Linear Motion And Velocity Time Graph

1.A particle starts from rest from a point A with uniform acceleration. particle passes a point B on its path and in another 6 (s) it travels 156(m) and then passes a point C on its path .In another 4 (s) it traveles 144(m). Find the acceleration of the particle. Also find the time taken to move from A to B and the distance AB.

2.Along a straight path the distance between the points A and B is 28 (m). A body moving with constant acceleration takes 4(s) to move from A to B. Soon after reaching point B the acceleration of the body is increased by $1(ms^{-2})$. After another 6(s) it passes a point C on its path with velocity $21(ms^{-1})$. Find the initial velocity of the body and its acceleration. Also find the distance between the points B and C.

3.i.A small body is projected with velocity $40ms^{-1}$ vertically upwards from a point on the ground. Find the times it takes to pass a point at a height 75(m) above the ground.

ii.From a point on the edge of a well a body is projected vertically upwards with velocity $30ms^{-1}$. After 4 (s) another body is dropped from the same point in to the well. If the two bodies collide each other, find the depth of the point of collision.

4.A helicoptor moving upwards with constant velocity $20ms^{-1}$ drops a bomb at a height 580(m) above the ground. when bomb is at its maximum height, a soldier on the ground fires a shell vertically upwards with the initial vertical velocity $120ms^{-1}$ with the intention of blasting the bomb in mid air. If the bomb blasts , find the height to the point of blasting, and the time since the moment of firing the shell.

5.A train starting from rest moves a distance 37.5 (m) and acquires a velocity $54(kmh^{-1})$. Find the constant acceleration of the train and time taken. Now the train maintains this velocity until it travels another 150(m) and then under constant retardation it comes to rest in 10(s). Find the constant retardation of the train and the distance travelled with constant retardation.

6.A train starting from rest moves with constant acceleration in first 2(min) and with uniform velocity 5(min). Finally the train comes to rest with constant retardation in 3 (min). If the maximum velocity of the train is $36(kmh^{-1})$,
Find

i) acceleration of the train
ii) distance travelled with acceleration
iii) distance travelled with uniform velocity
iv) retardation of the train
v) distance travelled with retardation

7.A train of length 66(m) passes a signal post in 6(s). If the rear of the train passes the signal post with $17(ms^{-1})$. Find the constant acceleration and initial velocity of the train.

8.A particle is projected vertically upwards with initial velocity $100(ms^{-1})$ from a point O on the ground. Find the maximum height ascended by the particle. Show that the particle passes a point a height 180(m) above O at times 2(s) and 1(s) of projection. Find the time taken by the particle to reach a point a depth 220(m) below the point O. Also find the velocity of the particle at this moment.

9.Initial velocity of a body moving with constant acceleration of $4cm/\sec^2$ is $10cm/\sec$. Find the velocity after 8 seconds and also the distance travelled in this time. What is the velocity after it has covered $19\frac{1}{2}cm$?

10.A train which is moving at the rate of $60m/\sec$ is brought to rest in 3 minutes with uniform retardation. Find the retardation and also the distance that the train travels before coming to rest.

11.If a body moving with uniform acceleration passes over *200 m* while the velocity increases from 30 to 50 *m/sec*, find the acceleration and the time of motion.

12.A particle starts with a velocity of *20 m/sec* and travels *300 meters* in *30 secs* along a straight line.
Find
(i). the acceleration supposed uniform,
(ii). the time at which it is $166\frac{2}{3}$ metres from the starting point,
(iii). the time when it comes to rest.

13.The velocity of a particle, which is moving in a straight line with contant retardation, decreases *10 cm/sec* while the particle travels *10 cm* and *15 cm/sec* while it travels $12\frac{1}{2}cm$ cm from the starting point. Find the distance the particle travels from the starting point till it comes to rest.

14.A particle starting with a given velocity moves for 3 seconds with constant acceleration during which time it decribes 81 *cm*. The acceleration then ceases and during the next 3 seconds, it describes 72 *cm*.
Find the initial velocity and acceleration.

15.A car is moving at 158.4 *km* per hour when passing one lamp- post and at 79.2 *km* per hour when passing the other. If the lamp-posts are 330 *metres* apart, how far will the car travel before it comes to rest if this retardation is maintained? How long after passing the first lamp-post will the car be moving?

16.A train is moving with a speed of 66 *m/s* and the brakes produce a retardation of 4 m/sec^2. At what distance from a station should the brakes be applied so that the train may stop at the station? If the brakes are put on at half the distance, with what speed will the train pass the station?

17.A bullet fired into a target loses half its velocity after penetrat ing *3 cm*. How much further will it penetrate ?

18.The brakes of a train reduce its speed from 60 *km p.h.* to 20 *km p.h.* while it runs 125 *m*. Assuming that they exert a constant retardation force, find how much further will the train run before coming to rest and how long will it take ?

19.The speed of a train is reduced from 60 meters per second to *15 metres* per second whilst it travels a distance of *450metres*. If the retardation be uniform, find how much further will it travel before coming to rest.

20.In a certain interval of 10 seconds, a point passes over 220 metres, in the next interval of *5 seconds*, it passes over *330 metres*. If the point is moving with uniform acceleration, find its velocity at the beginning of each of the two intervals.

21.A body starting with some initial velocity and moving with uniform acceleration acquires a velocity of 20 *cm/sec* after moving through 10 *cm* and a velocity of 30 *cm/sec*. After moving through a further 15 *cm*. When and where will its velocity be 40 *cm/sec*?

