Quadratic Equations

01.Sketch the graph for the following quadratic functions.

(i). $y=x^2+x-2$ (ii). $y=x^2+2x-8$ (iii). $y=-3x^2+5x-2$ (iv). $y=2x^2+3x-2$ (v). $y=-2x^2-7x+4$

02.Sketch the graph for the following quadratic functions.

(i) $y=x^2+4x+4$ (ii) $y=-x^2+6x-9$ (iii) $y=x^2+4x+8$ (iv) $y=-x^2-2x-5$ (v) $y=x^2+6x+18$

03.Sketch the graph of $y=x^2$ and hence sketch the graph of

(i) $y=x^2-1$ (ii) $y=x^2+1$ (iii) $y=(x-1)^2+1$ (iv) $y=(x+2)^2+1$

(v) $y=x^2-4x+5$ (vi) $y=x^2+4x+3$

04 ***a***.Sketch the graph of $y=x^2+2x-3$ and hence sketch the graph of

(i) $y=x^2+2x$ (ii) $y=x^2+2x+6$ (iii) $y=x^2-4$ (iv) $y=x^2+4x$

b.i.Using the substitution , $u=x^2$ solve $x^4-2x^2+1=0$

ii.Using substitution $u=x^{\frac{1}{3}}$ Solve the the equation

$x^{\frac{2}{3}}+2x^{\frac{1}{3}}-3=0$.

iii.Solve $\sqrt{\dfrac{2+x}{3+x}}-4\sqrt{\dfrac{3+x}{2+x}}=3$

c.i. Let $u=x+\dfrac{1}{x}$, Show that $x^2+\dfrac{1}{x^2}=u^2-2$

Hence Solve, $x^4-4x^3-3x^2-4x+1=0$

ii. Solve $\sqrt{x^2-3x+16}-\sqrt{x^2-3x+9}=1$.

05.Find the range of values of x that satisfy the following inequalities.

i. $(x-1)(x-2)>0$ *ii.* $(x+1)(x-2)>0$ *iii.* $(x-3)(x-5)<0$ *iv.* $(2x-1)(x+1)<0$

v. $x^2-4x>5$ *vi.* $4x^2<1$ *vii.* $5x^2>3x+2$ *viii.* $(2-x)(x+4)<0$

ix. $(x-1)^2>4x^2$ *x.* $(x-1)(x+2)<x(4-x)$

06.The expression $-x^2+4px-9$ is negative for all real values of x. Show that $\dfrac{-3}{2}<p<\dfrac{3}{2}$.

07Find the range of t for which $x^2+(t-1)x+t+2>0$ for all real values of x.

08.Find the range of k. So that $f(x)=2x^2+2kx+(k^2+3k+4)$ is positive for all real values of x

09.Find the range of values of p for the expression $px^2+(2p-1)x+1$ to be positive for all real values of x.

10.Let $f(x)=(p-2)x^2-3px+p-2$. Find the range of p such that $f(x)$ is negative for all real x.

11.Let $y = ax^2 + bx + c$ .Where $a(\neq 0), b, c$ are real numbers.

(i). Express y in the form $y = \lambda(x+\alpha)^2 + \beta$.

(ii). Hence determine coordinates of the vertex

(iii). Find the roots of y.

(iv). Determine conditions required for the roots of y to be,

(a). real distinct (b).real coincident (c). imaginary

12. Using algebraic methods, obtain conditions required for the roots of $y = ax^2 + bx + c$ to be,

(a). real distinct (b). real coincident (c). imaginary

13.Show that the roots of $x^2 - 2ax + a^2 - b^2 - c^2 = 0$ are real. Where a, b, c are real numbers.

14.Show that when $a < c < b$ the roots of $(a-b)^2 x^2 + 2(a+b-2c)x + 1 = 0$ are imaginary.

15.If the equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ has equal roots, find the value of k.

16.When the roots of $ax^2 + bx + c = 0$ are real, show that the roots of $x^2 + \left(2 - \frac{b^2}{ac}\right)x + 1 = 0$ are also real.

Where $abc \neq 0$.

17.If the roots of $ax^2 + bx + c = 0$ are real and positive then show that the roots of $a^2x^2 + (2ac - b^2)x + c^2 = 0$ are also real and positive.

18. Write down the condition required for the graph $y = ax^2 + bx + c$ and the line $y = mx + c'$

i. to intersect each other ii. to touch each other iii. to be in separate

a.show that $y = x^2 - 1$ and $y = x - 1$ intersect each other and find their points of intersection.

b.show that $y = 2x^2 + 4x + 1$ and the line $2y - 4x - 1 = 0$ touch each other and find the point of contact.

c.show that $y = x^2 - 2x + 2$ and the line $2y + x + 4 = 0$ donot intersect each other.

