BASIC MATHS

1) Simplify the following.

1. $\frac{x}{3}+\frac{y}{4}$ 2. $\frac{x+1}{5}+\frac{2x+5}{3}$ 3. $\frac{4x-5}{2}-\frac{3x-1}{6}$ 4. $\frac{1}{x}+\frac{3}{y}$ 5. $\frac{5}{m}-\frac{2}{n}$

6. $\frac{3}{a+2}+\frac{2}{a+1}$ 7. $\frac{10}{x}+\frac{10}{x+1}$ 8. $\frac{a}{a+b}+\frac{b}{a-b}$

2)Simplify the following.

1. $1+\frac{1}{a}+\frac{1}{a^2}$ 2. $\frac{1}{c^2}-\frac{1}{c}-1$ 3. $x+1-\frac{1}{x}$ 4. $1+\frac{1}{b}-\frac{b}{1-b}$

5. $\frac{1}{y+1}-\frac{2}{y-2}+\frac{1}{2}$ 6. $3-\frac{2}{x-2}+\frac{5}{2x+3}$

3)Factorise the following quadratic expressions.

1. p^2+2p+1 2. x^2+4x+4 3. x^2+6x+9 4. $x^2+8x+16$ 5. $x^2+10x+25$ 6. a^2+2a+1

4) Factorise the following.

1. $2a^2+11a+5$ 2. $3x^2+13x+4$ 3. $2p^2+7p+5$ 4. $3x^2+8x+4$ 5. $2a^2+7ab+6b^2$

5) Factorise the following.

1. $9x^2+6x+1$ 2. $4a^2+4a+1$ 3. $16a^2+24a+9$ 4. $4y^2+20y+25$

5. $24x^2-29xy-4y^2$ 6. $3x^2+7xy-10y^2$

6) Factorise the following.

1. $x^2-(3x+2)^2$ 2. $a^2-9(x+y)^2$ 3. $(5x+1)^2-(3x-1)^2$ 4. $(x+5)^2-(a-b)^2$

5. $25x^2-36y^2$ 6. $100x^2y^2-25a^2b^2$ 7. $\frac{16x^2}{y^2}-\frac{b^2}{25a^2}$ 8. $\frac{49b^2}{a^2}-\frac{x^2}{64y^2}$ 9. $\frac{81a^2}{25b^2}-\frac{100x^4}{64y^4}$

7) **Sum and difference of two cubes**

$$a^3+b^3=(a+b)\,(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)\,(a^2+ab+b^2)$$

Factorise the following.

1. $8x^3+64y^3$ 2. $27x^3y^3-8a^6b^6$ 3. $(1+a^3)$ 4. (x^3-1) 5. $1000x^3-125y^6$ 6. $216x^3+125y^6$

7. x^3+1 8. p^3-27 9. x^3-8 10. $1+27x^3$ 11. $(1-y^3)$ 12. $(8x^3-1)$ 13. $(27x^3+8)$ 14. $(64x^3-125)$

15. a^3b^3-1 16. x^3y^3+216 17. $125p^3-27q^3$ 18. $8x^4+x$ 19. $32a^3-4b^3$ 20. $16a^3b^3-2a^6$

21. x^4y-xy^4 22. x^6-y^6

8) Find the L.C.M of the following.

1. $x(x+5)$, $(x+5)(x-3)$ 2 $(3x+1)(4x-3)$, $(4x-3)(x+1)$ 3. $(5x-1)(3x-2)$, $(5x-1)(2x-7)$

9) Simplify the following till single fraction.

1. $\dfrac{2}{x+4}+\dfrac{x-5}{x^2+7x+12}$ 2. $\dfrac{1}{x+1}+\dfrac{3}{x^2-1}$ 3. $\dfrac{1}{x^2}-\dfrac{1}{x^2+x}$ 4. $\dfrac{3}{x^2+x-2}+\dfrac{2}{x^2+3x+2}$ 5. $\dfrac{1}{a^2-2a+1}-\dfrac{3}{a^2-1}$

10) Simplify the following

1. $\dfrac{8x-16y}{2}$ 2. $\dfrac{3c}{6ac+9bc}$ 3. $\dfrac{4a^2-8ab}{a-2b}$ 4. $\dfrac{2a-2b}{2(a-b)}$ 5. $\dfrac{4x+20}{3x+15}$ 6. $\dfrac{y-3}{4y-12}$

7. $\dfrac{12p-3q}{24p-6q}$ 8. $\dfrac{y^2-y-6}{y-3}$ 9. $\dfrac{ax+5a}{(x+5)(x+3)}$ 10. $\dfrac{x^2-1}{x^2+2x+1}$

11) Find the solutions of the following quadratic equations.