22.A particle moving in a straight line with constant acceleration passes in succession through three points *A, B, C* the distances *AB* and *AC* each being equal to *12 cm*. The particle takes 1*sec* to travel from *A* to *B,* and *2 secs* to travel from *B* to *C*. Determine the point at which the particle comes to rest and the point at which its velocity is *8 cm/sec*.

23.A point moves in a straight line with uniform acceleration and describes distances *a* and *b cm* in successive intervals of time t_1 and t_2 seconds. Prove that the acceleration is $\dfrac{2(bt_1 - at_2)}{t_1t_2(t_1+t_2)}$.

24.A particle moving with uniform acceleration in a straight line passes over *P, Q, R*. If $PQ = QR = b$ and if the time from *P* to *Q* is t_1 and from *Q* to *R* is t_2, prove that the acceleration is $\dfrac{2b(t_1 - t_2)}{t_1t_2(t_1+t_2)}$.

25.If a point moving under uniform acceleration describes successive equal distances in times t_1, t_2, t_3 respectively, then prove that $\dfrac{1}{t_1} - \dfrac{1}{t_2} + \dfrac{1}{t_3} = \dfrac{3}{t_1+t_2+t_3}$.

26.Two particles *P* and *Q* move along a straight line *AB*, starting from *A*. *P* moves with velocity *u* and acceleration *f*; *Q* with velocity u' and acceleration f'. If they both have the same velocity at the middle point of *AB*, prove that $AB = \dfrac{u'^2 - u^2}{f - f'}$.

27.A bullet travelling horizontally pierces in succession three screens placed at equal distances *'a'* apart. If the time from the first screen to the second be t_1 and from the second to the third be t_2, prove that the retardation [assumed to be constant] is $\dfrac{2a(t_2 - t_1)}{t_1t_2(t_1+t_2)}$ and that the velocity at the middle screen is $\dfrac{a(t_1^2 + t_2^2)}{t_1t_2(t_1+t_2)}$.

28.A particle is moving with uniform acceleration. In the eleventh and fifteenth seconds from the begining, it travels *33 metres* and *39 metres* respectively. Find tthe initial velocity and the acceleration with which it moves. Find also the distance covered by it in *20 seconds* and the distance covered in 20th second.

29.If *x, y, z* be the distances described by a particle during the pth, qth and rth second respectively, prove that

(i). $(q-r)x + (r-p)y + (p-q) = 0$

(ii). $p(y-z) + q(z-x) + r(x-y) = 0$

30.If x_1 and x_2 be the distances described by a particle during the pth and $(p+q)th$ second of its motion respectively, prove that its accleration is $\dfrac{(x_2 - x_1)}{q}$.

31.Two cars start off to race with velocities v_1 and v_2 and travel in a straight line with uniform accelerations f_1 and f_2. If the race ends in a dead heat, prove that the length of the course is $\dfrac{2(v_1 - v_2)(v_1f_2 - v_2f_1)}{(f_1 - f_2)^2}$.

32.A point moves with uniform acceleration. If v_1, v_2, v_3 be the average velocities in three successive intervals of time t_1, t_2, t_3 prove that $\dfrac{v_1 - v_2}{v_2 - v_3} = \dfrac{t_1 + t_2}{t_2 + t_3}$.

33.A particle is projected in a straight line with a certain velocity and a constant acceleration. One second later, another particle is projected after it with half the velocity and double the acceleration. When it overtakes the first, the velocities are 22 and *31 cm/sec* respectively. Prove that the distance travelled is *48 cm*.

34.A cyclist passes a car that is just starting with an acceleration of $2m/\sec^2$. The speed of the cyclist is *45 km* per hour and the car maintains its acceleration for *10 secs* and then moves uniformly. How far will the car have run before overtaking the cyclist ?

35.A bus is begining to move with an acceleration of $1m/\sec^2$, show that

(i).A man who can run at *9 m/sec* will catch the bus in *8 secs* if he is *40 metres* behind when it starts.

(ii).If the man is 40.5 metres behind, he will only just catch the bus.

(iii). If he is *50 metres* behind, he will never catch the bus.

36.A particle moves in a straight line with constant acceleration and its distances from the origin *O* on the line (not necessarily the position at time $t=0$) at times t_1, t_2, t_3 are x_1, x_2, x_3 respectively. Show that if t_1, t_2, t_3 form an *A.P.* whose common difference is *d* and x_1, x_2, x_3 are in *G.P.* then the acceleraton is $\dfrac{\left(\sqrt{x_1}-\sqrt{x_3}\right)^2}{d^2}$

37.A particle moving in a straight line with uniform acceleration describes 25 metres in the fifth second and 33 metres in the seventh second. Find the initial velocity and acceleration.

38.Two particles *A* and *B* distance 16 metres apart move towards each other. *A* moves with constant velocity of *5 m/sec* and *B* with constant acceleration of *3 m/sec²*. Find when and where they meet.

39.Two particles start moving in a straight line simultaneously from the same point. The first particle moves with a constant velocity *u* and the second starts from rest and moves with a constant acceleration *f*. Prove that before the second particle overtakes the first, the distance between the two particles is maximum after a time $\dfrac{u}{f}$ and is equal to $\dfrac{u^2}{2f}$.

40.A bullet is fired through three screens placed at equal intervals of a metres and the times of passing the screens are t_1, t_2, t_3 seconds from the moment the bulet leaves the gun. Assuming that retardation is uniform, prove that it is equal to $\dfrac{2a\left(t_1-2t_2+t_3\right)}{\left(t_3-t_1\right)\left(t_3-t_2\right)\left(t_2-t_1\right)}$.

