Limits and Differentiation

Limits.

1. Evaluate the limit of the following

(i). $\lim\limits_{x\to 1/2}\dfrac{4x^2-1}{2x-1}$ (ii). $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1}$ (iii). $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}$ (iv). $\lim\limits_{x\to 1}\dfrac{x-1}{2x^2-7x+5}$

02. Find the limit of the folliwng

(i). $\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{x^2+x-6}$ (ii). $\lim\limits_{x\to 2/3}\dfrac{3x^2-5x+2}{3x^2-4x-4}$ (iii). $\lim\limits_{x\to -3}\dfrac{x^3+4x^2+4x+3}{x^2+2x-3}$

(iv). $\lim\limits_{x\to -3}\dfrac{x^3-4x-15}{x^3+x^2-6x-18}$ (v). $\lim\limits_{x\to 1}\dfrac{x^4-3x+2}{x^3-5x^2+3x+1}$ (vi). $\lim\limits_{x\to 2}\dfrac{x^2-x\log x+2\log x-4}{x-2}$

03. Find the limits of the following

(i). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$ (ii). $\lim\limits_{x\to 3}\dfrac{3-x}{\sqrt{4+x}-\sqrt{1+2x}}$ (iii). $\lim\limits_{x\to 2}\dfrac{x-2}{2-\sqrt{6-x}}$ (iv). $\lim\limits_{x\to 2}\dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$

04. Find the limits of the following.

(i). $\lim\limits_{x\to 0}\dfrac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}$ (ii). $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ (iii). $\lim\limits_{h\to 0}\dfrac{1}{h}\left[\dfrac{1}{\sqrt{x+h}}-\dfrac{1}{\sqrt{x}}\right]$

(iv). $\lim\limits_{x\to 1}\dfrac{x^2-1}{\sqrt{x+8}-3}$ (v). $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$ (vi). $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{2+3x}-\sqrt{2-3x}}$

05. Find the limits of the flolloiwing

(i). $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1}$ (ii). $\lim\limits_{x\to 2}\dfrac{x^7-128}{x-2}$ (iii). $\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}$ (iv). $\lim\limits_{x\to a}\dfrac{x\sqrt{x}-a\sqrt{a}}{(x-a)}$

(v). $\lim\limits_{x\to 3}\dfrac{x^5-243}{x^3-27}$ (vi). $\lim\limits_{x\to a}\dfrac{x^m-a^m}{x^n-a^n}$ (vii). $\lim\limits_{x\to -1}\dfrac{x^3+1}{x^5+1}$ (viii). $\lim\limits_{x\to 2}\dfrac{x^{1/4}-2^{1/4}}{x^{1/3}-2^{1/3}}$

(ix). $\lim\limits_{x\to -1}\dfrac{1+3\sqrt{x}}{1+5\sqrt{x}}$ (x). $\lim\limits_{x\to 1}\dfrac{1-x^{-1/3}}{1-2^{-2/3}}$

06. Evaluate $\lim\limits_{x\to a}\dfrac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

07. If $\lim\limits_{x\to 2}\dfrac{x^n-2^n}{x-2}=80$ find n where n is an integer.

08. If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1}=\lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$. Find the value of k.

09.If $\lim\limits_{x\to a}\dfrac{x^9-a^9}{x-a}=9$ Find all possible values a can assume.

10. Evaluate $\lim\limits_{x\to 2}\dfrac{(2x+4)^{1/3}-2}{x-2}$.

11 **Find the limit of the following.**

(i). $\lim\limits_{x\to\infty}\dfrac{(2x-3)(3x+5)(4x-6)}{3x^3+x-1}$ (ii). $\lim\limits_{x\to\infty}\dfrac{5x^2+3x-6}{2x^2-5x+1}$ (iii). $\lim\limits_{x\to\infty}\dfrac{2x^3+3x^2-5}{-5x^3+8x-17}$

(iv). $\lim_{x\to\infty}\left(\sqrt{x^2+5x+4}-\sqrt{x^2-3x+4}\right)$ (v). $\lim_{x\to\infty}\frac{(3x-1)(4x-2)}{(x+8)(x+1)}$ *(vi).* $\lim_{x\to\infty}\frac{(3x-1)^{30}(2x-1)^{30}}{(2x+4)^{60}}$

12. Evaluate $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x+3}-\sqrt{x}\right)$ 13.Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$ 14. Evaluate $\lim_{x\to\infty}\frac{x}{\sqrt{4x^2+1}-1}$

15. Evaluate $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$ 16.Evaluate $\lim_{x\to\infty}2x\left(\sqrt{x^2+1}-x\right)$ 17.Evaluate $\lim_{x\to\infty}\frac{\sqrt{x^2+1}-3\sqrt{x^2+1}}{4\sqrt{x^4+1}-5\sqrt{x^4+1}}$

18. Evaluate $\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

19. Evaluate the limits of the following trigonometirc functions.

(i). $\lim_{x\to0}\frac{\sin 3x}{5x}$ *(ii).* $\lim_{x\to0}\frac{\sin x^2}{x}$ *(iii).* $\lim_{x\to0}\frac{\tan\frac{x}{2}}{3x}$ *(iv).* $\lim_{x\to0}\frac{\tan x^2}{2x}$

(v). $\lim_{x\to0}\frac{\sin 4x}{xCosx}$ *(vi).* $\lim_{\theta\to0}\theta\cos ec\theta$ *(vii).* $\lim_{\theta\to0}\frac{\sin a\theta}{\sin b\theta}$ *(viii).* $\lim_{\theta\to0}\frac{\sin 4\theta}{\tan 3\theta}$

20. Find the limits of the folloiwng trigonometric functions.

(i). $\lim_{x\to0}\frac{\tan 3x}{\tan 5x}$ *(ii).* $\lim_{x\to0}\frac{3\sin^2 x-2\sin x^2}{5x^2}$ *(iii).* $\lim_{x\to0}\frac{\sin x^0}{x}$ *(iv).* $\lim_{x\to0}\frac{\cos 2x\sin 5x}{\sin 2x}$

21. Evaluate the limits of the folloiwng trigonometic functions.

(i). $\lim_{\theta\to0}\frac{1-\cos\theta}{\theta^2}$ *(ii).* $\lim_{x\to0}\frac{1-\cos 2x}{x^2}$ *(iii).* $\lim_{\theta\to0}\frac{1-\cos\theta}{2\theta^2}$

(iv). $\lim_{x\to0}\frac{1-\cos x}{x\sin x}$ *(v).* $\lim_{\theta\to0}\frac{1-\cos 4\theta}{1-\cos 5\theta}$ *(vi).* $\lim_{\theta\to0}\frac{1-\cos m\theta}{1-\cos n\theta}$

22. Evaluate the limits of the folloiwng trigonometric functions.

(i). $\lim_{x\to0}\frac{1-\cos x}{\sin^2 x}$ *(ii).* $\lim_{x\to0}\frac{\sin 2x(1-\cos 2x)}{x^3}$ *(iii).* $\lim_{x\to0}\frac{3\sin x-\sin 3x}{x^3}$

(iv). $\lim_{x\to0}\frac{2\sin x-\sin 2x}{x^3}$ *(v).* $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}$ *(vi).* $\lim_{x\to0}\frac{\tan 2x-\sin 2x}{x^3}$

23. Evaluate the limits of the following trigonometric functions.

(i). $\lim_{x\to0}\frac{\cos ax-\cos bx}{x^2}$ *(ii).* $\lim_{x\to0}\frac{\sin(2+x)-\sin(2-x)}{x}$

(iii). $\lim_{x\to0}\frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}$ *(iv).* $\lim_{x\to0}\frac{\tan x-\sin x}{\sin^3 x}$

24. Evaluate the limits of the following trigonometric functions.

(i). $\lim_{x\to0}\frac{\tan 3x-2x}{3x-\sin^2 x}$ *(ii).* $\lim_{x\to0}\frac{x^3\cot x}{1-\cos x}$ *iii.* $\lim_{x\to0}\frac{\tan^{-1}2x}{\sin 3x}$ *iv.* $\lim_{x\to1}\frac{1-\sqrt{x}}{\left(\cos^{-1}x\right)^2}$

25. Evaluate the following limits.

01. $\lim\limits_{x\to \pi/3} \dfrac{Sin\left(\pi/3 - x\right)}{2\cos x - 1}$ *02.* $\lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x\sin x\cdot\cos x}$ *03.* $\lim\limits_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

04. $\lim\limits_{x\to \alpha} \dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$, where are α,β the distinct roots of $ax^2+bx+c=0$

05. Show that, $\lim\limits_{x\to \tan^{-1}3} \dfrac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3} = 2$ *06.* $\lim\limits_{x\to \pi/3} \dfrac{\tan^3 x - 3\tan x}{\cos\left(x+\pi/6\right)} = -24$

07. $\lim\limits_{x\to \pi/4} \dfrac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x} = 5\sqrt{2}$

26. Evaluate the limits of the following trigonometric functions.

i. $\lim\limits_{x\to 0} \dfrac{\cos ax - \cos bx}{x^2}$ *ii.* $\lim\limits_{x\to 0} \dfrac{\sin(2+x)-\sin(2-x)}{x}$ *iii.* $\lim\limits_{x\to 0} \dfrac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$ *iv.* $\lim\limits_{x\to 0} \dfrac{\tan x - \sin x}{\sin^3 x}$

27. Evaluate the following limits

a. i. $\lim\limits_{x\to 0} \dfrac{Sin\,x^0}{x}$ ii. $\lim\limits_{x\to 0} \dfrac{3\sin^2 x - 2\sin x^2}{3x^2}$ iii. $\lim\limits_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$ iv. $\lim\limits_{x\to 0} \dfrac{\tan 2x - \sin 2x}{x^3}$

v. $\lim\limits_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$ vi. $\lim\limits_{x\to 0} \dfrac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$ vii. $\lim\limits_{x\to 0} \dfrac{x^3\cot x}{1-\cos x}$ viii. $\lim\limits_{x\to 0} \dfrac{x\cos x + \sin x}{x^2 + \tan x}$

b. Evaluate the limits of the folloiwng trigonometric functions.

