Remainder Theorem, Factor Theorem, Partial Fractions

1.Using long division, find the quotient and remainder when the following polynomial is divided by the given polynomial.

(i) $x^3 - 4x^2 + 2x - 1$ by $x^2 - 1$ (ii) $2x^4 + x^3 - 2x^2 + 4$ by $x^3 - 1$, (iii) $x^3 - 4x^2 + x + 2$ by $x + 1$,

(iv) $x^5 - 6x^4 - 2x^3 + 8x^2 - 2$ by $x^2 + x - 2$ (v) $x^4 - 4x^2 - x + 1$ by $2x - 1$ vi) $x^3 + 3x^2 - 1$ by $(x-2)$

vii) $2x^4 - 6x^3 - 5x^2 + 2x + 1$ by $x^2 - 3$ viii) $5x^3 + 2x^2 - 1$ by $x^2 - 1$ ix) $3x^3 + x^2 - x + 1$ by $x^2 + 1$.

2Using algorithm of division, find the quotient and remainder when the following polynomial is divided by the given polynomial.

(i) $x^3 - 2x^2 + 1$ by $(x + 1)$ (ii) $2x^4 + x^3 - x^2 + 3$ by $(x^2 - 1)$ (iii) $x^3 - 4x^2 - x + 2$ by $x^2 - 3x + 2$

(iv) $x^4 - 4x^3 + 8x - 2$ by $x^2 - x$, (v) $x^4 - 1$ by $x^2 + 2$ (vi) $x^3 + 3x^2 + 5x + 9$ by $(x+2)$

vii) $x^4 + 2x + 1$ $x^2 + 3x + 1$ viii) $4x^3 - 6x - 8$ by $x^2 + 2$ ix) $x^4 - 3x^3 - 2x^2 + x - 1$ by $(x+1)(x-1)$

3. (*a*) State and prove remainder theorem.

(*b*) Find the remainder when the following polynomial is divided by the given linear polynomial.

(i) $x^2 - x + 2$ by $x + 1$ (ii) $x^3 - 3x^2 + x + 2$ by $x - 2$ (iii) $2x^4 + x^3 - x^2 + 8x + 2$ by $(2x-1)$

(iv) $2x^4 + x^3 - 3x^2 + x - 1$ by $(2x+1)$ (v) $x^3 + x^2 - x + 3$ by $(2x-2)$

4.(*a*) State and prove factor theorem.

(*b*)Write the following polynomials in factor form.

(i) $x^3 + 2x^2 - x - 2$ (ii) $2x^3 + x^2 - 5x + 2$ (iii) $x^4 + x^3 - x^2 + x - 2$

(iv) $x^4 - x^3 + 2x^2 - 4x - 8$ (v) $x^4 + 2x^3 - 4x^2 - 2x + 3$ (vi) $x^4 - 2x^3 - 3x^2 + 4x + 4$

c) Solve the following equations.

i) $x^3 + 3x^2 + 2x = 0$, ii) $x^3 - 7x^2 + 11x - 5 = 0$

iii) $x^3 - 7x^2 + 19x - 13 = 0$ iv) $x^4 + x^3 - 4x^2 + x + 1 = 0$.

d). Find the factors of the polynomial $f(x) = x^3 + 4x^2 + x - 6$. Hence,choosing x suitably deduce the factors of the follwing polynomials.

(i) $x^3 + x^2 - 4x - 4$ (ii) $x^3 + 7x^2 + 12x$ (iii) $x^3 + 8x^2 + 4x - 48$

5. State and prove Remainder theorem.

(*a*). Find the remainder when $x^4 - 3x^3 - 2x - 1$ is divided by $x + 1$

(*b*). When $ax^4 - 3x^3 - 2x - 3$ is divided by $(x-1)$ the remainder is *6*. Find *a*.

(*c*).When $x^3 + ax^2 + bx + c$ is divided by $(x+1)$ and $x^2 - x$ the remainders are 2 and $x + 2$ respectively. Find *a, b* and *c*.

(*d*). Given that $f(x) = x^2 + 3\alpha x - \beta$ when $f(x) + f(3x-2)$ is divided by $(x-1)$ and $(x+2)$ the reaminders are 2 and 0 respectively. Find α and β.