19.sketch the graph of $f_1(x) = x^2 - 6x + 8$ and find the minimum value of $f_1(x)$.Let $f_2(x) = -x^2 + 6x + k$ where k is constant. Express $f_2(x) = a(x-3)^2 + b$ where a and b are constants to be determined.

Hence evaluate the maximum value of $f_2(x)$and sketch its graph on the same diagram.Hence show that the equation $f_1(x) = f_2(x)$

(i) has two distinct real roots when $k > -10$

(ii) has a real root when $k = 10$

(iii) has no real roots when $k < -10$

20.(*a*) If $\Delta = b^2 - 4ac$ write down condition required for $y = ax^2 + bx + c$ to have distinct real roots and imaginary roots.Let $f(x) = (a-1)x^2 - 2(a-1)x + 2a + 3$

(*i*) find range of a such that $f(x) > 0$ for all $x \in \Re$,

(*ii*) show that the graph $f(x)$ touches $x-$ axis when $a = -4$

(*iii*) show that the equation $f(x) = 0$ has imaginary roots when $a < -4$ or $a > 1$

(*b*) Let $y_1 = x^2 - x + m$ and $y_2 = mx$ .Show that the quadratic equation $y_1 = y_2$ has distinct real roots if and only if $m \neq 1$

21.Show that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Where $a \neq 0$ is a real constant. Let $\Delta = b^2 - 4ac$. Give in terms of Δ , conditions required for the above equation to have

*i.*real distinct roots. *ii.*real coincident roots. *iii.* imaginary roots.

Solve the following equations.

a. $x^2 - 3x + 2 = 0$. *b.* $2x^2 + 11x + 5 = 0$. *c.* $x^2 + 6x + 9 = 0$.

22.If $x + \dfrac{1}{x} = t$, show that $x^2 + \dfrac{1}{x^2} = t^2 - 2$ using these substitutions, solve thefollowing equations completely.

$(i).\ 5x^4 - 11x^3 + 16x^2 - 11x + 5 = 0$ $(ii).\ x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ $(iii).\ x^4 - 2x^3 + 6x^2 - 2x + 1 = 0$

23.(*i*).If *x* is real, find the minimum and maximum value of $\dfrac{x^2 + 14x + 9}{x^2 + 2x + 9}$.

(*ii*).If $9x^2 + 2xy + y^2 - 92x - 20y + 244 = 0$. Show that $3 \le x \le 6$ and $1 \le y \le 10$ where *x, y* are real.

24.Find the following in terms of $\alpha + \beta$ and $\alpha\beta$.

$(i).\ \alpha^2 + \beta^2$ $(ii).\ \alpha^3 + \beta^3$ $(iii).\ \alpha^4 + \beta^4$ $(iv).\ \alpha - \beta$ $(v).\ \alpha^2 - \beta^2$ $(vi).\ \alpha^3 - \beta^3$

$(vii).\ \dfrac{1}{\alpha} + \dfrac{1}{\beta}$ $(viii).\ \alpha + \dfrac{1}{\alpha} + \beta + \dfrac{1}{\beta}$ $(ix).\ \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ $(x).\ \alpha^2 + \beta + \beta^2 + \alpha$ $(xi)\ \alpha^4 - \beta^4$

25.If the roots of $ax^2 + bx + c = 0$ are α, β , show that $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$. Hence find the following in terms of *a,b,c*

$(i).\ \alpha^2 + \beta^2$ $(ii).\ \alpha^3 + \beta^3$ $(iii).\ \alpha^4 + \beta^4$ $(iv).\ \alpha - \beta$ $(v).\ \alpha^2 - \beta^2$ $(vi).\ \alpha^3 - \beta^3$

$(vii).\ \dfrac{1}{\alpha} + \dfrac{1}{\beta}$ $(viii).\ \alpha + \dfrac{1}{\alpha} + \beta + \dfrac{1}{\beta}$ $(ix).\ \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ $(x).\ \alpha^2 + \beta + \beta^2 + \alpha$ $(xi)\ \alpha^4 - \beta^4$

26.The roots of $x^2 + 2ax + b^2 = 0$ are α_1, β_1 and that of $x^2 + 2cx + d^2 = 0$ are α_2, β_2. If $\alpha_1 + \alpha_2 = \beta_1 + \beta_2$. Show that $a^2 + d^2 = b^2 + c^2$.