1. $(x+2)(x+3)=0$ 2. $(x+4)(x+\frac{3}{7})=0$ 3. $(x+\frac{5}{2})(x+\frac{6}{7})=0$ 4. $(x-5)(x+4)=0$

5. $(x-3)(x+\frac{5}{11})=0$ 6. $(x-\frac{1}{2})(x+3)=0$ 7. $(x-\frac{8}{11})(x+\frac{3}{7})=0$ 8. $(a-3)(a-7)=0$

9. $(a-\frac{5}{12})(a-10)=0$ 10. $(a-\frac{2}{25})(a-\frac{11}{10})=0$ 11. $(x+5)^2=0$ 12. $(x+\frac{3}{4})^2=0$

13. $(x-7)^2=0$ 14. $(x-\frac{5}{6})^2=0$

12) Find the solutions of the following quadratic equations using factors orusing formula or using completing square.

1. $x^2+6x+8=0$ 2. $x^2+7x+12=0$ 3. $30+11x+x^2=0$ 4. $24+10p+p^2=0$

5. $x^2+6x+9=0$ 6. $x^2+16x+64=0$ 7. $x^2-7x+12=0$ 8. $p^2+5=6p$

9. $n^2-2n+1=0$ 10. $p^2-10p+25=0$ 11. $x^2-2x-35=0$ 12. $x^2-5x=14$

13. $y^2+2y-8=0$ 14. $a^2+7a=44$

13).**Solve the following equations.**

1. $(x-4)^2-9=0$ 2. $(x+3)^2-12=0$ 3. $(x-5)^2=9$ 4. $(x+4)^2=5$ 5. $(x-2)^2-\frac{9}{4}=0$

6. $(x-\frac{4}{3})^2-\frac{5}{4}=0$

14) **Proportions**

$a:b=c:d$ is written as $\frac{a}{b}=\frac{c}{d}$.

It is read as "a is to b" equal to "c is to d"

Properties of proportions

1. If $\frac{a}{b}=\frac{c}{d}$ then $ad=bc$.

2. If $\frac{a}{b}=\frac{c}{d}$ then *i.* $\frac{a}{c}=\frac{b}{d}$. *ii.* $\frac{b}{a}=\frac{d}{c}$.

3. If $\frac{a}{b}=\frac{c}{d}$ then $\dfrac{a+b}{b}=\dfrac{c+d}{d}$.

4. If $\frac{a}{b}=\frac{c}{d}$ then $\dfrac{a-b}{b}=\dfrac{c-d}{d}$.

5. If $\frac{a}{b}=\frac{c}{d}$ then $\dfrac{a+b}{a-b}=\dfrac{c+d}{c-d}$

15)1. If $x : y = 3 : 5$, find the following.

1. $\frac{x}{y}$ 2. $\frac{y}{x}$ 3. $\frac{x+y}{y}$ 4. $\frac{x-y}{y}$ 5. $\frac{x+y}{x-y}$

16).Find $a : b$ for the following proportions.

1. $4a = 3b$ 2. $a = b - \frac{b}{4}$ 3. $3a - 2b = a + 6b$ 4. $\frac{a+b}{a-b} = \frac{7}{3}$

17).Solve the following equations using the properties of proportions.

1. $\frac{2}{a} = \frac{3}{5}$ 2. $\frac{x-2}{2} = \frac{1}{3}$ 3. $\frac{1}{x-2} = \frac{1}{2x}$ 4. $\frac{2x+5}{2x-5} = \frac{1}{2}$

18) Solve the following simultaneous equations.

