COMBINED MATHS
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General Certificate of Education (Adv, Level) Examintaion, November 2024

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Combined Mathematics II

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∗ Answer **all** *Q*uestions in Part *A* and **five** Questions in Part *B*.

Test Paper 05

Three hours

PART A

1.It is required to keep a particle moving with a given uniform velocity v in a horizontal circle of given radius r by means of a string attached to the particle and to a fixed point vertically above the centre. Show that the point must be at a distance $\frac{gr^2}{v^2}$ above the circle.

2.A ball released from rest at a point A of height h rebounds to a height $\frac{2}{3}h$. Find the velocity with which it should be projected from A so as to just reach the point A after the rebound.

3.Two forces of magnitudes $3p$ and $2p$ respectively have resultant R. If the first force is doubled, the magnitude of the resultant is also doubled. Find the angle between the forces.

4.Engine of a car works at power $H(w)$. The resistance to motion is R. If the velocity of the car is v. Find its acceleration. Mass of car is m.

5.A particle of weight w is placed on a rough inclined plane, inclination α to the horizontal and coefficient of friction is μ. Find minimum horizontal force required to prevent the particle from sliping down the plane.

6. $\underline{i}$ and $\underline{j}$ are two perpendicular unit vectors. $\underline{p} = 3\underline{i} + 4\underline{j}$ and $q = 5\underline{i}$. If $4\underline{r} = \underline{p} + \underline{q}$ and $2\underline{S} = \underline{p} - \underline{q}$ show that $|\underline{r}| = |\underline{s}|$. show also that the vectors $\underline{r}$ and $\underline{s}$ are perpendicular each other.

7.*ABCDEF* is a lamina in the shape of a regular hexagon. Forces of magnitudes *4P, 5P, 2P, 3P, 6P, 3P* act along *AB, BC, CD, DE, EF, FA* respectively. Show that the system of forces is equivalent to a single force and find its moment about *A*.

8.Two particles are projected with the same speed from a point O on the ground inclined at α and β to the horizontal, so that they move in the same vertical plane. If the two particles fall at a point P on the ground, show that $cosec\,\alpha = sec\beta$.

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PART B

11).An express train travels from station A to its next stop at station B. The distance between the two stations is $d\ km$. The uniform acceleration and retardation of the train are $f\ kms^{-2}$ and $\lambda f\ kms^{-2}$ respectively, where λ is a positive constant. The greatest velocity that the train can attain and maintain is $v\ kms^{-1}$. The train starts from station A at rest and stops at station B in a minimum time.

(i).Draw the velocity - time graph for the motion of the train. Hence show that if $d > \frac{v^2}{2f}\left(1+\frac{1}{\lambda}\right)$ then the total time taken for the train to reach station B is $\frac{d}{v}+\frac{v}{2f}\left(1+\frac{1}{\lambda}\right)$.

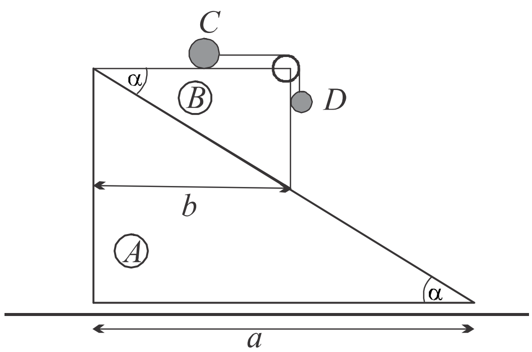
(ii).If $d < \frac{v^2}{2f}\left(1+\frac{1}{\lambda}\right)$ make the necessary modification in your velocity - time graph and hence show that in this case the total time for the train to reach station B is $\sqrt{\frac{2d}{f}\left(1+\frac{1}{\lambda}\right)}$. Find the average speed of the train.

12.An aeroplane A moves in a horizontal path with uniform acceleration f. At the moment the velocity of A is u, a shell S is fired with velocity v at an angle θ to the horizontal, from a point P on the ground which is a height h vertically below A. The areoplane and the shell both move in the same vertical plane.

After a time t_0 of fire of the shell , if A and S are in the same vertical line ,show that $t_0 = \frac{2(v\cos\theta - u)}{f}$.