27. α, β are the roots of $ax^2 + 2bx + c = 0$ and γ, δ are the roots of $px^2 + 2qx + r = 0$ If $\alpha, \beta, \gamma, \delta$ lie in geometric progression. Show that $\frac{ac}{b^2} = \frac{pr}{q^2}$.

28. The roots of $ax^2 + 8(b-a)x + 4(4a - 8b + c) = 0$ are $4 - 2\alpha, 4 - 2\beta$. Show that the quadratic equation with the roots α, β is $ax^2 - 4bx + c = 0$.

29. α and β are the roots of $x^2 + ax + b = 0$. Show that the roots of $bx^2 + (2b - a^2)x + b = 0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

30. *a*).If α, β are the roots of the equation $ax^2 - bx + c = 0$ form the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}$.

b).If the roots of $x^2 + px + q = 0$ are α and β where α and β non-zero form the equation whose roots are $\frac{2}{\alpha}, \frac{2}{\beta}$.

c). If α and β are the roots of the equation $ax^2 + bx + c = 0$. Show that the roots of the equation $acx^2 - (b^2 - 2ac)x + ac = 0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

31. The roots of the quadratic equation $x^2 - px + q = 0$ are α and β.

Form, in terms of *p* and *q*, the quadratic equation whose roots are $\alpha^3 - p\alpha^2$ and $\beta^3 - p\beta^2$.

32. *a*).Form a quadratic equaton wth roots which exceed by2 the roots of the quadratic equation $3x^2 - (p-4)x - (2p+1) = 0$. Find the values of *p* for which the given equatoin has equal roots.

b).Given that the roots of the equation $ax^2 + bx + c = 0$ are β and $n\beta$. Show that $(n+1)^2 ac = nb^2$.

33. *a*).Given that the roots of $x^2 + px + q = 0$ are α and β, form an equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

b).Given that α is a root of the equation $x^2 = 2x - 3$. Show that,

i. $\alpha^3 = \alpha - 6$ *ii*. $\alpha^2 - 2\alpha^3 = 9$

34. *a*).Find the set of values of k for which the equation $x^2 + kx + 2k - 3 = 0$ has no real roots.

When $k = 7$, the roots of the equaiton $x^2 + kx + 2k - 3 = 0$ are α and β where $\alpha > \beta$.

b). Write down the values of $(\alpha + \beta)$ and $\alpha\beta$.

c). Form an equation with integral coefficients whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

d).Prove that $\alpha - \beta = \sqrt{5}$.

35. Given that α and β are the roots of the equation $3x^2 + x + 2 = 0$.

i).Evaluate $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

ii). Find an equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

iii).Show that $27\alpha^4 = 11\alpha + 10$.

36.*a*).Show that the roots of $x^2+2(t+1)x-(2t+3)=0$ are real for all values of *t*.

b).Show that the roots of $(k^2+1)x^2-(2k-1)x-3=0$ are real for all values of *k*.

37.If the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are coincident, prove that $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in arithmatic progression.

38.If the roots of the equation $ax^2+bx+c=0$ are in the ratio $\frac{p}{q}$ prove that $ac(p+q)^2=b^2pq$.

39.The equations $ax^2+a^2x+1=0$ and $bx^2+b^2x+1=0$ have a common root. Show that the other two roots satisfy the equation $x^2-(a+b)x+ab=0$.

40.If the equations $ax^2+bx+c=0$ and $px^2+qx+r=0$ have a common root, show that $(ar-pc)^2=(aq-bp)(br-qc)$.

41.If the equations $x^2+ax+b=0$ and $x^2+bx+a=0$, $(a\neq b)$ have a common root show that the roots of $2x^2+(a+b)x=(a+b)^2$ are $x=1, x=-\frac{1}{2}$.

42.If the equations $kx^2+2x+1=0$ and $x^2+2x+k=0$ have a common root find *k*.

43.When $a\neq p$, if the equations $x^2+2ax+b=0$ and $x^2+2px+q=0$ have a common root, show that $(q-b)^2=4(p-a)(aq-bp)$.

44.The equations $x^2+ax+b=0$ and $cx^2+2ax-3b=0$ have a common root. When $a\neq 0, b\neq 0$,

show that $b=\frac{5a^2(c-2)}{(c+3)^2}$.

45.The equations $x^2-px+\lambda=0$ and $x^2-qx+\mu=0$ have a common root. If the roots of the second equation are equal, show that $2(\lambda+\mu)=pq$.