1. $2x - y = 3$, $x + y = 9$
2. $2a - b = 1$, $2a + b = 9$
3. $x + 3y = 8$, $x - 2y = 3$
4. $7a - b = 2$, $6a - b = 0$
5. $a - 2b = 3$, $2a + b = 1$
6. $x + 2y = 7$, $2x + 3y = 12$
7. $3x + 5y = 34$, $x + y = 12$
8. $3x + 9 = y$, $13 + 2x = 3y$
9. $\frac{1}{x} + \frac{2}{y} = 2\frac{1}{3}$, $\frac{2}{x} + \frac{3}{y} = 3\frac{2}{3}$
10. $\frac{2}{x} + \frac{5}{y} = 5$, $\frac{1}{x} - \frac{7}{y} = 12$
11. $3x - y = 1$, $\frac{2}{x} + \frac{3}{y} = 2\frac{1}{2}$
12. $\frac{x+2}{5} = \frac{8-y}{4} = \frac{x+y}{7}$
13. $\frac{x+y}{3} = \frac{3y-x}{5} = \frac{x+2y+2}{6}$
14. $\frac{3x-2}{5} = \frac{7-y}{2}$, $3x - 2y = 6$
15. $\frac{2x}{3} + \frac{3y}{2} = 4x - 5y - 23 = -1$

19)Solve the following simultaneous equations.

1. $x = 2$, $x + 2y = 4$, $x + y + 3z = 15$
2. $a = 3$, $2a - b = 8$, $2a - 2b + 3c = 25$
3. $a = -3$, $2b - 3c = 19$, $a + 2b + 3c = 10$
4. $x - 3y = 9$, $x + 2z = 9$, $2x + y + 3z = 13$
5. $2a + b = 8$, $a + 3c = 14$, $2a - 2b + 3c = 23$
6. $x + 2y = 8$, $x + 3y + z = 6$, $x + 2y - 3z = 23$

20) Solve the following simultaneous equations.

1. $x+y-z=4$: $2x+y+z=9$: $3x-y-z=6$
2. $a+b+c=8$: $a+2b-c=14$: $2a-b-3c=9$
3. $a+b+c=5$: $a-2b-c=-4$: $2a+3b+2c=11$
4. $2x+y+z=11$: $x-2y-z=6$: $3x+2y-z=8$
5. $x + y + z = 6$: $2x - y - 2z = 5$: $5x + 2y + 2z = 21$
6. $x - \frac{y}{3} = 4$: $y - \frac{z}{4} = 1$: $z - \frac{x}{5} = 7$
7. $x + y + z = x - y = 8$: $z + x = 2$
8 $x + y = 10$: $y + z = 8$: $z + x = 4$
9. $x - \frac{x}{2} + \frac{y}{3} + \frac{z}{2} = 7$: $\frac{x}{4} + \frac{y}{2} + z = 7$: $\frac{x}{8} + \frac{y}{3} + \frac{z}{2} = 4$
10 $a + c = 2b$: $9a + 3c = 8b$: $2a + 3b + 5c = 36$
11. $\frac{a+b}{2} = \frac{b+c}{4} = \frac{c+a}{3}$: $2a + b + c = 30$
12. $a + b = 3$: $2a + 3b = 3c + 7$: $3a - 2c = 8 - 2b$

21) Solve the following equations.

1. $2x + y = 4$, $x^2 + y^2 = 5$

2. $3x - y = 8$, $3x^2 - xy - y^2 = -9$

3. $x + y = 1$, $2x^2 - xy + y^2 = 53$

4. $x - 2y = 2$, $x^2 + 3xy = 4$

5. $\frac{x}{2} + \frac{y}{3} = 1$, $4xy = 3$

6. $x + 2y = 13$, $xy = 15$

22) **Simplify as far as possible.**

1. $5\sqrt{6} + 3\sqrt{6}$ 2. $4\sqrt{5} - 3\sqrt{5}$ 3. $8\sqrt{2} + 2\sqrt{3} - 3\sqrt{2} - 6\sqrt{2} + 5\sqrt{3}$ 4. $2\sqrt{7} + 3\sqrt{6} + 2\sqrt{6} - 5\sqrt{7} + 8\sqrt{7}$

5. $7\sqrt{28} - \sqrt{700} + \sqrt{63}$ 6. $\sqrt{20a^2} - \sqrt{5a^2}$ 7. $2\sqrt{90} + 3\sqrt{40} - \sqrt{160}$

8. $\sqrt{45} - 9\sqrt{5} + \sqrt{80}$ 9. $3\sqrt{7a} + 4\sqrt{28a}$

23) **Simplify.**

1. $\sqrt{5} \times \sqrt{8}$ 2. $\sqrt{8} \times \sqrt{4}$ 3. $3\sqrt{5} \times 2\sqrt{5}$ 4. $2\sqrt{3} \times 3\sqrt{33}$ 5. $\sqrt{\frac{7}{8}} \times \sqrt{\frac{8}{7}}$ 6. $\frac{\sqrt{10}}{\sqrt{5}}$ 7. $\frac{\sqrt{75}}{\sqrt{3}}$

8. $\frac{\sqrt{99}}{8\sqrt{11}}$ 9. $\frac{3\sqrt{60}}{4\sqrt{15}}$ 10. $\frac{\sqrt{80} - 3\sqrt{5}}{\sqrt{5}}$

State the following numbers with rational denominators.