1. $\lim\limits_{x\to 1} \dfrac{1+\cos\pi x}{(1-x)^2}$ 2. $\lim\limits_{x\to 1} \dfrac{1-x^2}{\sin \pi x}$ 3. $\lim\limits_{x\to \pi} \dfrac{1+\cos x}{(\pi - x)^2}$ 4. $\lim\limits_{x\to \pi/2} \dfrac{1-\sin x}{(\pi - 2x)^2}$

5. $\lim\limits_{x\to 1} (1-x)\dfrac{\tan\pi x}{2}$ 6. $\lim\limits_{\theta\to \pi/2} (\sec\theta - \tan\theta)$

c. Evaluate the limits of the folloiwng trigonometric functions.

1. $\lim\limits_{x\to 1} \dfrac{1-x}{\left(\cos^{-1} x\right)^2}$ 2. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sin^{-1} x}$ 3. $\lim\limits_{x\to 0} \dfrac{1}{x}.\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

4. $\lim\limits_{x\to 0} \dfrac{x\left(1-\sqrt{1-x^2}\right)}{\sqrt{1-x^2}.\left(\sin^{-1} x\right)^3}$ 5. $\lim\limits_{x\to 0} \dfrac{1-x}{\pi - 2\sin^{-1} x}$ 6. $\lim\limits_{x\to 1/\sqrt{2}} \dfrac{x-\cos\left(\sin^{-1} x\right)}{1-\tan\left(\sin^{-1} x\right)}$

d. Evaluate the limits of the folloiwng

1. $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{x}$ 2. $\lim\limits_{x\to 0} \dfrac{e^{\sin x}-1}{x}$ 3. $\lim\limits_{x\to 0} \dfrac{e^{x}-e^{-x}}{x}$ 4. $\lim\limits_{x\to 0} \dfrac{e^{x}-x-1}{x}$

f. Evaluate the limits of the folloiwng.

1. $\lim\limits_{x\to 0} \dfrac{x(e^x-1)}{1-\cos 2x}$ 2. $\lim\limits_{x\to 0} \dfrac{e^{ax}-e^{bx}}{x}$

Prove the following

28. $\lim_{y\to 0} \dfrac{(x+y)Sec(x+y)-x\,Secx}{y} = x\tan x\,Sec\,x + Secx$

29. $\lim_{x\to 0} \dfrac{1-\cos x\sqrt{\cos 2x}}{x^2} = \dfrac{3}{2}$

30. $\lim_{x\to 0} \dfrac{2\sin x^0(1-\cos x^0)}{x^3} = \left(\dfrac{\pi}{180}\right)^3$

31. $\lim_{x\to 0} \dfrac{1-\cos(1-\cos x)}{x^4} = \dfrac{1}{8}$

32. $\lim_{x\to 1} \dfrac{1+\cos\pi x}{(1-x)^2} = \dfrac{\pi^2}{2}$

33. $\lim_{x\to 1} (1-x)\tan\dfrac{\pi x}{2} = \dfrac{2}{\pi}$

34. $\lim_{\theta\to \pi/2} (Sec\,\theta - \tan\,\theta) = 0$

35. $\lim_{x\to 1} \dfrac{x^2-3x+2}{x^2-x+\sin(x-1)} = -\dfrac{1}{2}$

36. $\lim_{x\to\frac{\pi}{4}} \dfrac{\cos x-\sin x}{\cos 2x} = \dfrac{1}{\sqrt{2}}$

37. $\lim_{x\to 0} \dfrac{1}{x}\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = 2$

38. $\lim_{x\to 0} \dfrac{x\left(1-\sqrt{1-x^2}\right)}{\sqrt{1-x^2}\,(\sin^{-1}x)^3} = \dfrac{1}{2}$

39. $\lim_{x\to 1} \dfrac{1-x}{x-2\sin^{-1}x} = 0$

40. $\lim_{x\to\frac{1}{\sqrt{2}}} \dfrac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)} = -\dfrac{1}{\sqrt{2}}$

41.Evaluate : $\lim_{x\to 0} \dfrac{\sin^{-1}x-2x}{\sin^{-1}x-2\sin\left(\frac{1}{2}\sin^{-1}x\right)\left[3-4\sin^2\left(\frac{1}{2}\sin^{-1}x\right)\right]}$

42.Evaluate the following limits :

i) $\lim_{\theta\to\frac{\pi}{2}} \dfrac{\cos\theta}{\frac{\pi}{2}-\theta}$ ii) $\lim_{x\to\frac{\pi}{2}} \dfrac{\frac{\pi}{2}-x}{\cot x}$ iii) $\lim_{\theta\to\frac{\pi}{2}} \dfrac{\frac{\pi}{2}-\theta}{\sin 2\theta}$ iv) $\lim_{x\to\pi} \dfrac{\sin x}{x-\pi}$ v) $\lim_{x\to\frac{\pi}{4}} \dfrac{1-\tan x}{x-\frac{\pi}{4}}$ vi) $\lim_{\theta\to\frac{\pi}{4}} \dfrac{\sin\theta-\cos\theta}{\theta-\frac{\pi}{4}}$

43.Evaluate the following limits :

i) $\lim_{x\to 0} \dfrac{\sin^{-1}x}{x}$ ii) $\lim_{x\to 0} \dfrac{x}{\tan^{-1}2x}$ iii) $\lim_{x\to 0} \dfrac{1}{x}\cos ec^{-1}\left(\dfrac{1}{x}\right)$

iv) $\lim_{x\to 0} \dfrac{\sec^{-1}\sqrt{1+x^2}}{x}$ v) $\lim_{x\to 0} \dfrac{\tan^{-1}2x}{\sin 3x}$ vi) $\lim_{x\to 1} \dfrac{1-\sqrt{x}}{(\cos^{-1}x)^2}$

41 i. Let $y=f(x)$ Define $\dfrac{dy}{dx}$ at $P(x,y)$ using increments. Hence show that the derivative $f'(x)$ at $x=a$ is given by $f^{/}(a) = \lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$

ii Show that $\lim_{x\to a} \dfrac{x^2 f(a)-a^2 f(x)}{x-a} = 2a\,f(a)-a^2\,f'(a)$.

iii. If $f(2)=4$ and $f'(2)=1$ Show that $\lim_{x\to 2} \dfrac{xf(2)-2f(x)}{x-2} = 2$

iv. If $f(a)=2, f'(a)=1$ $g(a)=-1$ and $g'(a)=2$ Show that

$$\lim_{x\to a} \frac{g(x)f(a)-g(a)\,f(x)}{x-a} = 5.$$

Differentiation

42. Using first principle, find the differential coefficient $\left(\frac{dy}{dx}\right)$ of the following

i. $\sqrt{2x+3}$ ii. $\sqrt{x^2+1}$ iii. $(ax+b)^n$ iv. $\sin x$ v. $\cos x$ vi. $\tan x$ vii. $\cot x$ viii. $Sec\ x$

ix. $\cos ec\ x$ x. $\sin x^2$ xi. $\cos^2 x$ xii. $\sec(2x+3)$ xiii. $\tan^2 ax$ xiv. $\sin\sqrt{x}$

xv. $\sqrt{\cot x}$ xvi. $x^2\cos x$ xvii. e^x xviii. x^n

43. Using first principle obtain $\frac{dy}{dx}$ of the following

i. $\sin^{-1} x$ ii. $\cos^{-1} x$ iii. $\tan^{-1} x$

44.Using the derivative of e^x obtain the derivative of $\ln x\ (\log_e x)$

45. Find the derivative of the following functions using first principle.

i). $\sin 2x$ ii). $\cos 3x$ iii). $\tan kx$ iv). $\sec ax$ v). $\mathrm{cosec}(ax+b)$ vi). $\sec(2x+3)$

vii). $\cot(2x+1)$ viii). $\sin^2 x$ ix). $\tan^2 ax$ x). $\sin x^2$ xi). $\cos(x^2+1)$

46. Find the first differential coefficient of the following using first principles.

i). $\sqrt{\cos x}$ ii). $\sqrt{\sin 3x}$ iii). $\sqrt{\sec x}$ iv). $\sqrt{\cot x}$

v). $\sin\sqrt{x}$ vi). $\tan\sqrt{x}$ vii). $x\sin x$ viii). $x^2\cos x$

47. Differentiate the following functions.

i). $y=2x^3-4x^2+6x+8$ *ii).* $y=x+\frac{1}{x}$ *iii).* $y=3\cos x-4\ln x$ *iv).* $y=e^x+3\sin x$

v). $y=x^{3/2}+4x^2$ *vi).* $y=\frac{3}{x^7}$ *vii).* $y=x^4+2x^{1/4}+\frac{2}{3\sqrt{x}}+5$ *viii).* $y=4\cot x+\cos x-\ln x$