46.The roots of $x^2+px+q=0$ are α,β and the roots of $x^2+ax+b=0$ are $\frac{1}{\alpha},\gamma$

Show that $(p-aq)(a-pb)=(1-bq)^2$. Show also that the equation whose roots are β,γ is

$$x^2(1-bq)-x[(a+p)bq-(aq+bp)]+bq(1-bq)=0.$$

47.*i*.The equations $x^2+ax+b=0$ and $x^2+mx+n=0$ have a common root. Then show that the equations $x^2+ax+b=0$ and $x^2+(2a-m)x+a^2-am+n=0$ also have a common root

ii. If $\alpha^2+\beta^2=5,\ 3(\alpha^5+\beta^5)=11,\ (\alpha^3+\beta^3)$ where α,β are real, show that the quadratic equation whose roots are α,β is $x^2\pm 3x+2=0$

48.α and β are the roots of $ax^2+bx+c=0$ and $\alpha+h,\beta+h$ are the roots of $px^2+qx+r=0$ show that

$i.\ h=\frac{1}{2}\left[\frac{b}{a}-\frac{q}{p}\right]$ $\qquad ii.\ \frac{b^2-4ac}{a^2}=\frac{q^2-4pr}{p^2}$

49.*i*.The equation $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root. Show that this common root can be either $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$ where $p\neq p'$ and $q\neq q'$

$ii.\ \alpha,\beta$ are the roots of $px^2+qx+r=0$ If $f(x)=ax^2+bx+c$ show that

$$f(\alpha)f(\beta)=\frac{(cp-ar)^2-(bp-aq)(cq-br)}{p^2}$$

Hence or otherwise, if there exists a common root to the equations $ax^2+bx+c=0$ and $px^2+qx+r=0$ then show that $bp-aq$, $cp-ar$ and $cq-br$ lie in geometric progression.

50.If a,b,c are in geometric progression and if the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root then show that $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in arithmetic progression.

51(i).Using the substitution $t=x+x^{-1}$, solve $x^4-10x^3+26x^2-10x+1=0$.

(ii).If the roots of the equation $(q-r)x^2+(r-p)x+p-q=0$ are equal, show that $p,\ q,\ r$ lie in an arithmetic progression.

52.(i).If the roots of $x^2-px+q=0$ are α,β , find the quadratic equation whose roots are $\left(\alpha^3-p\alpha^2\right)$ and $\left(\beta^3-p\beta^2\right)$.

(ii).If the roots of $3x^2-5x+7=0$ are α,β . Show that

$3\left(\alpha^{2006}+\beta^{2006}\right)=5\left(\alpha^{2005}+\beta^{2005}\right)-7\left(\alpha^{2004}+\beta^{2004}\right)$.

(iii) α,β are the roots of $x^2-px-q=0$. When $n>1$ is an integer show that

$\alpha^n+\beta^n=p\left(\alpha^{n-1}+\beta^{n-1}\right)+q\left(\alpha^{n-2}+\beta^{n-2}\right)$. When α,β are the roots of $x^2-2x-1=0$, find $\alpha^5+\beta^5$.
Find the quadratic equation with the roots α^5,β^5 .

53.(i).The roots of $x^2-a(x-1)+b=0$ are α,β . Find the value of

$\frac{1}{\alpha^2-a\alpha}+\frac{1}{\beta^2-b\beta}+\frac{2}{a+b}$ where $a\neq 0$.

(ii).The roots of $x^2+2px+q^2=0$ and $x^2+2mx+n^2=0$ are α_1,β_1 and α_2,β_2 respectively.

(a). If $\alpha_1+\alpha_2=\beta_1+\beta_2$, show that $p^2+n^2=q^2+m^2$.

(b). If $\alpha_1\alpha_2+\beta_1\beta_2=0$,show that $p^2n^2=q^2m^2$.

54. Given that $f(x)=x^2+(k+2)x+2k$.

(i). Show that for all real values of k the roots of $f(x)=0$ are real.

(ii). Find the roots of $f(x-k)=0$.

(iii). If the roots of $f(x-k)-2x=0$ are $x=0$ and $x=7$, show that $k=7$.

55.The quadratic equations $a_r x^2+2b_r x+c_r=0$ where $r=1,2$ have a common root. a_1,b_1,c_1 lie in geometric progression. Show that $\frac{a_2}{a_1},\frac{b_2}{b_1},\frac{c_2}{c_1}$ lie in arithmetic progression a_1,a_2,b_1,b_2,c_1,c_2 are positive real numbers.

56. (α,β) are the roots of $ax^2+bx+c=0$ and (α,γ) are the roots of $a^1x^2+b^1x+c^1=0$. By considering the common root show that $(ca^1-c^1b)^2=(bc^1-b^1c)(ab^1-a^1b)$.