6. When a polynomial function in x is divided by $(x-a)$ and $(x-b)$ the remainders are R_1 and R_2 respectively. When this polynomial is divided by $(x-a)(x-b)$ show that the remainder is $\dfrac{(R_1-R_2)x+aR_2-bR_1}{a-b}$.

7. $f(x)$ is a polynomial function in x and $f(1)=a, f(-1)=b$ and $f(0)=c$. Show that when $f(x)$ is divided by (x^2-1) the remainder is $\frac{1}{2}(a-b)x+\frac{1}{2}(a+b)$.

8.The expression ax^3+bx+c has a factor in the form x^2+px+1 Show that $a^2-c^2=ab$ .Show also that in this case ax^3+bx+c and cx^3+bx^2+a have a common quadratic factor.

9.Let $f(x)=x^4-bx^3-11x^2+12x+a$. Where a and b are constants. $(x+2)$ is a facor of $f(x)$ and $f(x)$ is a perfect square of a quadratic expression. Find a and b.

10.Let $f(x)=2x^3+3x^2-3x+q$. Where q is non-zero integer. If $(x-q)$ is a factor of $f(x)$, find the value of q. For this value of q state $f(x)$ as a product of linear factors. Find constants a, b, c such that $f(x)=(x-a)(2x-1)(x+1)+bx+c$.

11.If x^3+px^2+qx+r is divisible by x^2+ax+b show that $q-b=a(p-a)$ and $r=b(p-a)$.

12.Let $f(x)=ax^3+bx^2-2x+c$. When $f(x)$ is divided by (x^2+x) the remainder is $6(x+1)$ and $(x-1)$ is a factor of $f(x)$. Find the values of a, b, c.

13.The polynomial $ax^4-6x^3+bx^2-cx+28$ is divisible by $(x-2)^2$ and on dividing by $(x+1)$ resulting a remainder 36. Find the constants a, b, c.
At these values of a, b, c when the polynomial is divided by $(x-1)$ find by using synthetic division the quotient and the remainder.

14.The polynomial $f(x)$ in x of degree three has the following properties.

i. $(x+3)$ is a factor of it.
ii. $f(x)-12$ is divisible by x without a remainder.
iii. When $g(x)=2x^2-11x+13$ the polynomial $[f(x)-g(x)]$ can be expressed only by the recurring factor of $(x-1)$. Find $f(x)$.

Hence find all factors of $f(x)$. (Ans : $f(x)=x^3-x^2-8x+12=(x+3)(x-2)^2$)

15.Let $f(x)=2x^3+rx^2-12x-7$. Where $x\in\Re$. $(x-\alpha)^2$ is a factor of $f(x)$. $(\alpha\in\Re)$ Show that $\alpha=-1$
Also find the value of r. State $f(x)$ interms of factors.

16. If the polynomials $P(x)$ and $Q(x)$ are divided by $3x^2+5x-2$ and x^2-4 the reaminders are $3x+5$ and $x+3$ respectively. Find a linear factor for the polynomial $P(x)+Q(x)$. Determine the remainder when the polynomial $P(x)-Q(x)$ is divided by the above linear factor.

17.Let $f(x)=x^3+ax^2+bx+c$. When $f(x)$ is divided by $(x-1)$ and $(x+2)$ the remainder is 4. Find the remainder when $f(x)$ is divided by $(x-1)(x+2)$. If $f(0)=2$, find the quotient when $f(x)$ is divided by $(x-1)$.

18.Let $f(x)=x^3-2ax^2+(ab+a^2-b^2)x-ab(a-b)$ where *a, b* are real numbers such that $a \neq b$. Show that $x-a+b$ is a factor of $f(x)$ and hence solve the equation $f(x)=0$. Deduce the values of *p, q* and *r* in x^3+px^2+qx+r so that 1,3 and 4 are its roots.

19.Let $Q(x)=2x^2+ax-7$ and $P(x)=(2x-1)Q(x)+b$ where *a* and *b* are real constants. $Q(x)$ is an even function and when $P(x)$ is divided by $\underline{x}$ the remainder is 12. Find the values of *a* and *b*. Express $P(x)$ as a product of linear factors in *x*.

20.Let $f(x)=x^3-3x^2+px+8$. Where *p* is a constant When this polynomial is divided by $x-2$ the remainder is 2. Find *p* Hence or otherwise when $g(x)=x^2-x-4$ if $H(x)=f(x)+g(x)$ find all solutions of $H(x)=0$.