48. Obtain the differential coefficient of the following functions.

i). $y=e^x$ ii). $y=\ln x$ iii). $y=a^x$, a constant

49 Differentiate the following inverse trigonometric functions with respec to x.

i). $y=\sin^{-1} x$ ii). $y=\cos^{-1} x$ iii). $y=\tan^{-1} x$

50. Differentiate the following functions with respect to x.

i). $y=\cos x\cot x$ ii). $y=x^4\sin x$ iii). $y=x^{1/5}e^x$ iv). $y=2^x\ln x$ v). $y=x^3\sin^{-1} x$

vi). $y=x^3\cos x+e^x\cot x$ vii). $y=x^3\ln x+(1+x^2)\tan^{-1} x$ viii). $y=x^3\sin x+2x\cos x-\cos ecx\ln x$

ix). $y=(x\sin x+\cos x)(x\cos x-\sin x)$

51. Differentiate the following function with respect to x.

i). $y=(x+1)(3x^2+2)(5x^2+3)$ ii). $y=x\sin x\ln x$ iii). $y=x\sin xe^x$

52. Differentiate the following functions with respect to *x*.

i). $y = \dfrac{\ln x}{x}$ ii). $y = \dfrac{x}{1+\tan x}$ iii). $y = \dfrac{x+\sin x}{x+\cos x}$ iv). $y = \dfrac{1+e^x}{1-e^x}$

v). $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$ vi). $y = \dfrac{\sin x + e^x}{1+\ln x}$ vii). $y = \dfrac{2x^2-4}{3x^2+7}$ viii). $y = \dfrac{3x^2-2}{x^2+7}$

ix). $y = \dfrac{\sec x - 1}{\sec x + 1}$ x). $y = \dfrac{\log x}{1 + x\log x}$ xi). $y = \dfrac{1}{ax^2+bx+c}$ xii). $y = \dfrac{1-\sqrt{x}}{1+\sqrt{x}}$

xiii). $y = \dfrac{\sec x + \tan x}{\sec x - \tan x}$ xiv). $y = \dfrac{x\sin x - \cos x}{x\cos x + \sin x}$

53. Differentiate the following functions with respect to x

i). $y = \left[\dfrac{3+4x}{2-x}\right]^2$ ii). $y = \sqrt{\sin x^3}$ iii). $y = \tan(\ln x)$ iv). $y = e^{\cos x}$ v). $y = \left(5x^2 - 7x + 13\right)^{2/5}$

vi). $y = \ln\sin x$ vii). $y = \sec\sqrt{x}$ viii). $y = \left(x^2 + \cos x\right)^2$ ix). $y = \ln(e^{nx} + e^{-nx})$ x). $y = \sin 2x \cos 3x$

xi). $y = \cos(1-x^2)^2$ xii). $y = \dfrac{1}{\ln\cos x}$ xiii). $y = \ln(\ln x)$ xiv). $y = \ln\cos\left(\sqrt{x}\right)$ xv). $y = e^{\sin\sqrt{x}}$

54. Differentiate the following functions with respect to x.

i). $y = \sqrt{\dfrac{1+x}{1-x}}$ ii). $y = \sqrt{\dfrac{a^2-x^2}{a^2+x^2}}$ iii). $y = \dfrac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

iv). $y = \ln\left(x + \sqrt{1+x^2}\right)$ v). $y = \sqrt{a^2 + \sqrt{a^2+x^2}}$ vi). $y = \dfrac{\sqrt{a^2+x^2}+\sqrt{a^2-x^2}}{\sqrt{a^2+x^2}-\sqrt{a^2-x^2}}$

vii). $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ viii). $y = \sin\left[\cos\left(\sin\sqrt{ax+b}\right)\right]$ ix). $y = \cos\left[\sin\sqrt{ax+b}\right]$

x). $y = \sqrt{\dfrac{1-\sin x}{1+\sin x}}$ *xi).* $y = \sqrt{\sin\sqrt{1+x^2}}$ *xii).* $y = \sqrt{\dfrac{\sec x + \tan x}{\sec x - \tan x}}$

55. Differentiate the following functions with respect to x.

i). $y = \ln\tan\left(\pi/4 + x/2\right)$ ii). $y = \ln\left(\sin x/2 + \cos x/2\right)$ iii). $y = \sin\left(\sqrt{\sin x + \cos x}\right)$

iv). $y = \ln\left(x + e^{\sqrt{x}}\right)$ v). $y = \ln\left[\ln x\right]$ vi). $y = \log\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

56. Differentiate the following functions with respect to x.

i). $y = \log_2 \cos x$ ii). $y = \log_{x^2} \sin x$ iii). $y = \log_{\cos x} \sin x$ iv). $y = \sqrt{\log\sin\left(\dfrac{x^2}{3} - 1\right)}$

v). $y = \left[x + \sqrt{x^2+a^2}\right]^n$ vi). $y = \dfrac{e^{2x}\cos x}{x\sin x}$ vii). $y = e^{-ax^2}\sin(\ln x)$ viii). $y = \log_{10}(\ln\cos x)$

57. Differentiate the following functions with respect to x.

i). $y=\sin^{-1}\sqrt{x}$ ii). $y=\tan^{-1}(\ln x)$ iii). $y=\cos^{-1}(\sin x)$ iv). $y=\sin(\tan^{-1}x)$

vi). $y=e^{\sin^{-1}(x-1)}$ vii). $y=\tan^{-1}(xe^{x})$ viii). $y=\cos^{-1}\left(\frac{x}{x+1}\right)$ x). $y=\left(\sin^{-1}x^{2}\right)^{3}$

58. Differentiate the following functions with respect to x.

i). $y=\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ ii). $y=\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$ iii). $y=\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$

vi). $y=\cos^{-1}\sqrt{\frac{1+\cos 2x}{2}}$ v). $y=\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ vi). $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

59. Differentiate with respect to x the functions given below.

i). $y=\tan^{-1}\left(\sec x+\tan x\right)$ ii). $y=\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

iii). $y=\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$ iv). $y=\cos^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{2}\right)$

60. Differentiate the functions given below with respect to x.

i). $y=x^{x}$ ii). $y=x^{\sin x}$ iii). $y=(\sin x)^{\tan x}$ iv). $y=(\log x)^{x}$ v). $y=x^{\log x}$ vi). $y=\left(\frac{1}{x}\right)^{x}$ vii). $y=x^{\sin^{-1}x}$

viii). $y=(x\sin x)^{x}$ ix). $y=\left(1+\frac{1}{x}\right)^{x}$ x). $y=x^{\sin 2x+\cos 2x}$ xi). $y=x^{x}\sqrt{x}$ xii). $y=\cos(x^{x})$

61. Differentiate the following functions with respect to x.

i). $y=x^{\tan x}+\left(\sin x\right)^{\cos x}$ ii). $y=\left(\sin x\right)^{\cos x}+\left(\cos x\right)^{\sin x}$

62. u and v are functions of x show by first principle that $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v.\frac{du}{dx}$

63. **Differentiate the following**

i. $(x\sin x+\cos x)\,(e^{x}+x^{2}\ln x)$ ii. $a^{x}\log_{a}x+e^{x}\log_{e}x$ iii. $x^{3}\sin x+2x\cos x-\operatorname{cosec}x\log_{e}x$

iv. $(x\sin x+\cos x)\,(x\cos x-\sin x)$ v. $x^{3}\sin x+2x\cos x-\operatorname{cosec}x\log_{e}x$

vi. If $y=x\tan x$ prove that $x\sin^{2}x\frac{dy}{dx}=y^{2}+y\sin^{2}x$

vii. If $y=x\sin x$ prove that $\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\cot x$

viii. If $f(x)=(ax+b)\sin x+(cx+d)\cos x$, find the values of a,b,c and d such that $f'(x)=x\cos x$ for all x

64. 1. When u and v are functions of x show that $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^{2}}$

2. Differentiate the following

i. $\dfrac{x\sin x-\cos x}{x\cos x-\sin x}$ ii. $\dfrac{\sec x+\tan x}{\sec x-\tan x}$ iii. $\dfrac{1+3^x}{1-3^x}$ iv. $\dfrac{x\ln x}{e^x\tan x}$ v. $\dfrac{e^x(x-1)}{x^2+1}$

vi. If $y=\dfrac{x}{x+5}$ prove that $x\dfrac{dy}{dx}=y(1-y)$

vii. If $f(x)=\dfrac{x\sin x}{\cos x-\sin x}$ find $f'(0)$

65. (i)If y is a differentiable function of u and u is a differentiable function of x show that $\dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}$

66. Differentiate the following

i. $\sqrt{a+\sqrt{a+x}}$ ii. $\log(x+\sqrt{1+x^2})$ iii. $\cos^2 x\, e^{\tan x}$ iv. $Sin(\cos(\sin(\tan 2x)))$

v. $\dfrac{\sin^2 x+\cos x}{1+x^2}$ vi. $\log_2\cos x$ vii. $\log_{x^2}\sin x$ viii. $\log_{\cos x}\sin x$