Also show that $\dfrac{\alpha}{aa^1(bc^1-b^1c)}=\dfrac{\beta}{ca^1(ab^1-a^1b)}=\dfrac{\gamma}{c^1a(ab^1-a^1b)}$.

57.*i.*If α,β are the roots of the quadratic equation $x^2+px+1=0$ and γ,δ are the roots of the quadratic equation $x^2+qx+1=0$ then show that, $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)=q^2-p^2$.

*ii.*Let α,β are the roots of the equation $x^2+qx+1=0$ and γ,δ are the roots of the equation $x^2+x+q=0$ show that $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)=(\gamma^2+q\gamma+1)(\delta^2+q\delta+1)$ determine all possible values for q such that the given equations have at least one real root in common.

58.Let α and β be the roots of the equation $x^2+px+1=0$ and let γ and δ be the roots of the equation $x^2+\frac{1}{p}x+1=0.$

Show that $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)=(\gamma^2+p\gamma+1)(\delta^2+p\delta+1)$

and deduce that $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)=\left(p-\frac{1}{p}\right)^2$.

59.*i.* Show that the solutions of the equation $\left(2+\sqrt{3}\right)^{x^2-2x+1}+\left(2-\sqrt{3}\right)^{x^2-2x-1}=\dfrac{2}{2-\sqrt{3}}$ are $x=0$, $x=2$.

ii. Solve $\left(3-2\sqrt{2}\right)^{x^2-2}+\left(3+2\sqrt{2}\right)^{x^2-2}-6=0\cdot$

60. Let $a<b<c$ be three real numbers. Given that $f(x)=(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)$. Show that $f(x)=0$ has two real distinct roots. If these roots are α,β , find $\alpha+\beta$ and $\alpha\beta$ in terms of *a, b,c*.

(i).If α,b,β lie in arithmetic progression then show that *c, b, a* alsolie in arithmetic progression.

(ii). Show also that $f(a)=f(c)=-2f(b)$.

(iii).If *c, b, a* lie in geometric progression, show that $\frac{1}{\alpha},\frac{1}{b},\frac{1}{\beta}$ lie in arithmetic progression.

61.(i).The roots of $k^2x^2+(kx+1)(x+k)+1=0$ are α,β. Write down expressions for $\alpha+\beta, \alpha\beta$ in terms of k.

Show that $\alpha^2\beta^2+(\alpha\beta+1)(\alpha+\beta)+1=0$.

(ii). α,β are the roots of the equation $x^2+bx+c=0$. Write down the equation whose roots are α^2 and β^2.

Hence if the roots of $x^2+x-1=0$ are α,β, find the equation with the roots α^{16},β^{16}.

Using this show that $(2207)^{\frac{1}{16}} \approx \dfrac{\sqrt{5}+1}{2}$.

62. Let $x^2-kx+4=0$.

(i). Find the range of k, such that the roots of this equation are real positive.

(ii). Show if the roots are positive and in 3 : 1, that the value of k is $\dfrac{8}{\sqrt{3}}$.

(iii). The roots of $x^2+bx+c=0$ are α,β. Where b and c are real. Obtain the equation whoose roots are α^3,β^3. If $b^3-6b+9=0$ and $c=2$, find the real values of α and β. Hence find the real root of $y^3-6y+9=0$.

63. Given that $f(x)=x^2+4x+a+2$.

(i). If the equation $f(x)=ax$ has two roots, then find the range of a.

(ii). If the roots of $f(x)=0$ are α,β find the equation whose roots are $\alpha+\dfrac{2}{\alpha}$ and $\beta+\dfrac{2}{\beta}$.

(iii). If $f(x)=(x^2+1)(2-a)$ has one real root, show that the values of a are -1 and 2.

64. (i). Show the equation $\dfrac{x}{x-a}+\dfrac{x}{x-b}=1$ to have real distince roots that a and b must take the same sign. $(a,b\in\Re)$

(ii).Show, for the roots of $\dfrac{x}{x-a}+\dfrac{x}{x-b}=1+c$ to be coincident that $c^2=\dfrac{-4ab}{(a-b)^2}$.

Deduce that $c^2=1-\left(\dfrac{a+b}{a-b}\right)^2$. Hence show that $0<c^2\le 1$ where $a,b,c\in\Re$

65. If p and q are real numbers, when $b^2-4ac<0$ express ax^2+bx+c in the form $a\{(x+p)^2+q^2\}$. Hence, when $a>0, b^2-4ac<0$, show that ax^2+bx+c is always positive.