21.When the polynomial $ax^5-2x^3+x^2+b$ is divided by (x^2-1) the quotient is $f(x)$ and remainder is $-x-2$. Find *a, b* and $f(x)$. Find also the
remainder when $f(x)$ is divided $x+2$.

22.If $n(>2)$ is an odd integer, show that when x^n+2 is divided by x^2-1 the reaminder is $x+2$.

23.Find 4[th] degree polynomial in *x*, which is divisible by x^2+1 and resulting a remainder $-10x+6$. When dividing by $(x-1)^2(x+1)$.

24. $f(x)$ is a quadratic function. when it is divided by $x-1,\ x-2,\ x-3$ and $x-4$ the corresponding reaminders are $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}$ respectively. A polynomial function $g(x)$ is defined as $g(x)=x^2f(x)-1$.
Show that $(x-1),(x-2),(x-3)$ and $(x-4)$ are factors of $g(x)$. Hence find $g(x)$.

25.State and prove factor therom. Find the factors of the following polynomials.

(i). x^3-2x^2-5x+6 (2). $x^3+7cx^2+11c^2x+2c^3$ (3). x^4-2x^3-6x-9

(4). $2x^3-3x^2-12x+20$ (5). $x^4+x^3-(a^2+1)x^2-a^2x+a^2$

26.If $(x-p)$ is a factor of $4x^3-(3p+2)x^2-(p^2-1)x+3$, find the possible values of *p*. Find the remaining factors for each value of *p*.

27.If x^3+lx^2+m and lx^3+mx^2+x-l have a common factor, then show that it is a factor of $(m-l^2)x^2+x-l(1+m)$.

28. If $(x+1)^2$ is a factor of x^5+2x^2+ax+b find *a* and *b*.

29.Show that when the polynomial $f(x)$ is divided by x^2-a^2 the remainder is $\frac{1}{2a}[f(a)-f(-a)]x+\frac{1}{2}[f(a)+f(-a)]$. Hence find the remainder when x^n-a^n divided by (x^2-a^2).

(i). if *n* is even (ii). If *n* is odd

30.If $(x-p)$ is a common factor for the polynomials $P(x)$ and $Q(x)$, show that $(x-p)$ is a factor of $[P(x)-Q(x)]$. If there exist a common factor for the polynomials $ax^3+4x^2-5x-10$ and ax^3-9x-2 then show that $a=2$ or $a=11$.

31.Find the remainder when $f(x)=2x^3-x^2-5x+3$ is divided by $(x-2)$. Hence deduce a factor of $f(x)-5$ and express $f(x)-5$ as a product of linear factors.

32. Show that $(x-a)$ is a factor of $f(x)=x^n-a^n$ show also that $f(x)=(x-a)(x^{n-1}+ax^{n-2}+a^2x^{n-3}+.....+a^{n-1})$. Hence find the factors of x^n-1 and show that all integers of the form 8^n-1 are divisible by 7. Using the above results, for odd numbers of *n*, show that the integers 10^n+1 are divisible by 11.

33.Given that $ax^5-2x^3+x^2+b=(x^2-1)f(x)-x-2$. Where *a* and *b* are constants and $f(x)$ is a polynomial function. Find *a* and *b*. Find the remainder when $f(x)$ is divided by $x+2$.

34.Given that $f(x)=x^2+3bx-a$. When $f(x)+f(3x-2)$ is divided by $(x-1)$ and $(x+2)$ the remainders are 2 and 0 respectively. Find *a* and *b*.

35. $f(x)$ and $g(x)$ are two polynomial functions such that when $f(x)$ is divided by $3x^2+x-2$ the remainder is $2x+1$ and $g(x)$ is divided by x^2-1 the remainder is $x+2$. Find a linear factor of $f(x)+g(x)$ and show when $f(x).g(x)$ is divided by this linear factor the remainder is -1.

36.Show that when the polynomial $f(x)$ is divided by x^2-a^2 the remainder is $\frac{1}{2a}[f(a)-f(-a)]x+\frac{1}{2}[f(a)+f(-a)]$. Hence if x^n-a^n is divided by x^2-a^2. Find the remainder when

(i). *n* is even (ii). *n* is odd

37. The expression $x^3-3b^2x+2c^3$ can be divided exactly by $(x-a)$ and $(x-b)$. Show that $a=b=c$.

38. when the polynomial *f*(*x*) is divided by x^2-1, the remainder is $3x+2$. When *f*(*x*) is divided by $(x-2)$ the remainder is1. Find the remainder when *f*(*x*) is divided by $(x-1)$ (x^2-x-2).