41. A body is projected vertically upwards with a velocity of *24.5 m/sec.*

(i). How high will it go ?

(ii). How long will it take to return to the point of projection ?

(iii). When will its velocity be *4.9 m/sec*?

(iv). At what time will be *29.4 m* above the point of projection?

(v). *29.4 m* below the point of projection. ($g=9.8ms^{-2}$)

42.A body projected vertically downwards describes *220.5 m* in *t secs* and *686 m* in *2t secs*. Find *t* and the velocity of projection.($g = 9.8ms^{-2}$)

43.An arrow is shot vertically upwards with a velocity of *49 metres/sec*. Find,

(a). the velocity at the end of 8 secs.

(b). the maximum height reached,

(c). the time taken to be at a height of 117.6 m.($g = 9.8ms^{-2}$)

44.A body falls freely from the top of a tower and during the last second of its motion, it is observed to fall $\frac{16}{25}$ of the whole distance. Find the height of the tower. $(g=10ms^{-2})$

45.A particle is projected vertically upwards with a velocity of *u m/sec* and after *t* secs another particle is projected upwards from the same point and with the same velocity. Prove that particles will meet at a height $\frac{4u^2 - g^2t^2}{8g}$ *metres* after a time $\left(\frac{t}{2} + \frac{u}{g}\right)$ *secs*.

46.A particle is dropped from the top of a tower *h metres* high and at the same moment, another particle is projected upwards from the bottom. They meet when the upper one has descended $\frac{1}{n}$ of the distance. Show that the velocities when they meet are in the ratio $2:(n-2)$ and that initial velocity of the lower is $\sqrt{\frac{1}{2}ngh}$.

47.*A, B, C, D* are points in a vertical line, the length $AB = BC = CD$. If a body falls from rest at *A*, prove that the times of describing *AB, BC, CD* are as $1:\sqrt{2}-1:\sqrt{3}-\sqrt{2}$.

48.A particle projected vertically upwards, takes *t secs* to reach a height *h ft*. If t' *secs* is the time from this point to the ground again, prove that

(i). $h = \frac{1}{2}gtt'$ and that the maximum height is $\frac{g(t+t')^2}{8}$.

(ii).Show also that the velocity of the particle at a height $\frac{1}{2}h$ is $\frac{1}{2}g\sqrt{t^2 + t'^2}$.

49.Two balls are projected simultaniously with the same velocity from the top of a tower, one vertically upwards and the other vertically downwards. If they reach the ground in times t_1, t_2 respectively, show that $\sqrt{t_1t_2}$ is the time which each will take to reach the ground if simply let drop from the top of the tower.

50.A particle takes *t* seconds less and acquires a velocity of *s metres/sec* more at one place than at another in falling from rest through the same distance. Show that the gravities g_1 and g_2 at the two places are related by $g_1g_2 = \frac{s^2}{t^2}$.

51. A body travels a distance s in t (s). It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. Show that $t=\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$.

52.A lift performs first part of its ascent with uniform acceleration f and remainder with uniform retardation $2f$. Prove that if h is the height ascended and t, the time of ascent then $h=\frac{1}{3}ft^2$.

35.A train takes time T to perform a journey. It travels for time $\frac{T}{n}$ (s) with uniform acceleration, then for time $(n-3)\frac{T}{n}$ (s) with uniform speed V and finally for time $\frac{2T}{n}$ (s) with constant reardation.

Prove that the average speed is $(2n-3).\frac{V}{2n}$.

53.For $\frac{1}{p}$ of the distance between two stations, a train is uniformly accelerated and for $\frac{1}{q}$ of the distance, it is uniformly retarded; it starts from rest at one station and comes to rest at another station. Prove that the ratio of its greatest velocity to its average velocity is $\left(1+\frac{1}{p}+\frac{1}{q}\right):1$.

54.The speed of a train increases at a constant rate α from 0 to v, then remains constant for an interval and finally decreases to 0 at a constant rate β. If l be the total distance described, prove that the total time occupied is

$$\frac{l}{v}+\frac{1}{2}v\left(\frac{1}{\alpha}+\frac{1}{\beta}\right).$$

55.A body is subject to uniform acceleration. If t repressents the whole time of motion to be considered and if a be the space described in the first m seconds and b the space described, in the last m seconds and s the total space described prove that $s=\frac{(a+b)t}{2m}$.

56. A train starting from rest moves with uniform acceleration f in the first stage, with uniform velocity v in the second stage.Finally with uniform retardation f in the third stage and comes to rest. If the average velocity for the whole journey is $\frac{7v}{8}$, show that the train travels with uniform velocity during the time $\frac{3}{4}$ of total time. Also find which fraction of total distance travelled by the train with constant veloctiy.

57. An intercity express train travels between the two stations A and B with unifrom velocity u (ms^{-1}). On a certain day due to repairs of the track, it retards with uniform rate $f_1$ms^{-2} and comes to rest at a signal post C between A and B. The train stops at C in time t_o and begins to travel with uniform acceleration f_2. Afterwards the train continues the journey with uniform velocity u (ms^{-1}). Sketch the velocity-time graph for the motion of the train. If the train delays a time T due to stop at C. Find an expression for T. If $f_1\le f, f_2\le f$,show that the minimum time of T is $t_o+\frac{u}{f}$

58.The front and the rear of a train moving with uniform acceleration, pass a signal post with velocity u and v respectively.Show that, in first half of the time taken by the train to pass the signal post, it moves a distance $\frac{3u+v}{4(u+v)}$ of its length.