11. $\frac{1}{\sqrt{3}}$ 12. $\frac{4}{\sqrt{8}}$ 13. $\frac{\sqrt{3}}{\sqrt{11}}$ 14. $\frac{5\sqrt{5}}{6\sqrt{3}}$ 15. $\frac{5}{2 + \sqrt{3}}$ 16. $\frac{7}{5 - \sqrt{7}}$

17. $\frac{3 + \sqrt{2}}{3 - \sqrt{2}}$ 18. $\frac{3 - \sqrt{7}}{2 + \sqrt{7}}$ 19. $\frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} + \sqrt{3}}$ 20. $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$

24). **Expand the following using "Pascal's"** triangle.

1. $(a + b)^4$ 2. $(a - b)^5$ 3. $(2x - y)^6$ 4. $(3x + 2y)^7$ 5. $\left(\frac{x}{2} - 3y\right)^4$ 6. $\left(\frac{x}{3} + \frac{y}{2}\right)^5$

7. $(2ab - 3xy)^5$ 8. $\left(\frac{3x}{2} + \frac{2y}{3}\right)^4$ 9. $\left(\frac{2a}{3} - \frac{1}{2}b\right)^5$ 10. $(x + y + z)^3$

11. Show that $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.

INDICES AND LOGARITHMS

i. Laws Of Indices

i. $a^m \times a^n = a^{m+n}$

ii. $\frac{a^m}{a^n} = a^{m-n}$

iii. $(a^m)^n = a^{mn}$

iv. $(ab)^n = a^n b^n$

v. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

vi. $a^n = \frac{1}{a^{-n}}$

vii. $a^0 = 1 \ (a \neq 0)$

viii. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

ix. $a^{\frac{1}{2}} = \sqrt{a}$

ii. Laws Of Logarithms

$i.\ \log_a^{(MN)} = \log_a^M + \log_a^N$ $\quad ii.\ \log_a^{(\frac{M}{N})} = \log_a^M - \log_a^N$ $\quad iii.\ \log_a M^r = r\log_a^M$ $\quad iv.\ \log_a^b = \dfrac{1}{\log_b^a}$

$v.\ \log_a^b = \dfrac{\log_c^b}{\log_c^a}$ $\quad vi.\ a^{\log_a^x} = x$ $\quad vii.\ a^{\log_b^x} = x^{\log_b^a}$ $\quad viii.\ \log_{a^m}^{b^n} = \dfrac{n}{m}.\log_a^b$ $\quad ix.\ \log_a^a = 1$

$x.\ \log_a^1 = 0$

1\. *a.* Prove that $\dfrac{1}{\log_x^{(xyz)}} + \dfrac{1}{\log_y^{(xyz)}} + \dfrac{1}{\log_z^{(xyz)}} = 1$

b. Prove that $\left(\log_{b^2}^a\right)\cdot\left(\log_{c^2}^b\right) = \dfrac{1}{4}\log_c^a$

c. If $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$ prove that $abc=1$ and $a^a b^b c^c = 1$

2\. *a.* If $\log_4^x = a,\ \log_{12}^x = b$ prove that $\log_3^4 = \dfrac{b}{a-b}$ and $\log_3^{48} = \dfrac{a+b}{a-b}$

b. If a, b, x are three positive numbers. show that $\log_{ab}^x = \dfrac{\log_b^x}{1+\log_b^a}$

c. If $p = \log_a^{(bc)},\ q = \log_b^{(ca)},\ r = \log_c^{(ab)}$ Show that $pqr=p+q+r+2$

3\. *a.* Prove that $\log_a^b \times \log_b^c \times \log_c^a = 1$

b. Prove that $2\log_c^{(a+b)} = 2\log_c^a + \log_c^{\left(1+\frac{2b}{a}+\frac{b^2}{a^2}\right)}$

c. If $\log_a^{\left(1+\frac{1}{8}\right)} = p$, $\log_a^{\left(1+\frac{1}{15}\right)} = q$ and $\log_a^{\left(1+\frac{1}{24}\right)} = r$. Show that $\log_a^{\left(1+\frac{1}{80}\right)} = p-q-r$

d. If $\log_a^n = x$, $\log_c^n = y$ and $n \neq 1$ then prove that $\dfrac{x-y}{x+y} = \dfrac{\log_b^c - \log_b^a}{\log_b^c + \log_b^a}$