67. Differentiate the following functions

i. $\sin^{-1}(\cos x)$ ii. $(\tan^{-1}x)^2$ iii. $\tan^{-1}(\ln x)$

iv. $(\sin^{-1}x^3)^3$ v. $\tan^{-1}(\sin x^2)$ vi. $\ln(\cos^{-1}x^3)$

68. If $y=\sin^{-1}(\cos x)+\cos^{-1}(\sin x)$ find $\dfrac{dy}{dx}$ and express the result free from trigonometrical symbols

69. Using the fact $\sin(A+B)=\sin A\cos B+\cos A\sin B$ and the technique of differentiation obtain that $\cos(A+B)=\cos A\cos B-\sin A\sin B$

70. Using suitable trigonometric substitutions differentiate the following

i. $\sin^{-1}\sqrt{1-x^2}$ ii. $\sin^{-1}(2x\sqrt{1-x^2})$ iii. $\tan^{-1}\sqrt{\dfrac{1+x}{1-x}}$ iv. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

v. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ vi. $\tan^{-1}\sqrt{\left(\dfrac{1-x}{1+x}\right)}$ vii. $\cos^{-1}\left(\dfrac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$ viii. $\tan^{-1}\left(\dfrac{x}{1+\sqrt{1-x^2}}\right)$

71. Differentiate the following functions and find $\dfrac{dy}{dx}$

i. $x^3y^2=\ln(x+y)+\sin e^x$ ii $\sin(xy)+\dfrac{x}{y}=x^2-y$ iii. $\sin^2 x+2\cos y+xy=0$ iv. $\tan(x+y)+\tan(x-y)=1$

72. Find $\dfrac{dy}{dx}$ by differentiating the following functions.

i). $x^2+y^2=a^2$ ii). $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ iii). $ax^2+2hxy+by^2+2gx++2fy+c=0$ iv). $xy=x+y$

v). $x^3+y^3=3axy$ vi). $(x^2+y^2)=xy$ vii). $xy^2-x^2y=4$ viii). $xy+xe^{-y}+ye^x=x^2$

ix). $x^3y^3=\log(x+y)+\sin e^x$ x). $\sin xy+\dfrac{x}{y}=x^2-y$

73. Find $\frac{dy}{dx}$ by differentiating the following functions.

i). If $y=\sqrt{x+\sqrt{x+\sqrt{x+.........\infty}}}$ show that $(2y-1)\frac{dy}{dx}=1$

ii). If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+.........\infty}}}$ show that $(2y-1)\frac{dy}{dx}=\cos x$

iii). If $y=x^{x^{x^{x^{\cdots\infty}}}}$ show that $x\frac{dy}{dx}=\frac{y^2}{1-y\log x}$ iv). If $y=e^{x+e^x}$ show that $x\frac{dy}{dx}=\frac{y}{1-y}$

74. If $\sin y = x\sin(a+y)$ prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$ 75. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ prove that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$

76. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

77. If $y=x^x\frac{2x-1}{2x+1}$ then prove that $\frac{dy}{dx}=x^x\frac{2x-1}{2x+1}\left(\frac{4x^2+3}{4x^2-1}+\log x\right)$

78. If $x^y=y^x$ prove that $\frac{dy}{dx}=\frac{y(x\log y-y)}{x(y\log x-x)}$ 79. If $x^y+y^x=2$ then prove that

80. Differentiate following parametric functions and find $\frac{dy}{dx}$

i. $x=2at^2, y=at^4$ ii. $x=\cos\theta-\cos 2\theta$, $y=\sin\theta-\sin 2\theta$

iii. If $x=\sin^{-1}\left(\frac{2t}{1+t^2}\right), y=\tan^{-1}\left(\frac{2t}{1-t^2}\right)$ prove that $\frac{dy}{dx}=1$

iv. If $x=a\sin 2\theta(1+\cos 2\theta)$ and $y=a\cos 2\theta(1-\cos 2\theta)$ show that $\frac{dy}{dx}=\tan\theta$

v. If $x^3+y^3=t-\frac{1}{t}$ and $x^6+y^6=t^2+\frac{1}{t^2}$. Show that $x^4y^2\frac{dy}{dx}=1$.

81. let $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1-x^2}+\sqrt{1-x^2}}\right)$ and $z=\cos^{-1}x^2$ then show that $\frac{dy}{dz}=-\frac{1}{2}$

82. let $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $z=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ prove that $\frac{dy}{dz}=\frac{1}{4}$

83. let $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $z=\tan^{-1}x$ Find $\frac{dy}{dz}$

84. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

85. i. If $y=\tan x+\sec x$ prove that $\frac{d^2y}{dx^2}=\frac{\cos x}{(1-\sin x)^2}$

ii. If $y = ae^{mx} + be^{-mx}$ prove that $\dfrac{d^2y}{dx^2} - m^2 y = 0$

iii. If $y = \sin(\log x)$ prove that $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

iv. If $y = (\tan^{-1} x)^2$ prove that $(x^2+1)^2 \dfrac{d^2y}{dx^2} + 2x(x^2+1)\dfrac{dy}{dx} = 2$

v. If $y = a\cos(\log x) + b\sin(\log x)$ then prove that $x^2 \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 0$

vi. If $y = \log(x + \sqrt{x^2+1})$ prove that $(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 0$

86. If $x = \sin t, y = \sin pt,$ prove that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + p^2 y = 0$

87. If $x = \log t$ and $y = \dfrac{1}{t}$, show that $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$

88. If $x = a\sin t - b\cos t,\ y = a\cos t + b\sin t,$ show that $\dfrac{d^2y}{dx^2} = -\dfrac{x^2+y^2}{y^3}$

89. If $x = a(1-\cos\theta), y = a(\theta + \sin\theta)$ then find $\dfrac{d^2y}{dx^2}$ at $\theta = \pi/2$

90. If $x = a(1-\cos^3 t), y = a\sin^3 t$ find $\dfrac{d^2y}{dx^2}$ at $t = \pi/6$

91. y is a function of x and $x = \sin\theta$ Express $\dfrac{d^2y}{dx^2}$ in terms of $\dfrac{dy}{d\theta}$ and $\dfrac{d^2y}{d\theta^2}$

If $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + ky = 0$ prove that $\dfrac{d^2y}{d\theta^2} + ky = 0$.

92. If $\sin y = 2\sin x,$ prove that

i. $\left(\dfrac{dy}{dx}\right)^2 = 1 + 3\sec^2 y$ ii. $\dfrac{d^2y}{dx^2} = 3\sec^2 y\ \tan y$ also prove that $\cot y\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^2 + 1 = 0$

93. i. If $y = e^{\cos x}$ find $\left(\dfrac{dy}{dx}\right)_{x=0}, \left(\dfrac{d^2y}{dx^2}\right)_{x=0}, \left(\dfrac{d^3y}{dx^3}\right)_{x=0}, \left(\dfrac{d^4y}{dx^4}\right)_{x=0}, \left(\dfrac{d^{5y}}{dx^5}\right)_{x=0}$

94. The transformation $z = \log\tan\left(\dfrac{x}{2}\right)$ reduces the differential equation $\dfrac{d^2y}{dx^2} + \cot x\dfrac{dy}{dx} + 4y\,\mathrm{cosec}^2 x = 0$

in to $\dfrac{d^2y}{dz^2} + Ay = 0$ .show that the value of A is 4.

95. The transformation $x = \cos\theta,$ reduces the differential equation.

$(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ into $\dfrac{d^2y}{d\theta^2} + \dfrac{A}{28}y = 0$. Show that A=28.

96. If $x = 2t^3 + 1$ and $y = 4t^4 - 1$ Show that $\left(\frac{dy}{dx}\right)\left(\frac{d^3y}{dx^3}\right) + 2\left(\frac{d^2y}{dx^2}\right)^2 = 0$

97. Establish the following differential equations.

1. If y=A sin mx + B cos mx, show that $\frac{d^2y}{dx^2} + m^2y = 0$ 2. If y=tan x + sec x, show that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$

3. If y=ae^{mx}+be^{-mx}, show that $\frac{d^2y}{dx^2} - m^2y = 0$ 4. If y=sin(log$x$), show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

98. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

99. If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2\frac{d^2y}{dx^2} - 2x(x^2+1)\frac{dy}{dx} = 2$

100. If $y = \sin(m\sin^{-1}x)$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

101. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

102.If $y = \log\left(x + \sqrt{x^2+1}\right)$, show that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

103. If $y = e^{\tan^{-1}x}$, show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$

104. Establish the following differential equations.

i. if $y = \log\left|\sqrt{1+x} + \sqrt{x-1}\right|$; $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

ii. If $y = \sin[\log|1+x|]$; $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 0$

iii. If $y = \sqrt{1-x^2}\sin^{-1}x$; $(1-x^2)\frac{dy}{dx} + xy + x^2 - 1 = 0$

iv. If $\tan y = \log x^2, x\frac{dy}{dx} = 2\cos^2 y$ and $x\frac{d^2y}{dx^2} + \frac{dy}{dx}(1 + 2\sin 2y) = 0$

v. If $y = ae^{px} + be^{-px}$, $\frac{d^2y}{dx^2} - p^2y = 0$ vi. If $y = (\cos^{-1}x)^2 - a\cos^{-1}x$, $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

vii. $y = a\cos(\log x) + b\sin(\log x)$, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ viii. If $y = \log_e(1+\sin x)$; $\frac{d^2y}{dx^2} + e^{-y} = 0$

105.If $x = 2\cos t - \cos 2t$ and $y = 2\sin t - \sin 2t$. Find $\frac{d^2y}{dx^2}$ at $t = \pi/2$

106.If $x = \sin t, y = \sin pt$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

107.If $u = u(x)$ and $v = v(x)$, prove the following formula.

i. $\dfrac{d(u+v)}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ ii. $\dfrac{d(uv)}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ iii. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

108. If $u = u(x)$ and $v = v(x)$, prove the following formula.

i. $\dfrac{d(u+v)}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$

109. If $x = \sin\theta, y = \sin p\theta$ prove that $\left(1-x^2\right)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + p^2y = 0$ p is a positive constant.