59. A train starts from rest at a station A and moves to another station B. The train moves $\frac{1}{3}$ of AB with uniform acceleration f_1, thereafter uniform velocity v, the final $\frac{2}{5}$ of AB moves with uniform retardation f_2 and comes to rest at station B. Show that the time taken by the train to reach station B is $\frac{13v}{11}\left[\frac{1}{f_1}+\frac{1}{f_2}\right]$

60. On a straight road, the distance between the points A and B is $2a$. There is a narrow groove at mid-point C of AB. A car moves at speed u ms^{-1} at the point A. It uniformly decreases the speed and comes to the point C at speed v (ms^{-1}). Due to the jerk in the groove at C, the speed is suddenly reduced by $w(<v)$ ms^{-1}. After wards the car moves with constant retardation in C-B and comes to rest at B. Draw velocity-time graph for the motion of the car. Show that the time taken by car for the journey A-B is $2a\left[\frac{1}{v+u}+\frac{1}{v-w}\right]$. Find the retardations of car in AC, CB and show that they become the same if $w = v-(u^2-v^2)^{\frac{1}{2}}$

61.A train moves between two positions a distance d apart. The train starts from rest and first part of its journey moves with uniform acceleration f, second part with uinform velocity u and final stage with uniform retardation f'. Show that the time the train moved with uniform velocity is $\frac{d}{u}-\frac{u}{2}\left[\frac{1}{f}+\frac{1}{f'}\right]$. If the train doesnot acquire the maximum velocity u, show that the total time for the journey is $\sqrt{2d\left(\frac{1}{f}+\frac{1}{f'}\right)}$.

62. A particle starting from rest, moves a distance a with uniform acceleration, and then uniform velocity and finally with uniform retardation. The retardation is twice the acceleration. The total distance travelled by the particle is b and total time taken is T. Sketch the velocity - time graph of the motion of the particle. Hence, show that

i) The maximum velocity of the particle is $\left(\frac{3a+2b}{2T}\right)$.

ii) The distance travelled with uniform velocity is $\left(\frac{2b-3a}{2}\right)$.

63. A motor car starting from rest moves along a straight track with a constant acceleration $a\ ms^{-2}$. At a point A, on the track ,the velocity of the car is $u\,(ms^{-1})$ and driver sees an obstacle ahead. But he further moves with the same acceleration another time T (s) and comes to rest at a point B with constant uniform retardation $2a(ms^{-2})$. If $AB = d\,(m)$, Show that $u^2+6aTu+3a^2T^2-4ad=0$.

64.A lift ascends from A to B. AB=h(m). In the first stage of the journey it moves with acceleration f_1 and attains the velocity $u\,(ms^{-1})$.Afterwards it ascends with velocity u (ms^{-1}) over a time t(s). Finally it ascends with retardation f_2(ms^{-2}) and comes to rest at B. If T is the total time taken by the lift for whole journey, show that

$$T=\frac{u}{2}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)+\frac{h}{u}$$

65. A lift ascends from rest with uniform acceleration a and then moves with uniform velocity u. Finally it comes to rest with retardation $3a$. The lift ascends a height h and total time taken is t. Show that the time during which the lift moved with uniform velocity is $\left(t^2 - \frac{8h}{3a}\right)^{\frac{1}{2}}$. Find also the time during which the lift moved with retardation.

66. A motor car moves with acceleration a_1 starting from rest attains its maximum velocity and immediately begins to move with retardation a_2. The car comes to rest after travelling a distance S. Show that the time elapsed is $\left[\frac{2S(a_1 + a_2)}{a_1 a_2}\right]^{\frac{1}{2}}$. Obtain the maximum velocity if the maximum acceleration and retardation is a, show that the minimum time for the motion is $2\sqrt{\frac{S}{a}}$.

67. *i* The distance between two bus stops is S. A bus starts from A and comes to rest at B. The maximum acceleration and retardation of the bus are a_1 and a_2 respectively. Show that the minimum time taken by the bus to travel the distance S is $\left[\frac{2S(a_1 + a_2)}{a_1 a_2}\right]^{\frac{1}{2}}$.

ii. In the above case, it is given that the maximum velocity of the bus is V. Due to repairs to road the drivers are asked not to travel with speed more than v_0 $(v_0 < v)$ Show that the minimum additional time taken to travel from A to B is $\frac{S(v - v_0)^2}{v_0 v^2}$.

68. As a train A passes a station P with its maximum velociy, a train B starts from rest and moves with uniform accelaration f. After wards it travels with maximum velicity and comes to rest at Q subject to uniform retardation f. The train A comes to Q with unifrom retardation f, a time t before B comes to Q. If the maximum velocity of the trains is V, by using a velociy time graph show that $t = \frac{V}{2f}$.

69. X is an intercity express train runs between the stations A and B. It starts with acceleration f_1 ms^{-2} and moves with maximum velocity v ms^{-1}. Finally it comes t_o rest with retardation f_1 ms^{-2}. Y is a semi-express train runs between A and B. It stops at the station C, between A and B for a time t_o. The train y starts with acceleration f_2 ms^{-2} and then moves with maximum velocity u ms^{-1}($>v$). there after comes to rest with retardation f_2 ms^{-2}. The trains X and Y leave simultaniously from A and reach on B simultaneously. If Y has travelled with maximm velocity a time t_1 between A and C and a time t_2 between C-B. Sketch velocity time graphs for X and Y on the same diagram.