4\. *a.* When $a,b,c \in R^+$, prove that $\log_a^b = \dfrac{1}{\log_b^a}$

b. If $a^2 + b^2 = c^2$, show that $\dfrac{1}{\log_{b+c}^a} + \dfrac{1}{\log_{c-b}^a} = 2$

c. Prove that $\log_a^b \cdot \log_b^c \cdot \log_c^a = 1$.

Hence deduce the value of $\log_5^{32} \cdot \log_4^7 \cdot \log_{49}^{125}$

d. Prove that $\left(\log_{10}^2\right)\left(2\log_4^5 + 1\right) = 1$

e. If $\log_b^a + \log_c^a = 2\log_b^a \cdot \log_c^a$ Prove that $a^2 = bc$

5. *a.* If $a^2+b^2=7ab$ prove that $2\log\left(\frac{a+b}{3}\right)=\log a+\log b$

b. The terms $\log_z^x, \log_y^z, \log_x^y$ are in arithmetic progression with middle term *a* and common difference *d.*

Prove that $ad^2=a^3-1$

c. Prove that $\log_a^N=\log_b^N\cdot\log_a^b=\frac{\log_b^N}{\log_b^a}$

6. *a.* If $2^x=3^y=12^z$, Prove that $xy=z(x+2y)$.

b. If $n>1$, then show that the value of

$\frac{1}{\log_2^n}+\frac{1}{\log_3^n}+......+\frac{1}{\log_{53}^n}$ *is* $\frac{1}{\log_{53!}^n}$

c. Find the value of $\log_3^{11}\log_{11}^{13}\log_{13}^{15}\log_{15}^{27}\log_{27}^{81}$

7. *a.* If $\frac{\log_{1/2}^x}{b-c}=\frac{\log_{1/2}^y}{c-a}=\frac{\log_{1/2}^z}{a-b}$ then find the value of $x^a y^b z^c$

b. If $\log_a bc=x,\ \log_b ca=y,\ \log_c ab=z$

then find the value of $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

c. If $\log_5^2,\ \log_5^{(2^x+1)}$ and $\log_5^{\left(2^x+\frac{5}{2}\right)}$ are in arithmetic progression, then find *x.*

8. *a.* if $a^2+4b^2=12ab$ then find *log (a+2b)*

b. *If* $\log_{12}^{18}=\alpha$ *and* $\log_{24}^{54}=\beta$ then find the value of $\alpha\beta+(\alpha-\beta)$

c. If $\log_a bc=x$, $\log_b ca=y$, $\log_c ab=z$ then find the value of $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

9. If $x=\log_a^{\left(\frac{9}{8}\right)}$, $y=\log_a^{\left(\frac{16}{15}\right)}$, $z=\log_a^{\left(\frac{24}{25}\right)}$

Prove that

i. $\log_a^2=3x+4y-2z$ *ii.* $\log_a^3=5x+6y-3z$ *iii.* $\log_a^5=7x+9y-5z$

10. If $xy^{p-1}=a,\ xy^{q-1}=b,\ xy^{r-1}=c,$

prove that $(q-r).\log a+(r-p)\log b+(p-q)\log c=0$.

11.If $a^x = b^y = (ab)^{xy}$, prove that $x + y = 1$.

12. Prove that $\log_{a^n}^{x^m} = \frac{m}{n}.\log_a^x$. Deduce that $\log_a^x + \log_{a^2}^{x^2} + \log_{a^3}^{x^3} + ... + \log_{a^{2009}}^{x^{2009}} = \log_a^{x^{2009}}$.

13.If $\frac{\log z}{a-b} = \frac{\log y}{c-a} = \frac{\log x}{b-c}$, show that $x^{b+c}.y^{c+a}.z^{a+b} = 1$.

14. Prove that $a^{\log\left(\frac{b}{c}\right)}.b^{\log\left(\frac{c}{a}\right)}.c^{\log\left(\frac{a}{b}\right)} = 1$.

15. *a.* Solve $7^{2x} - 7^{x+2} + 48 = 0$ *b.* Solve $5^{2x} - 5^{x+1} + 4 = 0$ *c.* Solve $9^x - 12(3^x) + 27 = 0$

16. *a.* Show that the solutions of

$\log_2(3^{2x-2} + 7) = 2 + \log_2(3^{x-1} + 1)$ are 1 and 2.