110. If $x = \tan\theta,\ \ y = \tan p\theta$ then show that, $\left(1+x^2\right)\dfrac{d^2y}{dx^2} = 2(py-x)\dfrac{dy}{dx}$ where p is a constant.

Application of derivatives

Rate of change

111.A balloon which always remains spherical is being inflated by pumping in $900\ \text{cm}^3$ of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is $150(cm)$

112. A particle moves along the curve $6\text{y} = \text{x}^3 + 2$ Find the points. On the curve at which the y-coordinate is changing 8 times as fast as the x- coordinate

113.A man $2\ m$, high walks at a uniform speed of $6\ \text{kmh}^{-1}$ away from a lamp post $6\ m$ high. Find the rate at which the length of his shadow increases.

114. A balloon gets inflated so that the rate of change of volume is proportional to its radius lnitially its radius is 2 units and after 1 second it is 3 units. Find its radius at time t.

115. Water is dripping out from a conical funnel of semi - vertical angle $\pi/4$ at the uniform rate $2\ cm^3/s$ through a tiny hole at the vertex at the bottom. When the slant height of the water is $4\ cm$, find the rate of decrease of the slant height of the water .

116. Find the equation of the tangent to the curve $x = a\cos\theta, y = b\sin\theta$ at $\theta = \dfrac{\pi}{4}$.

Equation of tangents and normals

117. Find the equation of tangent at the point "t" on the curve $x = a\sin^3 t, y = b\cos^3 t$

118. Find the point (points) on the curve $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$ where the tangent is perpendicular to x axis $(o \le \theta \le \pi)$.

119. Show that the carves $x = y^2$ and $x\ \ y = k$ intersect at right angles if $8k^2 = 1$.

120. Find the equation of tangents to the curve $y = \cos(x+y), -2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$.

Maxima and minima

121.Show that the surface area of a closed cuboid with square base and given volume is minimum , when it is a cube

122.An Open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least, when depth of the tank is half of its width.

123.A jet of an enemy is flying along the curve $y = x^2 + 2$ A soldier is placed at the point (3,2) what is the nearest distance between the soldier and the jet ?

124. A closed right circular cylinder has volume 2156 cubic units. What should be the radius of the base so that the total surface area may be minimum ?

125. A given quantity of metal is to be cast into half cylinder with a rectangular base and semi- circular ends. show that in order that total surface area is minimum the ratio of length of cylinder to the diameter of its semi - circular ends is $\pi : (\pi + 2)$

126.The cost of fuel for running a bus is proportional to the square of the speed generated in km/hr It costs Rs 48 per hour when the bus is moving at a speed of 20 km/hr what is the most economical speed if the fixed charges are Rs. 108 for an hour

127.Show that among all the rectangles of given perimeter, the square has the greatest area.

128.Prove that among all the traingles of given hypotenuse, the isosceles traiangle has the maximum area.

129.Prove that the perimeter of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.

130.Prove that the triangle of maximum area inscribed in a circle must be equilateral.

131.Prove that among all the rectangles that can be inscribed in a given circle, the square has the maximum area.

132. An open box with a square base is to be made out of given quantity of sheet of area k^2. Show that the maximum volume of the box is $\frac{k^2}{6\sqrt{3}}$

133.Show that the semi-vertical angle of the cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

134.Prove that the semi-vertical angle of a cone of given total surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

135.Show that the height of a closed right circular cylinder of given surface area and maximum volume is equal to the diameter of its base.

136.Show that the height of the right circular cylinder of maximum volume is equal to the that can be inscribed in a given right circular cone of height H is (H/3)

137.Prove that the height of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius r is $\left(2r/\sqrt{3}\right)$.

138.Prove that the height of a cone of maximum volume that can be incribed in a sphere of radius r is $\frac{4r}{3}$.

139. Prove that the volume of the largest cone that can be in scribed in a sphere is (8/27) of the volume of the sphere.

Curve sketching

133.Sketch the graph of $y=\dfrac{x}{x^2+1}$ by finding maximum and minium values. Show that this graph is completely lying between $y=\pm\frac{1}{2}$

134.The function $y=\dfrac{ax+b}{(x-1)(x-4)}$ has a stationary point at P(2,-1) Find the values of *a* and *b* and show that y is maximum at p sketch the graph.

135. Trace the graph of $y=\dfrac{x}{(x-1)(x-4)}$ Deduce that when $-1<k<-\frac{1}{9}$ the equation $k(x-1)(x-4)-x=0$ has no real solution.

136.Sketch the graph of $y=\dfrac{2x^2}{x^2-1}$. Hence prove that when $0<k<2$ the equation $(2-k)x^2+k=0$ has no real solution.

137.Sketch the graph of $y=\dfrac{x^2}{1+x^2}$.Hence prove that when $0<k<1$ the equation $x^2(1-k)-k=0$ has two distinct real roots.

138.Sketch the graph of $y=\dfrac{x}{2}+\dfrac{2}{x}$. Hence prove that when $-2<k<2$ the equation $x^2-2kx+4=0$ has no real solutions.

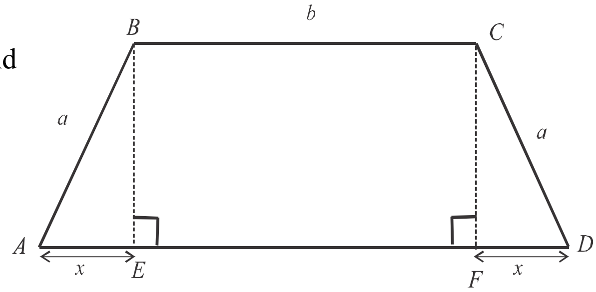
139. *a*.Let $y = x\sin\dfrac{1}{x}$ for $x\neq 0$. Show that

i. $x\dfrac{dy}{dx} = y - \cos\dfrac{1}{x}$ and ii. $x^4\dfrac{d^2y}{dx^2} + y = 0$

b.Let $f(x) = \dfrac{2x^2+1}{(x-1)^2}$ for $x\neq 1$.Find the first derivative and the turning point of $f(x)$. Sketch the graph of $y=f(x)$ indicating the turning point and asymptotes.

c. In the given figure, *ABCD* is a trapezium with parallel sides *BC* and *AD*. Lengths of its sides, measured in centimetres are given by $AB = CD = a$, $BC = b$ and $AD = b+2x$, where $0 < x < a$.

BE and *CF* are the perpendiculars drawn from the vertices *B* and *C*, respectively, on to the side *AD*.



Show that the area *S*(*x*) of the trapezium *ABCD* is given by $S(x) = (b+x)\sqrt{a^2 - x^2}$ in square centimetres. If $a = \sqrt{6}$ and $b=4$, show further that *S*(*x*) is maximum for a certain value of *x*, and find this value of *x* and the maximum area of the trapezium.

140.*a*. Let $f(x) = \dfrac{x^2}{(x-1)(x-2)}$ for $x\neq 1,2$.

Show that $f'(x)$, the derivative of *f*(*x*), is given by

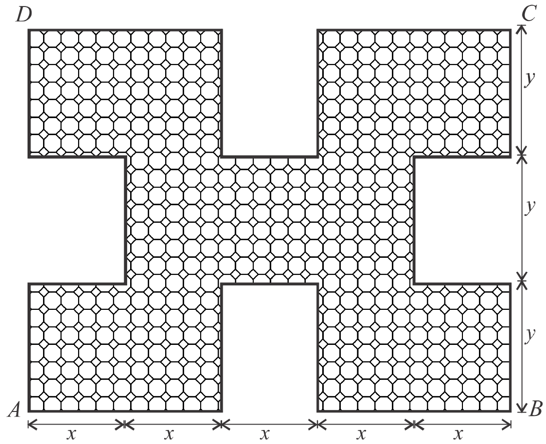
$$f'(x) = \frac{x(4-3x)}{(x-1)^2(x-2)^2} \text{ for } x \neq 1,2 .$$

Sketch the graph of $y = f(x)$ indicating the asymptotes and the turning points. Using the graph, solve the inequality $\dfrac{x^2}{(x-1)(x-2)} \leq 0.$

b.The shaded region shown in the adjoining figure is of area $385\ m^2$. This region is obtained by removing four identical rectangles each of length y metres and width x metres from a rectangle $ABCD$ of length $5x$ metres and width $3y$ metres. Show that $y = \dfrac{35}{x}$ and that the perimeter P of the shaded region, measured in metres, is given by

$P = 14x + \dfrac{350}{x}$ for $x > 0.$

Find the value of x such that P is minimum.