Hence show that $(t_1 + t_2)(u - v) = \frac{v^2(2f_1 - f_2)}{f_1 f_2} + t_o v - \frac{2(u - v)^2}{f_2}$

70. Two trains A,B moving on stranight parallel lines to the same direction with uniform accelerations $3f$ (ms^{-2}), f ms^{-2} respectively, pass a station S_1 at time t_1 with velocities u ms^{-1} and $2u$ ms^{-1} respecitvely. The train A maintains the acceleration $3f$ in the time $(t_2$-$t_1)$ and it travells with uniform velocity it has acquired at time t_2(s). At time t_2(s) the trains A and B pass together a station S_2. Afterwards at time t_3(s) the trains pass together the station S_3. Sketch the velocity-time graphs of the trains and hence show that.

i. $t_2 - t_1 = \frac{u}{f}(s)$

ii. The velocities at time t_2(s) of the trains A and B are $4u$, $3u$

iii. $t_3 - t_2 = \frac{2u}{f}(s)$

iv. The velocity of B at time t_3(s) is $5u(ms^{-1})$

v. The distance between the stations S_1 and S_3 is $\frac{21u^2}{2f}(m)$

71.A bus moves with velocity u on a straight road. When the bus reaches a point A on the road the driver sees a pedestrian requesting to board the bus at a point H a ditance $4l$ ahead. The driver applies brakes at the points A,B,C successively. So that the retardations in the intervals AB, BC, CH are a, $2a$, $3a$ respectively and stops at H. Where $AB = CH = l$ and $BC = 2l$. Draw velocity time graph for the motion of the bus.

Hence show that $u = 4\sqrt{al}$. Show also that the total time from A to H is $\left[24 - 3\sqrt{14} - \sqrt{6}\right]\frac{2l}{3u}$.

72. A train of length l is travelling along a straight track with constant acceleration f and has a maximum speed $2v$. A car travelling along a road parallel to the track in the same direction as the train has a constant acceleration $2f$ and a maximum speed $3v$. Initially the rear of the train is level with the front of the car and the speeds of the train and the car are v and $\frac{v}{2}$ respectively. The train and the car attain their maximum speeds at times t_1 and t_2 respectively and the front of the train is level with the front of thecar at time $t_3(>t_2)$. Draw the velocity-time curves for the motions of the train and the car on the same diagramand hence deduce that the rear of the train is again level with the front of the car at time t_1.

Show that $t_3 = \frac{l}{v} + \frac{17v}{16f}$ and that $3v^2 < 16fl$.

73.Two stations A and B are at a distance c (km) apart. A train P passes the station A with velociy v kmh^{-1} and maintains this velocity a distance a (km). After wards it comes to rest at B with uniform retardation. Another train Q starts from rest at the station A, a time T (h) before the train P leaving station A. The train Q accelerates uniformly until it acquires the velocity $\lambda v(kmh^{-1})$ and it comes to rest at B under a uniform retardation as the train P stops at B. Sketch v - t graph for the motion of A and B. Hence

(i). Show that $T = \frac{2c}{\lambda v} + \frac{a}{v} - \frac{2c}{v}$ and deduce $\lambda < \frac{2c}{2c - a}$

(ii). Show that the retardation of P is $\frac{v^2}{2(c-a)}kmh^{-2}$.

(iii).If the retardation and acceleration of the train Q are f_2 and f_1 show that $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2c}{\lambda^2 v^2}$

74.A van is travelling along the straight highway from a town A to town B. At time $t=0$, the van starts from rest at A and then moves with constant acceleration f to attain its maximum speed $2u$. When $t = \frac{u}{2f}$, the van passes a traffic police car at C between A and B. Immdiately the driver of the police car notices the van and after a further time $\frac{u}{2f}$ the police car starts from rest at C nd moves with constant acceleration $2f$ to catch the van as early as possible. The van maintains its maximum speed for a period of time $t_o (> 0)$ and then it comes to rest at B under a constant retardation $2f$. If the maximum speed of the police car is $3u$ which it maintains until the van is overtaken, draw the velocity-time curves for the motion of the van and police car on the same diagram. Hence show that the police car can over-take the van during their maximum speeds if $t_0 \geq \frac{9u}{8f}$. If $t_0 < \frac{9u}{8f}$ and the police car over-takes the van when $t = \frac{2u}{f} + t_o + t_1$, where $O < t_1 < \frac{u}{f}$ Show that $t_1 = \left(\frac{11u^2 - 8fut_o}{8f^2} \right)^{\frac{1}{2}} - \frac{u}{2f}$.

75. A motor car A, starting from rest at a point O, moves with constant acceleration f and acquires the maximum velocity $2u$. At the moment A starts at the point O, a motor car B moving with uniform velocity u, passes A.
Sketch velocity- time graphs for the motions of the motor cars.

If A and B meet after travelling a distance $x \left(> \frac{2u^2}{f} \right)$, show that,

i. the motor car A, has travelled with uniform velocity a time $\frac{xf - 2u^2}{uf}$.

ii. $2u^2 = xf$

76. A policeman notices a motor car travels with high speed u. As the car passes him, he sets off to chase the car by his motor-bicycle. He rides with uniform acceleration f until attains the maximum velocity v. Policeman overtakes the car having travelled a distance $a \left(> \frac{v^2}{2f} \right)$ from his initial point of start. Show that, by using velocity-time graph or otherwise the policeman has travelled with maximum velocity a time $\left(\frac{a}{u} - \frac{v}{f} \right)$.