39. Find a fourth degree polynomial divisible by (x^2+1) and gives remainder $-10x+6$ when divied by $(x-1)^2$ $(x+1)$.

40. Given that $px^5-2x^3+x^2+q=(x^2-1).f(x)-x-2,$ where *p*,*q* are constants and *f*(*x*) is a polynomial. Find *p* and *q*. find the remainder when *f*(*x*) is divided by $(x+2)$.

41.Let $f(x)=x^8+px^2+qx+r$ .When *f*(*x*) is divided by $(x-1)$ $(x+1)$ and *x* the remainders are 3, 5 and 2 respectively. Find *p*, *q*, *r* and find the remainder when it is divided by $x+2$.

42.Given that $f(x)=x^2+3\alpha x-\beta$. When the polynomial $f(x)+f(3x-2)$ is divided by $(x-1)$ and $(x+2)$, the remainders are 2 and 0 respectively. Find α and β .

43. If $(x-a)^2$ is a factor of $x^3+3px+q$, show that its other factor is $(x+2a)$ and $q^2+4p^3=0$.

44. If x^2+2 is a factor of x^4-6x^2+q .find q. Find other factors of x^4-6x^2+q.

45. Solve the following equations.

i) $x^3+3x^2+2x=0,$ ii) $x^3-7x^2+11x-5=0$

iii) $x^3-7x^2+19x-13=0$ iv) $x^4+x^3-4x^2+x+1=0$.

46. Let $f(x)=x^3+2x^2+x+1$. Show that when *f*(*x*) is divided by $(x-1)\ (x+1)$ the remainder is $2x+3$. Without using remainder theorem directly and using only the above result show that when *f*(*x*) is divided by (*x*-1) and $(x+1)$ the remainders are 5 and 1 respectively.

47.*f*(*x*) is a polynomial function. $f(3)=a,\quad f(-3)=b$. Show that when *f*(*x*) is divided by (x^2-9) the remainder is given by $\left(\frac{a-b}{6}\right)x+\left(\frac{a+b}{2}\right)$.

48.Show that when the polynomial $f(x)=x^3+x^2-x+2$ is divided by (x^2+1), the quotient is $(x+1)$ and the remainder is $-2x+1$. Hence, by considering the function $f(x-1)$, show that when the polynomial $g(x)=x^3-2x^2+3$ is divided by (x^2-2x+2) the remainder is $-2x+3$. Show also that when the function $f(x+1)$ is divided by $(x+1)^2\ (x+2)$ the remainder is $(1-x)$. Hence show that when $f(x+1)$ is divided by $(x+1)$ the quotient is (x^2+3x+1) and the remainder is 2.

49. When the function *f*(*x*) is divided by (x^2-1) and (x^2+1), the remainder is $(x+1)$. Show that when *f*(*x*) is divided by $(x^2-1)(x^2+1)$, the remainder is $(x+1)$. Hence show that $(x+1)$ is a factor of *f*(*x*).

50. *f*(*x*) is a polynomial function of degree two. When *f*(*x*) is divided by $(x-1)$, $(x-2)$, $(x-3)$ the remainders are $1,\frac{1}{2},\frac{1}{3}$ respectively. Another polynomial *g*(*x*) is defined as $g(x)=x.f(x)-1$. Show that $(x-1)$, $(x-2)$ and $(x-3)$ are factors of *g*(*x*). Hence find *g*(*x*).

51. Let $q(x)=x^2-\alpha x+4$ and $h(x)=x^3-\beta x^2+x+11$. When *q*(*x*) is divided by $(x-1)$ the remainder is 2 and $h(x)$ is divided by $(x+1)$ the remainder is 7. Find α and β. For these values of α and β, verify that $h(x)=(x+1)\,q(x)\ +7$. Express *q*(*x*) in the form $(x-2)\ (x+\lambda)+\mu$, where λ and μ are constants. Hence show that when *h*(*x*) is divided by $(x+1)\ (x-2)$ the remainder is $(2x+9)$.

