity terms of the infinite series $1.2 + \dfrac{2.3}{1!} + \dfrac{3.4}{2!} + ... + \dfrac{n(n+1)}{(n-1)!} + ...$ is $7\,e$.

Where $e = \sum_{r=0}^{\infty} \dfrac{1}{r\,!}$.

13. (i).Using first principles, show that the derivative of e^x is e^x.

Differentiate $x.e^{2x^2}$ with respect to e^{x^2}.

(ii). If $x^2 y = a\cos nx$, show that $x^2 \dfrac{d^2 y}{dx^2} + 4x\dfrac{dy}{dx} + \left(n^2 x^2 + 2\right)y = 0$.

(iii). The parametric equation of curve C is given by $x = t, y = \dfrac{1}{t}$. The tangent drawn to the curve at

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a point A on the curve cuts the x-axis at B. Show that $OA = AB$. Where O is the origin.

Let $P \equiv (\sqrt{2}, \sqrt{2})$. The point Q lies on the tangent drawn to the curve C at A and PQ is perpendicular to AQ. Show that the coordinates of Q satisfy the equations $t^2 y + x = 2t$ and $y - t^2 x = \sqrt{2}\left(1 - t^2\right)$.

14. (i). Let $z = x + iy$ If $z^2 = a + ib$ show that $2x^2 = \sqrt{a^2 + b^2} + a$ Where x,y,a,b are real.

(ii). Let point P represents the complex number z. Sketch the path of P in the same diagram given by $|z + 2 - i| = \sqrt{5}$ and $\arg(2 + z) = \frac{\pi}{2}$. Find the complex number representing by the common point of this paths.

15. (i). Using suitable substitution find $\int_1^8 \frac{1}{x\left(1 + \sqrt[3]{x}\right)} \, dx$.

(ii). Write partial fractions of $\frac{2}{(x-1)(x-3)}$. Hence deduce the partical fractions of $\frac{4}{(x-1)^2(x-3)^2}$,

Hence find $\int \left[\frac{2}{(x-1)(x-3)}\right]^2 dx$.

(iii). Let $I_n = \int_0^2 x^n e^{4x} \, dx$. Show that $I_n = 2^{n-2} e^8 - \frac{n}{4} I_{n-1}$. Hence evaluate $\int_0^2 x^3 e^{4x} dx$.

16. (i).Given that $A = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$. Using the principle of mathematical induction prove that $A^n = \begin{pmatrix} 2^n & 2^n - 1 \\ 0 & 1 \end{pmatrix}$

(ii). $A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$ and I is the identity matrix of order 2.

(a). Show that $A^2 = 3A - 2I$.

(b). by writing $A^3 = A \times A^2$, show that $A^3 = 7A - 6I$

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17. (i). State the formula $\tan(x+y)$. When $2x+y=\frac{\pi}{4}$, show that $\tan y=\frac{1-2\tan x-\tan^2 x}{1+2\tan x-\tan^2 x}$.

Hence deduce that one of the roots of the equation $t^2+2t-1=0$ is $\tan \pi/8$ and

show that $\tan\frac{\pi}{8}=\sqrt{2}-1$.

(ii). Solve the equation $\sin^{-1}x+\cos^{-1}\frac{x}{2}=\frac{5\pi}{6}$.

(iii). Find maximum and minimum values of $\frac{1}{4\sin\theta-3\cos\theta+6}$.

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