Find a relation among u, v, a and f and hence, show that $v = \frac{af}{u} \left[1 - \left(1 - \frac{2u^2}{af} \right)^{\frac{1}{2}} \right]$ explain clearly the way you attained to this answer.

77. A lorry is moving with uniform velocity u. As it passes a point A, a motor car moving to the same direction with uniform acceleration f, passes the point A with velocity $\frac{u}{2}$. The motor car accelerates untill its velocity λu ($\lambda > 1$) and then moves with the retardation f. Show that if $\lambda < 1 + \frac{\sqrt{2}}{4}$ the motor car cannot overtake the lorry.

78. At the instant train B leaves a station with unifrom acceleration f, another train A moving with uniform velocity u passes the station. Both trains travel in the same direction on parallel lines. The train B is accelerated until its velocity ku ($k>1$) and retarded with uniform retardation f to stop at next station. By sketching v-t graphs for the trains, show that, if $k < 1 + \frac{1}{\sqrt{2}}$, B cannot overtake A.

79.Two bodies A, B move to the same direction, pass together a point with velocities u and v ($<u$) respectively and with uniform retardations f and f' ($<f$) respectively. Show that after a time $\frac{2(u-v)}{f-f'}$, B overtakes A.

Show also that the maximum distance between A and B during this period of time is $\frac{(u-v)^2}{2(f-f')}$

80.Two motor cars A and B move in opposite directions along a narrow road with velocities u and $2u$ respectively. When the distance between them is d, the drivers realise that the cars are going to collide.Now they apply brakes and moving with uniform retardations, they just prevent the collision.

By using a velocity time graph , find

i. the time taken by the cars to come to rest.

ii. uniform retardations of the cars.

iii. the distances travelled by the cars before coming to rest.

81.A particle started from rest from point O at t=0 moves on a straight line. It moves a time t_1 with acceleration a_1, time t_2 with retardation a_2 and again with acceleration a_1. If $a_2t_2 > 2a_1t_1$, sketch the graph of the motion. Hence show that particle moves twice across the point O. If $a_2t_2 = 2a_1t_1$ deduce by using the graph that the time taken by the particle to come to O at second time is $2t_1 + t_2$.

82.Trials are being undertaken on a horizontal road to test the performance of an electrically powered car. The car has a top velocity v. In a test run the car moves from rest with unifrom acceleration a and is brought to rest with uniform retardarion r.

1.If the car is to achieve top velocity during a test run, by using a velocity- time sketch, show that the length of the test run must be at least $\frac{v^2(a+r)}{2ar}$

2. Find the least time taken for a test run of length

a. $\frac{2v^2(a+r)}{9ar}$ *b.* $\frac{2v^2(a+r)}{3ar}$

3. Find, in terms of v the average velocity of the car for the test run described in 2. *b*.

83.A particle is projected vertically upwards with velocity u from the ground at time t=0. If t_1 and t_2 are the times at which it passes a point at a height h above the ground in ascending and decending respectively.

Sketch the velocity-time curve for the motion of the particle. Hence find the velocity of the particle at time $\frac{t_1+t_2}{2}$

and deduce that $t_1t_2 = \frac{2h}{g}$.

84.A is a point on the ground. B is a point in the space vertically above A at a height $\frac{23u^2}{2g}$. At t=0 a particle P is projected with velocity $4u$ vertically upwards from A. At $t = \frac{u}{g}$ another particle Q is projected downwards with velocity u

from B. Sketch velocity time graph for P and Q in the same diagram. Hence show that P and Q meet at a height $\frac{15u^2}{2g}$ from A.

85. A helicopter moves vertically upwards with uniform acceleration $\frac{g}{2}$ starting from rest at a point O on the ground. After a time t a bomb is dropped from the helicopter. When the bomb falls to the ground, show that by using velocity-time graph the height to the helicopter from O is $\frac{3(2+\sqrt{3})t^2 g}{8}$.

86.A balloon begins to ascend vertically upwards from rest with uniform acceleration $\frac{g}{8}$ from a point on the ground. After a time t a particle is released from the balloon and due to this the acceleration of the balloon increases up to $\frac{g}{4}$. After another time $\frac{t}{8}$ the power of the balloon is cut and it begins to move under gravity. Using same axes, sketch v - t graphs for the motion of the balloon and the particle. Hence show that the particle and the balloon acquire the same velocity after a time $\frac{77t}{64}$ from the start.

87.At time $t = 0$ a balloon is released from rest from a point on the ground. It rises vertically upwards with uniform acceleration g. At time $t = T$, a stone is thrown vertically upwards from the same point on the ground with velocity u. Sketch the velocity - time graphs for the motions of the balloon and the stone on the same diagram and hence show that the stone just touches the balloon if $u = (1+\sqrt{2})\,gT$. Find the height to the balloon from the ground when the stone falls back to the ground.

88.A particle A is projected vertically upwards with velocity u, from a point on the ground. As it reaches half of its maximum height, it collides with a stationary particle and coaleses. After the collision, if the velocity of the combined object reduces to half, Show that the combined object reaches the ground after time $\left[\frac{\sqrt{5}+2\sqrt{2}-1}{2\sqrt{2}}\right]\frac{u}{g}$.

89.A rocket is set to move vertically upwards with uniform acceleration g from rest at a point on the ground. After a time t a particle A is released from the rocket. when the particle A reaches its maximum height another particle B is released from the rocket. Sketch the v - t graphs for the motions of the rocket and the particles A, B. Hence find the velocity of A when B is at the maximum height. Show also that after a time $3t$ from start, the distance between A and B is $3gt^2$.