52. Let $c(\neq 0)$ and *d* be real numbers, and let $f(x)=x^3+4x^2+cx+d$. The remainder when *f*(*x*) is divided by $(x+c)$ is $-c^3$. Also, $(x-c)$ is a factor of *f*(*x*) . Show that $c=-2$ and $d=-12$. For these values of *c* and *d*, find the remainder when *f*(*x*) is divided by (x^2-4).

53.An even function $f(x)$ is given as $f(x)=3x^2+px+8$. It is defined $g(x)=(x+2)f(x)+q$ Where p , q are real values.If the remainder when $g(x)$ is divided by x is 5,find p and q Express $g(x)$ as a product of two factors.

54.Seperate $2x^2-5xy-x-25y-3y^2-28$ in to two linear factors.

55.If $2x^2+3y^2+7xy+4x+ky+2$ can be expressed as a product of two linear factors, find *k*. Obtain these factors.

56.Find λ such that $2x^2+xy+\lambda x-6y-y^2-5$ can be expressed as a product of two linear factors.

57.Find λ such that $f(x,y)=x^2+8xy-5y^2-\lambda(x^2+y^2)$ can be expressed in the form $a(x+by)^2$.
For each value of λ, obtain the values of *a* and *b*.

58.Find λ such that $f(x,y)=2x^2+\lambda xy+3y^2-5y-2$ is expressible as a product of two linear factors.

59. $f(x)$ is a polynomial in *x* of degree greater than 3. When $f(x)$ is divided by $(x-1),(x-2)$ and $(x-3)$ the reaminders are *a, b, c* respectively.
By repeated application of algorithm of division, show that when $f(x)$ is divided by $(x-1)(x-2)(x-3)$ the remainder can be expressed as $\lambda(x-1)(x-2)+\mu(x-1)+v$ where λ,μ,v are constants.
Find λ,μ,v interms of *a, b* and *c*.

60. (*a*) State remainder theorem.
If $b=8a$ in the polinomial $f(x)=ax^{2010}+bx^{2007}$ then show that $(x+2)$ is a factor of it.
find the remainder when $f(x)$ is divided by $(x-1)$ in terms of a.

(*b*) Resolve in to partial fractions. $\dfrac{2x+1}{(x-1)(x^2+2)}$

61.(*a*) Let $f(x)=2x^3+3x^2-3x+p$ where p is non-zero integer.
Find p so that $(x-p)$ is a factor of $f(x)$. Hence express $f(x)$ as a product of linear factors.

(*b*) Find partial fractions of $\dfrac{x^3+2x^2-x-3}{(x+1)^2(x^2+2)}$

62.(*a*) If $(x-a)^2$ is a factor of the polinomial $x^3+3px+q$ then show that its other factor is $(x+2a)$.
Show also that $q^2+4p^3=0$

(*b*) Find partial fractions of $\dfrac{x^3+2x^2-x-3}{(x+1)(x^2+2)}$

63.(*a*) Given that $px^5-2x^3+x^2+q=(x^2-1)f(x)-x-2$ where p and q are constants and $f(x)$ is a polynomial function. Find the value of p and q. Find the remainder when $f(x)$ is divided by $(x-2)$

(*b*) Find partial fractions of $\dfrac{2x^3+1}{x(x-1)^2}$

64.(a) The remainders when $f(x)$ is divided by $(x-1)$ and $(x-2)$ are2 and 3 respectively.Find the remainder when $f(x)$ is divided by $(x-1)(x-2)$

(b) A polynomial in x of degree 3 has the following properties.

(i) The remainder when it is divided by x^2+x-2 is $5x-1$

(ii) The remainder when it is divided by x^2-x-2 is $12x-1$

find the polynomial.

65.(a)A polynomial $f(x)$ of degree two gives remainders-1,2,4 when it is divided by $(x-1),(x+2)$ and $(x-2)$ respectively. It is defined $g(x)=ax^3+bx^2+cx-5=2x\,f(x)-1$. Find the values of a,b,c and find $g(x)$.

(b) Find A and $f(x)$ so that $\dfrac{1-8x-x^2}{(x+1)(x-1)^2}=\dfrac{A}{x+1}+\dfrac{f(x)}{(x-1)^2}$.

By writing the polinomial $f(x)$ in terms of $(x-1)$,express the partial fractions in the simplest form of the rational function on the left given.