141.a.Let $f(x) = \dfrac{(x-3)^2}{x^2-1}$ *for* $x \neq \pm 1.$

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \dfrac{2(x-3)(3x-1)}{(x^2-1)^2}$

Write down the equations of the asymptotes of $y = f(x)$.

Find the coordinates of the point at which the horizontal asymptote intersects the curve $y = f(x)$.

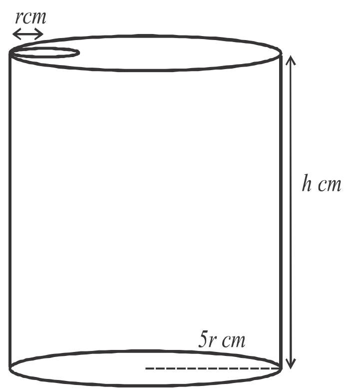
Sketch the graph of $y = f(x)$ indicating the asymptotes and the turning points.

b.A thin metal container, in the shape of a right circular cylinder of radius $5r$ *cm* and height h *cm* has a circular lid of radius $5r$ *cm* with a circular hole of radius r *cm*. (See the figure .)

The volume of the container is given to be $245\pi\ cm^3$.

Show that the surface area $S\ cm^2$ of the container with the lid containing the hole is given by $S = 49\pi\left(r^2 + \dfrac{2}{r}\right)$ for $r > 0$

Find the value of r such that S is minimum.



142.(a).By considering only the first derivative find the minimum and maximum values of $\dfrac{x^3}{x^4+27}$.

Sketch the graph of $y = \dfrac{x^3}{x^4+27}$.

Hence, find for what values of k, the equation $kx^4 - x^3 + 27k = 0$, where k is real, has

i. two coincident real roots ii. three coincident real roots.
iii. two distinct real roots. iv. no real roots

b).Consider a rectangle $ABCD$ with $AB = a$ and $BC = b(< a)$. Let P be a movable point on CD. The length of $AP + PB$ is $L(x)$, where $DP = x$. Show that $L(x) = \sqrt{x^2+b^2} + \sqrt{(a-x)^2+b^2}$.

Find the minimum length of $L(x)$ and the position of P on CD

correspondingto this minimum length. Also, find the maximum length of $L(x)$.

143. (a). Show that $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos 5x}{\sqrt{1+x^2}-\sqrt{1-x^2}}=8$.

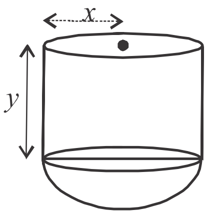
(b). Let $x=3-2\sin^2\theta$ and $y=2\cot\theta$ for $0<\theta<\dfrac{\pi}{2}$.

Find $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ and hence show that $\dfrac{dy}{dx}=\dfrac{1}{2\sin^3\theta\cos\theta}$. Find $\dfrac{d^2y}{dx^2}$ in terms of θ.

c). Let ABC be a triangle such that $AB=AC=40m$ and $BC=64m$.
A point P inside the triangle ABC is equidistant from B and C. By taking x to be the perpendicular distance from P to BC in metres,
express $2AP+3BP+3CP$ in terms of x. Hence, find the position of P such that $2AP+3BP+3CP$ is minimum.

144. a) Given that $y=\ln\left|x+\sqrt{1+x^2}\right|$; Prove that, $(1+x^2)(dy/dx)^2$ is independent of x.

Hence, show that $(1+x^2)\dfrac{d^3y}{dx^3}+3x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=0$.



b) A closed container takes the form of a cylinder of radius $x(cm)$ and height $y(cm)$ surmounted by a hemispherical cap as shown in the figure. Given that the volume of this container is $45\pi(cm^3)$, express y in terms of x and show that the total surface area of the container is given by $A(x)=\dfrac{5}{3}\pi x^2+\dfrac{90\pi}{x}$. Find the radius of the cylinder for the total surface area is to be minimum.

c)Let $f(x)=\dfrac{p}{x+2}+\dfrac{q}{x-1}$ for $x\neq -2,1$. where $p,\ q\in\Re$.

The graph of $y=f(x)$ has a turning point at (-1,3). Find the values of p and q.

Show that $f^1(x)=\dfrac{3(x+5)\,(x+1)}{(x+2)^2\,(x-1)^2}$; Sketch the graph of $y=f(x)$.

Indicating the local maximum and minimum and the asymptotes. Using the above graph, sketch the graph of $y=f(-x)$.

145. a) i. By differentiating suitable functions, prove that $\sin x<x<\tan x$ for every $x\in\left(0,\dfrac{\pi}{2}\right)$

Hence show that $\lim\limits_{x\to 0^+}\dfrac{sinx}{x}=1$ Deduce that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$.

ii) Show that $\lim\limits_{x\to 0}\dfrac{1-\cos^2(2\sin x)}{1-\cos 2x}=1$

b) Prove that $\dfrac{d}{dx}\{\ln|x|\}=\dfrac{1}{x};x\neq 0;$

If $y=\left[\ln\left|x+\sqrt{1+x^2}\right|\right]^2$ show that $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=2$

c) A man wishes to enclose a rectangular plot by a fence and then divide it into two plots by another fence parallel to one of the sides.

Find the dimensions of the rectangular plot that would enclose the largest area if the total length of the fences is 1200 (m).Also find the maximum area.

146. *a*) Using first principles obtain $\dfrac{d}{dx}(\cos x)$. Hence, show that $\dfrac{d}{dx}(\sin x)=\cos x$

b) If $x=\cos t$ and $y=\cos kt$

Prove that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+k^2y=0$. Where *k* is a real constant and *t* is a parameter.

c). Let $f(x)=x^4+qx^2+r$; where *q,r* are real constants.

The graph of $y=f(x)$ passes through the point (2,-16) and has a horizontal tangent there.

Find the values of *q* and *r*.

For these values of *q* and *r*, sketch the graph of $y=f(x)$ indicating,

(i).the intervals on which the graph of $y=f(x)$ is increasing or decreasing.

(ii).the coordinates of the local maximum point and local minimum point.

Hence sketch the graph of $y=\dfrac{1}{f(x)}$. Find the least number of zero values for the equation $e^x.f(x)-1=0$.

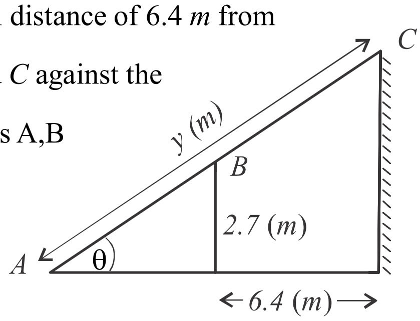
147. *a*) If $y=x\sin\left(\dfrac{1}{x}\right)$ for $x\neq 0$; show that $x^4\dfrac{d^2y}{dx^2}+y=0$.

b) Let $f(x)=2+\dfrac{a}{(x-1)}+\dfrac{b}{(x+1)}; x\neq\pm 1$ and *a* and *b* are real constants. If it is given that (0,0) is a turning point of the graph of $y=f(x)$, determine the values of *a* and *b* and show that (0, 0) is the only turning point.

Sketch the graph of $y=f(x)$, clearly indicating the turning points and the asymptotes.

Hence show that the equation $f(x)-\ln(x-1)=0$ has only one real root.

c) A vertical wall 2.7 *m* high runs parallel to the wall of a house and is at a horizontal distance of 6.4 *m* from the house. An extending ladder is placed to rest on the top *B* of the wall with one end *C* against the house and the other end *A*, resting on horizontal ground as shown in figure.The points A,B and C are in a vertical plane at right angles to the wall and ladder makes an angle θ with yhe horizontal. Show that the length *y* metrs of the ladder is given by



$y=\dfrac{2.7}{\sin\theta}+\dfrac{6.4}{\cos\theta}$. As θ varies, find the value of $\tan\theta$ for which *y* is a minimum.

Hence find the minimum value of *y*..

148.*a*)Let $f(x)=\dfrac{3-4x}{x^2+1}$. where $x\in\Re$. Using the knowledge of the first derivative show that the function *f* has two turning points. Sketch the rough graph of $y=f(x)$. Using this graph sketch the graph of $y=|f(x)|$ on a seperate

xy - plane. Hence show that the equation $|3-4x|e^{x}-x^{2}-1=0$ has at least three real roots.

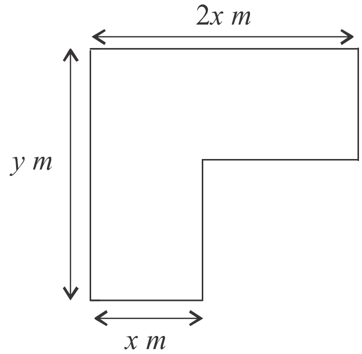
b) In a triangle ABC, $AB=AC$. The perimeter of it is $2s$ where s is a constant. This triangle is rotated about BC. Find the length of AB in terms of s which maximises the volume of sold generated.