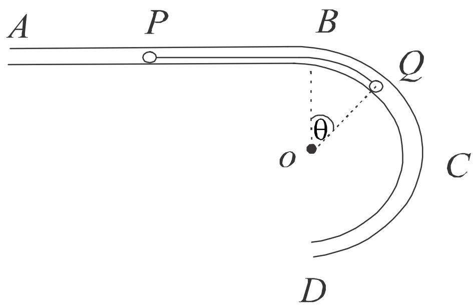
Find the vertical height ascended by S at this moment. when $u = \frac{v}{2}$, at time t_0 if the shell passes just above A, at a point Q in the space , show that $h = \frac{2u^2\,(2\cos\theta - 1)}{f^2}\left[2\,(f\sin\theta - g\cos\theta) + g\right]$.

13.A smooth wedge A of mass 4m is placed on a smooth horizontal table. Another smooth wedge B of mass $2m$ is placed on the inclined face of wedge A as shown in the diagram .



Two particles C and D each of mass m are joined to the ends of a light inextensible string passing over a smooth pulley fixed at right angled corner of wedge B. (see the diagram) The particles C and D and centres of gravity of wedges A and B lie in the same vertical plane.

The system is released from rest as in the diagram. Show that when the wedge B reaches the foot of wedge A , the distance travelled by the wedge A is $\frac{6(a-b)}{(13-\cot\alpha)}$. you may assume during the motion particle D doesnot reach the inclined face of wedge A.



14.A narrow smooth tube *ABCD* is bent into the form indicated in the figure below. The portion *AB* of the tube is straight. The portion *BCD* is of semi-circular shape with radius *a,* centre O and the diameter *BD* perpendicular to *AB*. The tube is fixed in a vertical plane with *AB* horizontal and uppermost.In side the tube there is a particle *P* of maass *m* and a particle *Q* of mass 3*m* connected by a light inextensible string of length $l\left(>\frac{\pi a}{2}\right)$. Initially, the string is taut, lying in *AB,* with the particle *Q* at the point *B*.The particle *Q* is slightly displaced from this position, and in time *t*, radius*OQ* turns through an acute angle θ.

Applying the principle of conservation of energy, show that $\left(\frac{d\theta}{dt}\right)^2=\frac{3g}{2a}(1-\cos\theta)$. Hence, or otherwise, show that the acceleration of the particle *P* is $\frac{3g}{4}\sin\theta$. Find the reactiion from the tube on the particle *Q* and the tension in the string, at time *t.*

15.A heavy ring of radius *a,* mass *M* hangs in a vertical plane from a rough peg. An insect creeps up along the ring starting from the lowest point of the ring. Show that, the insect can reach the peg only if its mass is smaller than $\frac{M\sin\lambda}{(1-\sin\lambda)}$. Where λ is the angle of friction between the peg and the ring. If the mass of the insect is greater than $\frac{M\sin\lambda}{1-\sin\lambda}$, show that the distance the inscect can go along the ring, before slipping the ring is

$$a\left\{\lambda+\sin^{-1}\left(\frac{(m+M)\sin\lambda}{m}\right)\right\}$$

16.Three weightless rods , *AB, BC*, *CD* of length *l* , *2d*, *l* are smoothly jointed at *B* and *C*. Two particles each of weight *w* are suspended from *A* and *D*. The system is placed symetrically on the smooth curved surface of a fixed cylinder of radius $d\sqrt{3}$ and axis is horizontal, so that *BC* in contact with the cylinder and horizontal. Find the reaction on *BC* by the cylinder and show that the condition required for the rod *BC* in contact is $l<4d$. If $l>4d$ and *AB* makes an angle α with the horizontal , show that $l\cos^3\alpha=d\left(\sqrt{3}\sin\alpha-1\right)$. Find the reaction at the joints *B* or *C*.

17. State (i). The law of conservation of momentum.

(ii). The Newton's law of restitution

as applied to the collision of particles.

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Three equal smooth elastic particles P, Q and R are at rest at the points A, B and C respectively on a smooth horizontal table so that B is the mid-point of AC. The particle P is projected along the table with velocity u and collides directly with the particle Q. If the particle P is involved in two collisions and that the particle R is involved in only one collision show that $e > 2 - \sqrt{3}$. Where e is the coefficient of restitution.

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