66. Let $f(x)=x^3+x^2+2x-1$ .Write down $f(x)$ as a polynomial in $(x+1)$. Hence show that the partial fractions of

$$\frac{x^3+x^2+2x-1}{(x+1)^3}=1-\frac{2}{(x+1)}+\frac{3}{(x+1)^2}-\frac{3}{(x+1)^3}.$$

67. The polynomial $2x^3-3ax^2+ax+b$ gives remainders -54 and 0 when it is divided by $(x+2)$ and $(x-1)$ respectively. Find the values of a,b and find its all factors.Hence find factors of

(i) $2x^6-9x^4+3x^2+4$ (ii) $4x^3+3x^2-9x+2$

68. Let $f(x)=2x^4+7x^3+\alpha x^2-x-2$ .Here $\alpha\in R$. Given that $(x+1)$ is a factor of $f(x)$.Find the value of α. Express $f(x)=(2x-1)(x+1)^2(x+\beta)$ .where β is a real constant to be determined. By writting $(2x-1)$ in the above expression such that $a(x+1)+b$ show that when $f(x)$ is divided by $(x+1)^3(x+2)$ the remainder is $-3(x+1)^2(x+2)$. Hence, Deduce that ,when $g(x)=2x^2+x-1$ is divided by $(x+1)^2$ remainder is $-3(x+1)$.

69. Given that $f(x)=x^4-x^3-x+1$. By applying remainder theorem repeatedly, show that $(x-1)^2$ is a factor of $f(x)$.

Express $f(x)=(x-1)^2(x^2+\alpha x+\beta)$ Where α and β are constants to be dertermined. Hence deduce that ,for all real x, $f(x)\geq 0$

70.An even function $f(x)$ is given as $f(x)=3x^2+px+8$. It is defined $g(x)=(x+2)f(x)+q$ Where p , q are real values.If the remainder when $g(x)$ is divided by x is 5. Find p and q Express $g(x)$ as a product of two factors.

71.(i) Let $f(x)=2x^4+\gamma x^3+\delta x+1$ where γ and δ are real constants.

Given that $f\left(-\frac{1}{2}\right)=0$ and $f(-2)=21$, find the two real linear factors of $f(x)$

(ii) Find the two linear expressions $P(x)$ and $Q(x)$ satisfying the equation

$\left(x^2+x+1\right)P(x)+\left(x^2-1\right)Q(x)=3x$ for all real x.

72. Let $g(x)=x^4+4x^3+7x^2+6x+2$ Using Remainder theorem repeatedly show that $(x+1)^2$ is a factor of $g(x)$.Express $g(x)$ in the form $(x-a)^2\left(x^2+bx+c\right)$where a, b and c are constants to be determined.

Deduce that $g(x)\geq 0$ for all real values of x.

73. Let $a\in R$ and let $f(x)=3x^3+5x^2+ax-1$ It is given that $(3x-1)$ is a factor of $f(x)$ Find the value of a.

Express $f(x)$ in the form $(3x-1)(x+k)^2$ where k is a constant.

By writting $(3x-1)$ in the above expression in the form $b(x+1)+c$ where b and c constants.Find the remainder when $f(x)$ is divided by $(x+1)^3$.

74.Let $f(x)=3x^3+ax^2+bx-1$ and $g(x)=x^3+cx^2+ax+1$, $a,b,c\in\mathbb{R}$. It is given that the remainder when $f(x)$ is divided by x^2+2x-3 is $16x-13$, and that the remainder when $g(x)$ is divided by $(x+1)$ is -1.

Find the values of a, b and c. For these values of a, b and c show that $f(x)+g(x)$ can be written as $x(\lambda x^2+\mu x+1)$, where $\lambda,\mu\in\mathbb{R}$ are to be determined.

75.Let $f(x)=x^3+ax^2+bx+c$, where $a,b,c\in\mathbb{R}$. It is given that x^2-4 is a factor of $f(x)$. Show that $b=-4$.

It is also given that the remainder when $f(x)$ is divided by x^2-x is $2x+k$. Show that $k=-20$

Show also that $f(x)$ can be written as $(x+\lambda)(x^2-4)$ where $\lambda\in\mathbb{R}$.

76.Let m and n be two non - zero real numbers and let $f(x)=x^3-2x^2-nx+mn$. It is given that $(x-m)$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $(x-n)$ is mn. Find the values of m and n. Find the quotient and the remainder when $f(x)$ is divided by $(x-2)^2$.