148. a. For $x\neq -1$, let $f(x)=\dfrac{(x+2)}{(x+1)^{2}}$. Show that $f'(x)$ the derivative of $f(x)$ is given by $f'(x)=\dfrac{-(x+3)}{(x+1)^{3}}$, for $x\neq -1$

Given that for $x\neq -1$, $f''(x)=\dfrac{2(x+4)}{(x+1)^{4}}$. where $f''(x)$ denotes the second derivattive of $f(x)$. Indicating,

asymptotes, turning points and points of inflection, sketch the graph of $y=f(x)$. Hence find the range of x, the graph is concave up and concave down.

b) The encloced region of the diagram shows a garden of total perimeter 20 (m). It is constructed by removing a square of side x from a corner of a rectangle of length $2x$ and width y.



i) Show that $0<x<5$

ii) If the total area of the garden is A Show that $A=20x-5x^{2}$.

iii) Find the value of x which maximises the area of the garden and show that the maximum area is $20(m^{2})$.

149. .i) when $0<x<\dfrac{\pi}{2}$, find $\dfrac{dy}{dx}$ of $y=\tan^{-1}\left\{\dfrac{1+\sin x}{1-\sin x}\right\}^{\frac{1}{2}}$.

ii) If $y=(\sin^{-1}x)^{2}+a\sin^{-1}x+b$,

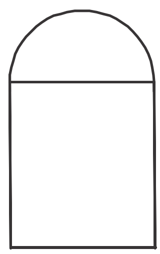
Show that $(1-x^{2}).\dfrac{d^{2}y}{dx^{2}}-x.\dfrac{dy}{dx}=2$ where a and b are constants.

iii) If $x^{2}y=a\cos nx$, where a and n are constants, show that $x^{2}.\dfrac{d^{2}y}{dx^{2}}+4x\dfrac{dy}{dx}+(n^{2}x^{2}+2)y=0$.

150.i) If $y=2x\,Cos\left(\dfrac{3}{x}\right)$, Show that

a) $x\dfrac{dy}{dx}=y+6\sin\left(\dfrac{3}{x}\right)$

b) $x^{4}\dfrac{d^{2}y}{dx^{2}}+9y=0$



ii) Sketch the rough graph of $y=\dfrac{(x-1)^{3}}{x^{2}}$.

iii) A window consisting of semi-circular are has the total permeter $10(m)$. Find its length and breadth so that its area is maximum.

151. *a*) Using first principles show that $\frac{d}{dx}(\sin 3x) = 3\cos 3x$.

b) If $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$,

show that $y\left(\frac{d^3y}{dx^3}\right) + 2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$ is independent of θ. where θ is a parameter such that $\theta \neq 2n\pi, n \in Z$.

c) A wire of length l is bent in the shape of a sector. Show that the area of the sector is $\left(\frac{1}{2}\right) r(l-2r)$. where r is the radius of the circle. Find r so that the area of the sector is maximum. Also find the angle of the sector in radians.

152.*a*. Let $f(x) = \frac{ax+b}{(x+2)^2}$ for $x \neq -2$. The graph of $y = f(x)$ has a turning point at $\left(0, \frac{1}{4}\right)$. Find the values of *a* and *b*. By assuming that $f''(x) = \frac{2(x-1)}{(x+2)^4}$; $x \neq -2$ and indicating asymptotes, turning points and points of inflection sketch the rough graph of $y = f(x)$. where $f''(x)$ is the second derivative of $f(x)$.

c. A variable straight line passes through $A(1,8)$ and cuts positive *OX.OY* axes at *P* and *Q* respectively. when it is given that $O\hat{P}Q = \theta$ and $PQ = l$, express l as a function of θ where O is the origin. Show that $\frac{dl}{d\theta} = 0$ iff $\theta = \tan^{-1}(2)$. Find the minimum of *PQ* by considering the sign of $\frac{dl}{d\theta}$ in suitable ranges.

153.*a*.*i*) Show that $\frac{d}{dx}\{ln|x|\} = \frac{1}{x}; x \neq 0$.

ii) If $xlny - ylnx = 1$, find $\frac{dy}{dx}$ at $x = 1$.

b) If it is given that $y = xe^{-\frac{1}{x}}$, show that $x^3\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y$.

c) A continuous function *f* satisfies the following conditions.

i) $f(-1)=0, f(0)=4, f(2)=1, f(5)=6, f(6)=0$. and $f'(x)=0$ .only at $x = 0,2,5$,

ii) $f'(x)<0$ when $0 < x < 2$ and $5 < x < \infty$,

iii) $f'(x) > 0$ when $-\infty < x < 0$ and $2 < x < 5$.

Sketch the graph of $y = f(x)$. Using the graph of $y = f(x)$, sketch the rough graph of $y = \frac{1}{f(x)}$ clearly indicating relative maximum, relative minimum points and horizontal and vertical asymptotes. Hence find the number of roots of the equation $lnx\ f(x) - 1 = 0$.

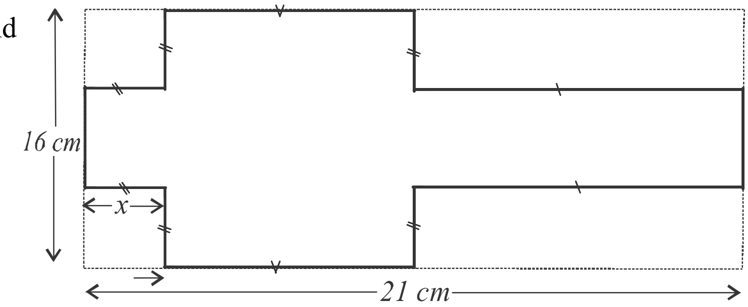
154.*a*) Let $f(x) = \frac{16x}{(x-2)^2(3x-2)}$ for $x \neq 2$ and $x \neq \frac{2}{3}$.

Show that the derivative $f'(x)$ of $f(x)$ is given by $f'(x) = \frac{-32(x-1)(3x+2)}{(x-2)^3(3x-2)^2}$.

Write down the equations of the asymptotes.

Find the coordinates of the point the horizontal asymptote cuts the graph $y = f(x)$. Indicating the turning points and asymptotes sketch the rough graph of $y = f(x)$.

b) It is required to form a closed box using a rectangular cardboard of width 16(cm) and length 21(cm), by cutting off two squares from the two corners of the side 16(cm) and two rectangles from the corners of the side 21 (cm) as shown in the figure.



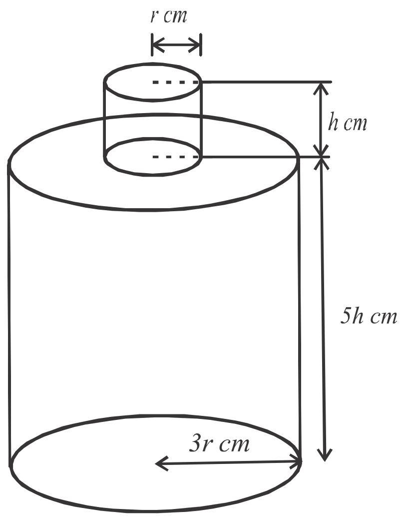
i) Obtain an expression for the volume $v(x)$ of the box in cm^3.

ii) Find the side of the square which should be cut off, so as to maximise the volume of the box to be formed.

155.*a*.Let $f(x)=\dfrac{16(x-1)}{(x+1)^2(3x-1)}$ for $x\neq -1, \dfrac{1}{3}$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{-32x(3x-5)}{(x+1)^3(3x-1)^2}$ for $x\neq -1,\dfrac{1}{3}$.

Sketch the graph of $y=f(x)$ indicating the asymptotes and the turning points.Using the graph, find the values of $k\in\Re$ such that the equation $k(x+1)^2(3x-1)=16(x-1)$ has exactly one root.



b. A bottle with a volume of $391\pi cm^3$ is to be made by removing a disc of radius *r cm* from the top face of a closed hollow right circular cylinder of radiues 3*r cm* and height 5*h cm*, and fixing an open hollow right circular cylinder of radiues *r cm* and height *h cm*, as shown in the figure. It is given that the total surface area $S\,cm^2$ of the bottle is $S=\pi r(32h+17r)$. Find the value of *r* such that *S* is minimum.

156.*a*.Show that $\lim\limits_{x\to 3}\dfrac{\sqrt{x-2}-1}{\sin(\pi(x-3))}=\dfrac{1}{2\pi}$.

b.Let *C* be the parabola parametrically given by $x=at^2$ and $y=2at$ for $t\in\Re$, where $a\neq 0$. Show that the equation of the normal line to the parabola *C* at the point $(at^2,2at)$ is given by $y+tx=2at+at^3$.

The normal line at the point $P\equiv(4a,4a)$ on the parabola *C* meets this parabola again at a point $Q\equiv(aT^2,2aT)$. Show that *T* = -3.

157.*a*.Let $f(x)=\dfrac{9(x^2-4x-1)}{(x-3)^3}$ for $x\neq 3$.

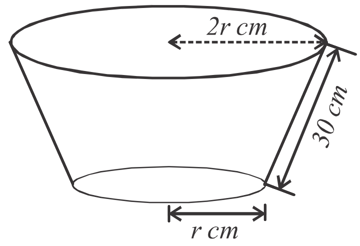
Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=-\dfrac{9(x+3)(x-5)}{(x-3)^4}$ for $x\neq 3$.

Sketch the graph of $y=f(x)$ indicating the asymptotes, y-intercept and the turning points.

It is given that $f''(x)=\dfrac{18(x^2-33)}{(x-3)^5}$ for $x\neq 3$.