90. A helicopter descends vertically downwards with uniform velocity u. At time $t = 0$ a particle P is projected from helicopter vertically downwards with velocity v relative to the helicopter. At time $t = T$ another particle Q is projected vertically down wards from the helicopter with velocity $2v$.

At time $t = \frac{8T}{3}$, the particles P and Q meet each other: In separate diagrams sketch

(i) v - t graph of P relative to helicopter in the interval $0 \le t \le T$

(ii) v - t graph of Q relative to P in the interval $T \le t \le \frac{8T}{3}$. Hence prove that $T = \frac{4v}{13g}$. when the particles meet, find the velocities of the particles by using equations of motion.

91.A particle is projected vertically upwards with velocity u. After a time T, another particle is projected vertically upwards with the same velocity from the same point. For the cases $O < T < \frac{u}{g}$ and $\frac{u}{g} < T < \frac{2u}{g}$

Sketch v - t graphs of the motions of the two particles in separate diagrams.
Hence show that

(i) After a time $\frac{u}{g} - \frac{T}{2}$ of projection of the second particle, the particles meet .

(ii) The speeds of the particles just before the collision are equal and independent of u.

(iii) The height to the point where the particles collide, from the ground is $\left(\frac{u^2}{2g} - \frac{gT^2}{8}\right)$.

92.A particle P is projected vertically upwards with velocity $4u$ from a point O on the ground. As the particle P reaches its maximum height, it is given a velocity u vertically downwards. At the same instant another particle Q is projected vertically upwards with velocity v ($< 4u$) from the same point O on the ground After a time $\frac{2u}{g}$ of projection of Q, the particles meet each other.

Sketch the velocity - time graph of the motions of the particles and hence show that $v = 3u$.
Deduce that when both the particles P and Q move the sum of their speeds is $4u$. show that, when P and Q collide, the velocity of Q is u and deduce the velocity of P at this moment. show also that the particles collide at a height half of the maximum height of particle P.

93.An elevator starts its motion from rest at time $t = 0$ and moves vertically upwards with uniform acceleration a.A man who is in elevator releases a particle P from rest under gravity at time $t = t_0$ .At the instant when the particle P reaches its maximum height, a second partricle Q is released from rest under gravity . Sketch the velocity time graphs for the motions of the elevator and the two particles P and Q on the same diagram.

Hence , show that at the instant when Q comes to instantaneous rest, the velocity of P is $at_0\left(\frac{a}{g} + 1\right)$

94.A particle P of mass M is projected at time $t = 0$ vertically up wards under gravity with velocity u from a point on the ground. Three particles P_1, P_2 and P_3 each of very small mass m($<<M$) are projected from the particle P horizontally in the same sense with velocities $2v$, $3v$ and $6v$ realtive to the particle P at times $t = \frac{u}{2g}$, $t = \frac{u}{g}$ and $t = \frac{3u}{2g}$ respectively.

Draw the velocity-time graph for the velocity of the particle P. Show that the velocity-time graphs for each of the vertical components of the velocities of the particles P_1, P_2 and P_3 coincide with portions of the velocity-time graph of the particle P and identify these portions. In a sperate diagram, draw the velocity-time graphs for each of the horizontal components of the velocities of the particles P_1, P_2 and P_3. Using the velocity-time graphs,

show that,

1.The four particles reach the ground at the same time $t = \frac{2u}{g}$

2.The three particles P_1, P_2 and P_3 fall on the ground at the same position.

95.A praticle P is projected vertically upwards from a point O in space with velocity $2u$. At the same instant, another particle Q is projected vertically downwards from the same point O with the velcity u. Both particles move under gravity. Draw the velocity-time graphs for the motions of the particles P and Q in the same figure and show that the speed of the particle Q when the particle P reaches its maximum hight is $3u$.

96.Two particles P and Q are placed at a point O on a fixed smooth plane inclined at an angle $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ to the horizontal. The particle P is given a velocity u upwards along the line of greatest slope through O, and at the same instant the particle Q is released from rest. Assuming that the two particles do not leave the inclined plane, sketch the velocity time graphs for the motions of P and Q on the same diagram.

Using these graphs, show that, at the instant the particle P returns to the point O the particle Q is at a distance $\frac{2u^2}{g \sin \alpha}$ from O.

97.A slow train A travelling in a stratight track starts to travel from rest at a station S_1 at time $t = 0$. It travels with a constant acceleration a until its velocity reaches a value u. Afterwards it travels a distance d with this constant velocity and then travels with constant retardation a until it comes to rest at a station S_2. An express train B travelling with uniform velocity $2u$ in the same direction along a parallel track passes the station S_1 at time $t = \frac{d}{u}$. Assuming that $\frac{1}{3}ad < u^2 < ad$, sketch the velocity-time graphs for both trains A and B for the motion between S_1 and S_2 in the same diagram, indicating the time at which each train reaches S_2.

98.A particle is projected vertically upwards with a velocity u from a point on a fixed rigid horizontal floor. After moving under gravity it strikes the floor. The coefficient of restitution between the particle and the floor is $e(0 < e < 1)$.

(i).Sketch the velocity-time graph for the motion of the particle until the third impact.

(ii). Show that the time taken by the particle until the third impact is $\frac{2u}{g}\left(1 + e + e^2\right)$.

(iii).Show further that the total time taken by the particle to come to rest is $\frac{2u}{g(1-e)}$.