77.Let $h(x)=x^3+ax^2+bx+c$, where $a, b, c\in\mathbb{R}$. It is given that x^2-1 is a factor of $h(x)$. Show that $b=-1$.

It is also given that the remainder when $h(x)$ is divided by x^2-2x is $5x+k$, where $k\in\mathbb{R}$. Find the value of k and show that $h(x)$ can be written in the form $(x-\lambda)^2(x-\mu)$, where $\lambda,\mu\in\mathbb{R}$.

78.Let c and d be two non - zero real numbers and let $f(x)=x^3+2x^2-dx+cd$. It is given that $(x-c)$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $(x-d)$ is cd. Find the values of c and d.

For these values of c and d, find the remainder when $f(x)$ is divided by $(x+2)^2$.

79. Let $f(x)=2x^3+ax^2+bx+1$ and $g(x)=x^3+cx^2+ax+1$, where $a, b, c\in\mathbb{R}$. It is given that the remainder when $f(x)$ is divided by $(x-1)$ is 5, and that the remainder when $g(x)$ is divided by x^2+x-2 is $x+1$.

Find the values of a, b and c.

Also, with these values for a,b and c , show that $f(x)-2g(x)\leq\frac{13}{12}$ for all $x\in\mathbb{R}$.

PARTIAL FRACTIONS

80. Find partial fractions of the followings.

01. $\dfrac{x+1}{(x-1)(x+2)}$

02. $\dfrac{2x+1}{x(x+1)(x-1)}$

03. $\dfrac{x^2-1}{x(x+2)(x+3)}$

04. $\dfrac{x^2+1}{(x+1)(x-1)^2}$

05. $\dfrac{x}{(x+1)^2(x+2)^2}$

06. $\dfrac{x^2+x-1}{x^2(x-1)^2}$

07. $\dfrac{x}{(x+1)(x^2-4)}$

08. $\dfrac{x^2+x+1}{(x-1)^2(x^2-1)}$

09. $\dfrac{x+3}{(x^2+1)(x^2-4)}$

10. $\dfrac{x^2}{(x^2+1)(x^2+2)}$

11. $\dfrac{2x^2+1}{(x^2-1)(x^2+2)}$

12. $\dfrac{x^2-x-1}{x^2(x^2+1)}$

13. $\dfrac{x^3}{(x+1)(x-1)^2}$

14. $\dfrac{x^3+1}{(x-1)(x+2)}$

15. $\dfrac{x^4+8}{x^3(x-2)}$

16. $\dfrac{2x^2+x-5}{(x+2)(x+1)}$

17. $\dfrac{x^4+8}{x^3(x-2)}$

18. $\dfrac{x}{(x-1)(x^2+x+1)}$

19. $\dfrac{x^2}{x^3-2x^2+2x-1}$

20. $\dfrac{5-7x}{2x^3-x^2-2x+1}$

21. $\dfrac{x^5-1}{(x^2-1)x}$.

22. $\dfrac{x}{(x+1)\ (x-1)\ (x+2)\ (x+3)}$

23. $\dfrac{2x^2+1}{(x+2)\ (x+3)(x-2)}$

24. $\dfrac{x+1}{(x-1)^2\ \ (x+2)}$

25. $\dfrac{x}{(x^2+1)\ \ (x+1)}$

26. $\dfrac{2x-1}{(x^2-1)\ \ (x^2+1)}$

27. $\dfrac{-3x^2+2x-3}{x^2-1}$

81. Find partial fractions of $\dfrac{x^2}{(x-a)(x-b)}$ where $a \neq b$. Hence when $a \neq b \neq c$ deduce that

$$\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}+\frac{c^2}{(c-a)(c-b)}=1.$$

82. *i.* Using suitable substitution find the partial fractions of the following.

a. $\dfrac{x^2-x+2}{(x-1)^2}$ *b.* $\dfrac{x^3}{(x+1)^2}$

ii. Using the substitution $y=(x-a)$

Show that $\dfrac{px^2+qx+r}{(x-a)^3}=\dfrac{p}{x-a}+\dfrac{2pa+q}{(x-a)^2}+\dfrac{pa^2+qa+r}{(x-a)^3}$.

83. Find the constant *k* and function *f(x)* so that $\dfrac{1}{(x-2)(x-1)^3}=\dfrac{k}{(x-2)}+\dfrac{f(x)}{(x-1)^3}$.

Express $f(x)$ as a polynomial of $(x-1)$. Hence find the partial fractions of $\dfrac{1}{(x-2)(x-1)^3}$.