If $f(x)=3x^2-5x-k$ find the value of k, so that $f(x)>1$ for all values of x. Show also that $f(x)$ assumes its minimum at $x=\dfrac{5}{6}$ and find the value of k corresponding to mimimum zero.

66. Express ax^2+bx+c in the form $a\{(x+m)^2+n\}$. Where m, n are to be determined interms of a, b, c.

ax^2+bx+c assumes its minimum value -9 at $x=-\dfrac{1}{3}$. If a root of $ax^2+bx+c=0$ is $\dfrac{8}{3}$ find the values of a, b and c.

67.i).The roots of $x^2+bx+c=0$ are α_1,β_1 and the roots of $x^2+kbx+k^2c=0$ are α_2,β_2.

Show that the equation with the roots $\alpha_1\alpha_2+\beta_1\beta_2$ and $\alpha_1\beta_2+\beta_1\alpha_2$ is $x^2-kb^2x+2k^2c(b^2-2c)=0$.

Show also that the roots of this equation are always real.

(ii). The roots of $2x^2 - qx + r = 0$ are $\alpha+1, \beta+2$. Where α, β are the real roots of the equation $x^2 - bx + c = 0$. Given that $\alpha \geq \beta$. Find q, r interms of b, c. When $\alpha = \beta$, show that $q^2 = 4(2r+1)$.

68.i).The roots of $(k-1)x^2 + kx + k - 2 = 0$ are α, β. Show that

$(\alpha - 2\beta)(2\alpha - \beta) = \dfrac{27k - 7k^2 - 18}{(k-1)^2}$. If α, β are real and α lies between $\beta/2$ and 2β, find the possible values of k.

ii). The roots of $ax^2 + bx + c = 0$ are α, β. Show that $as_r + bs_{r-1} + cs_{r-2} = 0$. Where $s_r = \alpha^r + \beta^r$.

Find $\alpha^4 + \beta^4$ interms of a, b, c.

69.Let α and β be the roots of the quadratic equation $x^2 + qx + r = 0$. Show that $\alpha + \beta = -q$ and $\alpha\beta = r$.

Let $\alpha = 1 + \dfrac{1}{p}$ and $\beta = 1 + \dfrac{1}{p+1}$, where $p(\neq 0, -1)$ is a real number.

(i). Show that $(q + r + 1)^2 = q^2 - 4r$ and $r \neq -1$.

(ii). Find the quadratic equation with coefficients in terms of q and r whose roots are $1 - \dfrac{1}{p}$ and $1 - \dfrac{1}{p+1}$.

70. Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, where a, b and c are real numbers. Show that α and β are both

(i). real, if and only if $b^2 - 4ac \geq 0$.

(ii). purely imaginary, if and only if $b = 0$ and $ac > 0$.

Find the quadratic equation whose roots are α^2 and β^2.

Show that the roots of this quadratic equation are both real, if and only if either α and β are both real or α and β are both purely imaginary.

71. α and β are the roots of the equation $x^2 + bx + c = 0$, where $c \neq 0$. Find the quadratic equation in terms of b and c, whose roots are $\alpha^3\beta^2$ and $\alpha^2\beta^3$. Hence, find thequadratic equation, in terms of b and c, whose roots are $\alpha^3\beta^2 + \dfrac{1}{\alpha^2\beta^3}$ and $\alpha^2\beta^3 + \dfrac{1}{\alpha^3\beta^2}$.

72. Let $f(x) = x^2 + 2kx + k + 2$, where k is a real constant.

(i).Express $f(x)$ in the form $(x-a)^2 + b$, where a and b are constant to be determined in terms of k. Find the turning point of $f(x)$ without using calculus and show that this point is a minimum point. Find the minimum value of $f(x)$ in terms of k. Hence, show that the curve $y = f(x)$.

(a). lies entirely above the x-axis if $-1 < k < 2$.

(b). touches the x-axis if $k = -1$ or $k = 2$.

(c). cuts the x-axis in two distinct points if $k < -1$ or $k > 2$.

(ii). Prove that the straight line $y = mx$ intersects the curve $y = f(x)$ in two real and distinct points for all real and finite values of m if and only if $k < -2$.

73.Let α and β be the roots of the equation $x^2+bx+c=0$, and γ and δ be the roots of the equation $x^2+mx+n=0$ where $b,c,m,n\in\mathbb{R}$.

(i).Find $(\alpha-\beta)^2$ in terms of b and c, and hence write down $(\gamma-\delta)^2$ in terms of m and n.
Deduce that if $\alpha+\gamma=\beta+\delta$ then $b^2-4c=m^2-4n$.