99.Two equal particles *P* and *Q* are at fixed points *A* and *B* respectively on a smooth horizontal plane such that $AB = d$. At time $t = 0$, the particle *P* starts from rest at the point *A* and moves in the direction *AB* with constant acceleration *a*, until it reaches the speed *u* ($<\sqrt{2ad}$) and then moves uniformly with this speed *u*. At the instant the particle *P* reaches the speed *u*, the particle *Q* starts with speed *u* at the point *B* and moves uniformly in the direction *BA*. Draw a velocity-time graph for the motion of the particle *P*.

Draw, in the same figure a velocity time graph for the motion of the particle *Q*. Using these velocity time graphs.

(i). Find the distance travelled by the particle *P* with constant acceleration.

(ii). Show that the two particles *P* and *Q* pass each other at time $\frac{1}{2}\left(\frac{d}{u} + \frac{3u}{2a}\right)$.

(iii). Find the total distance travelled by the particle *P* to meet the particle *Q*.

100.A particle *P* is projected at a point *O* vertically upward under the gravity with velocity *u*. After time $\frac{u}{2g}$ another particle *Q* is projected at the point *O* vertically upward under the gravity with velocity $v(> u)$. Let *A* be the highest point that the particle *P* reaches. The particles *P* and *Q* meet at the point *A*. Draw the velocity-time graphs for the complete motions of the particles *P* and *Q* in the same figure. Using these velocity-time graphs show that

(i). $OA = \frac{u^2}{2g}$.

(ii). $v = \frac{5u}{4}$ and the velocity of the particle *Q* at the point *A* is $\frac{3u}{4}$.

(iii). When the particle *Q* reaches the highest point the height of the particle *P* from the point *O* is $\frac{7u^2}{32g}$.

101.A particle is projected vertically upwards with velocity 2*u* from a point *A* on the ground. After a time *T* another particle is projected vertically up wards with velocity *u* from a point *B* which a height *h* vertically above *A*.

Draw velocity time graphs for the particles on the same diagram. If the particles collide before reaching the second particle to its maximum height, show that the time the second particle taken untill collision is

$$\frac{h}{u - gT} - \frac{(4u - gT)T}{2(u - gT)}.$$

102.At the instant a particle *A* is projected vertically upwards with velocity 3*u* from a point on the ground another particle *B* is projected vertically down wards with velocity 2*u* from a point a height *h* above the ground. *A* and *B* collide head on. Show that by using a velocity time graph at the collision the velocities of *A* and *B* are $\frac{15u^2 - gh}{5u}$ and $\frac{10u^2 + gh}{5u}$.

Also find the height to the point of collision from the ground.

103.A balloon is rising vertically upwards with uniform acceleration f from a point on the ground starting from rest. Afrer a time t, a particle A is released from the balloon. After another time $\frac{tf}{g}$ a particle B is also released.particles A and B move under gravity. Draw v-t graphs for the ballon and the particles A and B. If it is given that $f \leq \frac{9}{2}(\sqrt{5}-1)$,

Show that when B is at the maximum height the distance between A and B is $\frac{t^2 f^2}{2g}\left(1+\frac{f}{g}\right)\left(3+\frac{2f}{g}\right)$.

Where g is acceleration due to gravity.

104. Two particles P and Q are simultaneously projected vertically upwards with speeds u and $\frac{u}{\sqrt{2}}$ respectively, from two points on a fixed horizontal floor. There is a fixed smooth horizontal ceiling at a height $\frac{u^2}{4g}$ from the floor. The coefficient of restitution between the ceiling and the particle P which strikes it is $\frac{1}{\sqrt{2}}$, and the two particles move upwards and down wards only under gravity.

i.Find the speed of the particle P just before it strikes the ceiling and the time T_1 up to the instant of collision.

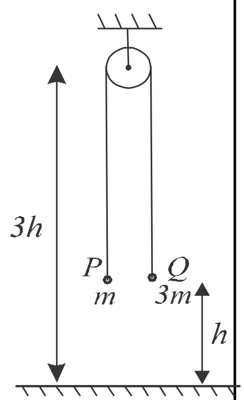
Show that the particle P returns to its point of projection with speed $\frac{u\sqrt{3}}{2}$.

ii .Show that the particle Q just reaches the ceiling, and find the time T_2 up to that instant.

iii.Sketch, on the same diagram, the velocity- time graphs for the motions of the two particles P and Q from the instant of projection until they return to the respective points of projection.

iv. Using the velocity-time graphs show that, at the instant when P strikes the ceiling, Q is at a verticle distance $\frac{u^2}{2g}\left(\sqrt{2}-1\right)^2$ below the ceiling.

105. A particle P of mass m is connected to a particle Q of mass $3m$ by a light inextensible string passing over a small smooth pulley fixed at a height $3h$ above an inelastic horizontal floor. Initially the two particles are held at a height h above the floor with the string taut, and released from rest. (See the adjoining figure.) Applying Newton's second law separately to the motions of P and Q, show that the magnitude of acceleration of each particle is $\frac{g}{2}$.After a time t_0 the particle Q strikes the floor, comes to rest instantly, remains at rest for a further time t_1 and begins to move up.



Sketch the velocity-time graphs separately for the motions of the two patricles P and Q until the particle Q begins to move up.

Using these graphs, show that $t_0 = 2\sqrt{\frac{h}{g}}$ and find t_1 in trems of g and h.

Show further that the particle P reaches a maximum height $\frac{5h}{2}$ above the floor.