84. Resolve $\frac{1}{(x-1)(x-2)}$ into partial fractions. Hence find partial fractions of

(i). $\frac{1}{(x-1)^3(x-2)}$ (ii). $\frac{1}{(x-1)^2(x-2)^2}$ (iii). $\frac{1}{(x-1)^2(x-2)^3}$

85. If $\frac{1}{(1-ax)(1-bx)}=\frac{A}{1-ax}+\frac{B}{1-bx}$ without solving for A and B,

show that

(i). $A+B=1$ (ii). $\frac{b}{a}A+\frac{a}{b}B=-1$ (iii). $\frac{1}{(1-ax)^2(1-bx)}=\frac{A}{(1-ax)^2}+\frac{AB}{(1-ax)}+\frac{B^2}{(1-bx)}$.

86.Find *A, B, C* such that $x^2=A(x+1)(x+2)+B(x+2)+C(x+1)^2$.

Hence find partial fractions of $\frac{x^2}{(x+1)^2(x+2)}$.

87.By writting $x(x-1)$ interms of (x^2+1) and $(x+1)$ find partial fractions of $\frac{x(x-1)}{(x^2+1)(x+1)}$.

88.Find *A, B, C, D* such that

$x^3=(Ax+B)(x+1)(x-2)+C(x^2+1)(x-2)+D(x^2+1)(x+1)$

Hence find the partial fractions of $\frac{x^3}{(x^2+1)(x+1)(x-2)}$.

89.Find the values of the constants *A, B,* and *C* such that $x^2+x+3=A(x^2-x+1)+(Bx+C)(x+1)$

Hence write down $\frac{x^2+x+3}{(x^2-x+1)(x+1)}$ in partial fractions

90.Comparing the coefficients of x^2, x^1 and x^0 , find the values of the constants *A, B* and *C* such that

$Ax(x+1)+B(x+1)+Cx^2=1$. For all $x\in\mathbb{R}$

Hence write down $\frac{1}{x^2(x+1)}$ in partial fractions.

91.Show that $(x+1)^2-(x-1)^2=4x$

Hence write down $\frac{4x}{(x+1)^2(x-1)^2}$ in partial fractions.

92.Show that $\frac{x}{(x+1)(x-2)}=\frac{1}{3(x+1)}+\frac{2}{3(x-2)}$

Hence write down partial fractions of

$i. \frac{x}{(1-x)(x+2)}$ $ii. \frac{x+1}{(x+2)(x-1)}$ $iii. \frac{2x}{(x+2)(x-4)}$

1. *Greek alphabet*

α	alpha	β	beta
γ	gamma	Δ, δ	Delta
ε	epsilon	η	eta
θ	Theta	κ	kappa
λ	Lambda	μ	mu
υ	nu	$\pi, \prod$	pi
ρ	rho	$\sum, \sigma$	Sigma
ϕ	Phi (Fie)	φ	psi
Ω, ω	Omega	τ	tau
χ	chi		

2. *Abbreviations*

//	Parallel	සමාන්තර
+ ve	Positive	ධන
- ve	Negative	සෘණ
w. r. t	With respect to.	සාපේක්ෂව
s. t	Such that (Sothat)	වනඅයුරින්
i.e. (idest)	That is	එනම්

3. *Useful signs and notations*

$\therefore$	Therefore	එමනිසා
$\because$	Because	මක්නිසාදයත්
...	And soon	යනාදිලෙස
$a \equiv b$	*a* is identical to *b*.	*a, b* ට සර්වසම වේ.
$a \,\alpha\, b$	*a* is proportional to *b*.	*a* සමානුපාතික වේ *b* ට
$a \neq b$	*a* is not equal to *b*.	*a, b* ට අසමාන වේ.

4. *factors*

1. ***Difference of two squares***

$$a^2 - b^2 = (a-b)(a+b)$$

2. ***Difference of two cubes***

$$a^3 - b^3 = (a-b)\ (a^2 + ab + b^2)$$

3. ***Sum of two cubes***

$$a^3 + b^3 = (a+b)\ (a^2 - ab + b^2)\cdot$$

4. ***Expansions***

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.

5. ***Sum of two squares***

$$a^2 + b^2 = (a+b)^2 - 2ab.$$