(ii).Show that $(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)=(c-n)^2+(b-m)(bn-cm)$.
Deduce that the equations $x^2+bx+c=0$ and $x^2+mx+n=0$ have a common root if and only if $(c-n)^2=(m-b)(bn-cm)$.
The equations $x^2+10x+k=0$ and $x^2+kx+10=0$ have a common root, where k is a real constant.
Find the values of k.

74.(a). α and β are the roots of the quadratic equation $f(x)\equiv x^2+px+q=0$ where p and q are real and $2p^2+q\neq 0$. If $y(p-x)=p+x$ substituting for x in $f(x)=0$ or otherwise,
show that $g(y)=(2p^2+q)y^2+2(q-p^2)y+q=0$ where $y\neq -1$.
Hence, find the roots of the equation $g(y)=0$ in terms of α and β.

Express $\left(\dfrac{\alpha}{2\beta+\alpha}\right)^2+\left(\dfrac{\beta}{2\alpha+\beta}\right)^2$ in terms of p and q.

b). If a, b, c and m are constants such that $a+b+c=0$ and $ab+bc+ca+3m=0$,
prove that $(y+ax)(y+bx)(y+cx)=y(y^2-3mx^2)+abcx^3$.
If $y=x^2+m$, show that $(x^2+ax+m)(x^2+bx+m)(x^2+cx+m)=x^6+abcx^3+m^3$

If $g(x)=x^6+16x^3+64$ has factors $(x^2-2x+m),(x^2+ax+m)$ and (x^2+bx+m) find the values of m, a and b.
Hence,
(i). Show that $g(x)$ is non-negative for all x,
(ii). Find the roots of the equatioon $g(x)=0$.

75. Let $f(x)=x^2+2x+9;\ x\in\mathbb{R}$.

i .If α,β are the roots of $f(x)=0$ obtain the quadratic equation whose roots are α^2-1 and β^2-1
ii.Find the value of a real constant k for which the equation $f(x)=k$ has exactly one real root for x.
iii.Find the greatest value of $\dfrac{1}{f(x)}$ giving the value of x for which it is attained.
iv.Determine the set of values of a real constant λ for which the equation $f(x)=\lambda x$ has no real solution for x.

76.Let $\lambda\in IR$ and $p(x)=(\lambda-2)x^2-3(\lambda+2)x+6\lambda$
i.Find the least integral value of λ for which p(x) is positive for all $x\in\mathbb{R}$.
ii .For what values of λ does the equation $p(x)=0$ have two distinct real roots ?
iii.If the roots of $p(x)=0$ are real and if the difference of the roots is equal to 3, find λ

77.Let $\lambda \in IR$ and $p(x)=x^2-2\lambda(x-1)-1$ show that the roots of *p(x)=0* are real. Find all the values of λ such that the sum of the roots of *p(x)*=0 is equal to the sum of the squares of the roots.

78.Let $f(x)=x^2+bx+c$ and $g(x)=x^2+qx+r$ where $b,c,q,r \in IR$ and $c \neq r$

Let α,β be the roots of *g(x)=0.*

Show that $f(\alpha)f(\beta)=(c-r)^2-(b-q)(cq-br)$. Hence, or otherwise. Prove that if *f(x)=0* and *g(x)=0* have a common root, then *b-q, c-r* and *cq-br,* are in Geometric proression.

If α,γ are the roots of *f(x)=0*, Show that the quadratic equation whose roots are β,γ is

$$x^2-\frac{(c+r)(q-b)}{(c-r)}x+\frac{cr(q-b)^2}{(c-r)^2}=0$$

79. α and β are the roots of the equation $x^2+bx+c=0$ find the quadratic equation in terms of *b* and *c*, whose roots are α^3 and β^3 hence, find the quadratic equation, in terms of *b* and *c*, whose roots are

$\alpha^3+\frac{1}{\beta^3}$ and $\beta^3+\frac{1}{\alpha^3}$

80. α and β are the roots of the equation $x^2+bx+c=0$, where $c \neq 0$. Find the quadratic equation in terms of *b* and *c,* whose roots are α^4 and β^4. Hence, find the quadrtic equation in terms of *b* and *c*, whose roots are

$\frac{\alpha^4}{\beta^4}+1$ and $\frac{\beta^4}{\alpha^4}+1$

81. α and β are the roots of the equation $x^2+bx+c=0$, , where $c \neq 0$. Find the quadratic equation, in terms of *b* and *c*, whose roots are $\alpha^3\beta^2$ and $\alpha^2\beta^3$.