. *b.* Find the value of x which satisfy the equation $3^{\log_5(x-7)} = \log_5(125)$

c. Find the value of x which satisfy the equation

$\log_2(x^2 - 3) - \log_2(6x - 10) + 1 = 0$

17.*a.* If $\log_5^2$, $\log_5^{(2^x+1)}$ *and* $\log_5^{\left(2^x + 5/2\right)}$ are in arithmetic progression then find x.

b. If $x = \log_{24}^{12}$, $y = \log_{36}^{24}$, $z = \log_{144}^{36}$ then find the value of $1 + 2xyz$.

18. *a.* Express $3^{2y} - 3^{y+1} - 3^y + 3$ in terms of z, where $z = 3^y$

Hence solve the equation $3^{2y} - 3^{y+1} - 3^y + 3 = 0$

b. Given that $\log_x u + \log_x v = p$ and $\log_x u - \log_x v = q$ prove that $u = x^{\frac{1}{2}(p+q)}$ and find v.

c. If $\frac{x(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y} = \frac{z(x+y-z)}{\log z}$ then show that $x^y y^x + z^y y^z - 2x^z z^x = 0$

d. If $xy^2 = 4$ and $\log_3(\log_2 x) + \log_{1/3}(\log_{1/2} y) = 1$ then $x = 64$

e. If $\log_3^2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, determine the value x.

19. Find the values of x and y of the following.

i. $x^2 + y = 12$, $\log_x^y + \log_y^x = 2$

ii. $xy = 64$, $\log_x^y + \log_y^x = \frac{5}{2}$

iii. $xy = 8$, $\log_8^x - 2\log_8^y = 1$

20. Solve the following equations

i. $\log_3^x - 4\log_x^3 + 3 = 0$ *ii.* $\log_3^x + 2\log_x^3 - 3 = 0$ *iii.* $\log_2^x + 6\log_x^2 - 5 = 0$

iv. $\log_3^{(2-3x)} = \log_9^{(6x^2-19x+2)}$

21. *a, b, c* are the p^{th}, q^{th} and r^{th} terms respectively of a geometric progression.
Prove that $(q-r)\log a+(r-p)\log b+(p-q)\log c=0$.

22. Prove that,

$$\frac{1}{1+\log_b a+\log_b c}+\frac{1}{1+\log_C a+\log_C b}+\frac{1}{1+\log_a b+\log_a c}=1$$

23. Prove that,

i. $\log_{27} 81=\frac{4}{3}$ *ii.* $\log_2\left(\frac{2}{3}\right)+\log_4\left(\frac{9}{4}\right)=0$ *iii.* $\log_{16} 32=\frac{5}{4}$

24. If $a=\log_2 3,\ b=\log_3 5,\ c=\log_7 2$. Show that $\log_{140} 63=\frac{1+2ac}{2c+abc+1}$.

25. *i.* If *a, b, c* in geometric progression then show that
$\log_x a,\ \log_x b,\ \log_x c$ in arithmetic progression.

ii. If $\log_e 2.\log_x 625=\log_{10} 16.\log_e 10$. show that $x=5$.

26. *i.* If $4^{\log_9 3}+9^{\log_2 4}=10^{\log_x 83}$ then show that $x=10$

ii. If $\log_{(2x+3)}(6x^2+23x+21)=4-\log_{(3x+7)}(4x^2+12x+9)$ then show that $x=-\frac{1}{4}$.

iii. Show that the solutions of the equations
$x^{\log_x 2}=\log_3(x+y)$ and $x^2+y^2=65$ are $x=8,\ y=1$.

27. *a.* If $2\log_y^x+2\log_x^y=5$, show that $\log_y^x$ is either $\frac{1}{2}$ or *2*.
Hence find all pairs of values of x and y which satisfy simultaneously the equation above and th eequation $xy=27$.

b. Prove that if $x=\log_{10}^{(a-by)}-\log_{10}^{a}$ where *a* and *b* are constants, then $y=\left(\frac{a}{b}\right)\left(1-10^{x}\right)$

c. Given that $\log_2^x+2\log_4^y=4$ show that $xy=16$.

Hence solve for *x* and *y* the simultaneous equations,

$\log_{10}(x+y)=1,\quad \log_2 x+2\log_4^y=4$