Find the *x*-coordinates of the points of inflection of the graph of $y=f(x)$.

b.The adjoining figure shows a basin in the form of a frustum of a right circular cone with a bottom. The slant length of the basin is 30 *cm* and the radius of the upper circular edge is twice the radius of the bottom. Let the radius of the bottom be *r* cm.



Show that the volume $V cm^3$ of the basin is given by

$V = \frac{7}{3}\pi r^2\sqrt{900 - r^2}$ for $0 < r < 30$.

Find the value of *r* such that volume of the basin is maximum.

158.*a*.Let $f(x) = \frac{x(2x-3)}{(x-3)^2}$ for $x \neq 3$.

Show that $f'(x)$, the derivative of *f*(*x*), is given by $f'(x) = \frac{9(1-x)}{(x-3)^3}$ for $x \neq 3$.

Hence, find the interval on which *f*(*x*) is increasing and the intervals on which *f*(*x*) is decreasing. Also, find the coordinates of the turning point of *f*(*x*).

It is given that $f''(x) = \frac{18x}{(x-3)^4}$ for $x \neq 3$.

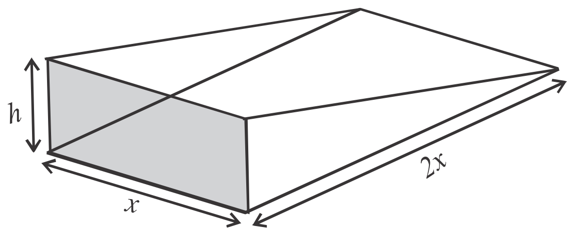
Find the coordinates of the point of inflection of the graph of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

b.The adjoining figure shows the portion of a dust pan without its handle. Its dimensions in centimetres, are shown in the figure. It is given that its volume $x^2 h\, cm^3$ is $4500 cm^3$.

Its surface area S cm^2 is given by $S = 2x^2 + 3xh$.

Show that S is minimum when $x = 15$.



159.*a*.Let $x = \cot(kt)$ and $y = e^{\operatorname{cosec}(kt)}$, where $k \in \Re$.

Show that $(1+x^2)\frac{d^2y}{dx^2} + x.\frac{dy}{dx} - \sqrt{1+x^2}.\left(x\frac{dy}{dx} + y\right) = 0$

b.Let $y = \frac{x-2}{(x-1)^2}$, for $x \neq 1$

Find

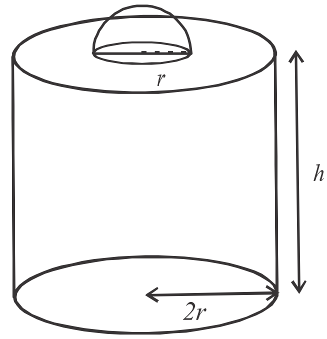
i. asymptotes

ii. stationary poins

iii.points of inflection

Showing clearly the above properties sketch a rough graph of $y = f(x)$. Also , deduce the number of solutions of the equation $(x-4)(x-1)^2 - 2(x-2) = 0$.

c.A casket shown in the figure is to be formed with thin tin. The casket consists of a hollow hemisphere of radius r and hollow cylinder of radius $2r$. The hemispherical part is joined to the upper flat surface of the cylinder by removing common area for both from the upper flat surface of the cylinder.



If the area of casket is A, show that $3r < \sqrt{\dfrac{A}{\pi}}$. Also find the values of r and h which maximise the volume of the casket.

Continuity

160.Discuss the continuity of the function

$$f(x) = \begin{cases} x-1, \text{ when } x<0 \\ \dfrac{1}{4}, \text{ when } x=0 \\ x^2, \text{ when } x>0 \end{cases} \quad \text{at } x=0.$$

161.Discuss the continuity of the function

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x^2}, \text{ when } x\neq 0 \\ 1, \text{ when } x=0 \end{cases} \quad \text{at } x=0.$$

162.For what value of k is the function.

$$f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x\neq 0 \\ k, & x=0 \end{cases} \quad \text{at the point } x=0$$

163. Examine the continuity of the function

$$f(x) = \begin{cases} \dfrac{3}{2}-x, \dfrac{1}{2}\leq x<1 \\ \dfrac{3}{2}, \quad x=1 \\ \dfrac{3}{2}+x, 1<x\leq 2 \end{cases} \quad \text{at the point } x=1$$

164.Test the continuity of the function $f(x) = \begin{cases} \dfrac{x}{|x|}, x\neq 0 \\ 1, \ x=0 \end{cases}$ at the origin.

165. If the function $f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, \text{ when } x\neq 5 \\ k, \text{ when } x=5 \end{cases}$ is continuous at $x=5$, find the value of k.

166. Find the value of k for which the function

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi-2x}, x\neq \dfrac{\pi}{2} \\ 3, \quad x=\dfrac{\pi}{2} \end{cases} \quad \text{is continuous at } x=\dfrac{\pi}{2}?$$

167. For what value of k is the function

$$f(x) = \begin{cases} \dfrac{x^2-2x-3}{x+1}, & x\neq -1 \\ k, & x=-1 \end{cases} \quad \text{is continuous at } x=-1.$$

168. Show that $f(x) = \begin{cases} \dfrac{|x-4|}{4-x}, x\neq 4 \\ 0 \quad x=4 \end{cases}$ is continuous at all points except at $x=4$.

169. Find k so that $f(x) = \begin{cases} kx^2, \text{ if } x\leq 2 \\ 3, \quad \text{if } x>2 \end{cases}$ is continuous at $x=2$.

170. Locate the point of discontinuity of the function

$$f(x)=\begin{cases}\dfrac{x^4-16}{x-2}, \text{ when } x\neq 2\\ 16, \qquad \text{when } x=2\end{cases}$$

171. Examine the continuity of the function

$$f(x)=\begin{cases}x\sin\dfrac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$$

at $x=0$.

172. Show that the function $f(x)=|x|$ is continuous at the origin.

173. Show that $f(x)=\begin{cases}\cos x, \text{ when } x\geq 0\\ -\cos x, \text{ when } x<0\end{cases}$ is discontinuous at $x=0$.

174. Show that the function

$$f(x)=\begin{cases}\dfrac{\sin x}{x}+\cos x, \text{ when } x\neq 0\\ 2, \quad \text{when } x=0\end{cases}$$ is continuous at $x=0$.

175. Show that $f(x)=\begin{cases}|x|, \text{ when } x\leq 2\\ |x|, \text{ when } x>2\end{cases}$ is continuous at $x=2$.

176. Examine the continuity of the function

$$f(x)=\begin{cases}\dfrac{x^n-1}{x-1}, & x\neq 1\\ n, & x=1\end{cases}$$ at $x=1$.

177. For what value of k, the function

$$f(x)=\begin{cases}|x-3|, x\neq 3\\ k, \quad x=3\end{cases}$$

is continuous at $x=3$?

19.For what value of k, the function

$$f(x)=\begin{cases}k+x, \; x<1\\ 4x+3, \; x\geq 1\end{cases}$$

is continuous at $x=1$.

178.Show that the function $f(x)=\begin{cases}5x-4, \text{ when } 0<x\leq 1\\ 4x^3-3x, \text{ when } 1<x\leq 2\end{cases}$ is continuous at $x=1$.

179.Let $$f(x)=\begin{cases}\dfrac{(1-\cos 4x)\tan x}{x^3} & \text{if } x<0\\ 2a^2 & \text{if } x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x>0\end{cases}$$

Show that the values of a for the function $f(x)$ to be continuous at $x=0$ are 2 and -2.

180. Let $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & ; x<0 \\ a & ; x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ; x>0 \end{cases}$

Find the value of a so that the function is continuous at $x=0$.

181.Let $g(x) = \begin{cases} x^2+5 & ; \text{ if } x<2 \\ 10 & ; \text{ if } x=2 \\ \dfrac{(1+x^3)}{1-x} & ; \text{ if } x>2 \end{cases}$

Show that $g(x)$ is not continuous at $x=2$.

182. Let $f(x) = \begin{cases} \dfrac{1}{x}-x & ; 0<x<\dfrac{1}{2} \\ \dfrac{1}{2} & ; x=0 \\ \dfrac{1}{2} & ; x=\dfrac{1}{2} \\ \dfrac{3}{2}-x & ; \dfrac{1}{2}<x<1 \\ 1 & ; x=1 \end{cases}$

Show that the function is not continuous at $x=0$, $x=1$ and continuous at $x=\dfrac{1}{4}$.

183. Let $f(x) = \begin{cases} \dfrac{x^3+x^2-16x+20}{(x-2)^2} & ; \text{ if } x \neq 2 \\ k & ; \text{ if } x=2 \end{cases}$

Show that when $k=7$, $f(x)$ is continuous at $x=2$

184. Discuss the continuity of the following functions at indicated points.

i) $f(x)= \begin{cases} \dfrac{3}{2}-x, & \text{if } \dfrac{1}{2} \le x < 1 \\ \dfrac{3}{2}, & \text{if } x=1 \\ \dfrac{3}{2}+x, & \text{if } 1<x<2 \end{cases}$

at $x=1$

ii) $f(x)= \begin{cases} \dfrac{1}{2}-x, & \text{if } 0 \le x < \dfrac{1}{2} \\ 1, & \text{if } x=\dfrac{1}{2} \\ \dfrac{3}{2}-x, & \text{if } \dfrac{1}{2}<x\le 2 \end{cases}$

at $x=\dfrac{1}{2}$