Hence, find the quadratic equation in terms of *b* and *c* whose roots are $\alpha^3\beta^2+\frac{1}{\alpha^2\beta^3}$ and $\alpha^2\beta^3+\frac{1}{\alpha^3\beta^2}$

82.The roots of the quadratic equation $ax^2+bx+c=0$ are α,β write down the values of $\alpha+\beta$ and $\alpha\beta$.

Find $\alpha^2+\beta^2$ in terms of *a,b* and *c*. Find the quadratic equation whose roots are $\frac{\alpha^2}{\beta^2}$ and $\frac{\beta^2}{\alpha^2}$.

Hence find the quadratic equation whose roots are $\frac{\alpha^2}{\alpha^2+\beta^2}$ and $\frac{\beta^2}{\alpha^2+\beta^2}$.

82.*a.* α and β are the roots of the equation $x^2+px+q=0$. Find the quadratic equation in terms of *p* and *q* whose roots are $\alpha-\frac{2}{\beta}$ and $\beta-\frac{2}{\alpha}$ where $q \neq 0$. Hence find the quadratic equation in terms of *p* and *q* whose roots are $\frac{\beta}{\beta(2-\alpha)+2}$ and $\frac{\alpha}{\alpha(2-\beta)+2}$

b. Using a suitable substitution, solve the equation $\left(x-\frac{x}{x+1}\right)^2+2x\left(\frac{x}{x+1}\right)=3$

83.*a*. If the quadratic equations $ax^2+2cx+b=0$ and $ax^2+2bx+c=0$ $(b\neq c)$ have a common root. Show that $a+4b+4c=0$.

b. If *a,b* and *c* are real numbers with $a\neq 0$ α is a root of $a^2x^2+bx+c=0$ and β is a root of $a^2x^2-bx-c=0$ and $0<\alpha<\beta$, show that the equation $a^2x^2+2bx+2c=0$ has a root γ such that $\alpha<\gamma<\beta$

84.*a*. If $m\left(ax^2+2bx+c\right)+px^2+2qx+r$ can be expressed in the form $n(x+k)^2$. Show that $(ak-b)(qk-r)=(pk-q)(bk-c)$

b.If every pair from among the equations $x^2+px+qr=0, x^2+qx+rp=0$ and $x^2+rx+pq=0$ has a common root, then show that the sum of the three common roots is $p+q+r$

c. Show that the solutions of the equation

$$10^{2/x}+25^{1/x}=(4.25)(50^{1/x})$$ are $-\dfrac{1}{2}$ and $\dfrac{1}{2}$

85.*a*. If $a(p+q)^2+2bpq+c=0$ and $a(p+r)^2+2bpr+c=0$ then show that $qr=p^2+\dfrac{c}{a}$

b. If α,β are the roots of the equation $ax^2+bx+c=0$ and $\alpha_1,-\beta$ those of $a_1x^2+b_1x+c_1=0$ Show that α,α_1 are the roots of $$\frac{x^2}{\left(\frac{b}{a}\right)+\left(\frac{b_1}{a_1}\right)}+x+\frac{1}{\left(\frac{b}{c}\right)+\left(\frac{b_1}{c_1}\right)}=0$$

86.Let $a,b,c\in\Re$ and $ac\neq 0$. Show that zero is not a root of the equation $ax^2+bx+c=0$.

Let α and β be the roots of this equation, and let $\lambda=\dfrac{\alpha}{\beta}$.

Show that $ac(\lambda+1)^2=b^2\lambda$.

Let $p,q,r\in\Re$ and $pr\neq 0$. Also, let γ and δ be the roots of the equation $px^2+qx+r=0$, and let $\mu=\dfrac{\gamma}{\delta}$. Show that $\lambda=\mu$ or $\lambda=\dfrac{1}{\mu}$ holds if and only if $acq^2=prb^2$.

It is given that the roots of the equations $kx^2-3x+2=0$ and $8x^2+6kx+1=0$ are in the same ratio, where $k\in\Re$ .Find the value of *k*.

87.Polynomials $F(x), G(x)$ and $H(x)$ of degree 4 in *x* are given as follows:

$F(x)\equiv(x^2-\alpha x+1)(x^2-\beta x+1)$,where α and β are real constants;

$G(x)\equiv 6x^4-35x^3+62x^2-35x+6$,

$H(x)\equiv x^4+x^2+1$.

(*i*) If both $F(x)=0$ and $G(x)=0$ have the same roots, show that the quadratic equation with α and β as its roots is $6x^2-35x+50=0$.**Hence** find all the roots of the equation $G(x)=0$.

(*ii*) If $F(x)=H(x)$, find possible values of α and β, and show that the roots of the equation $H(x)=0$ are not real.

88.Let $a,b,c \in \Re$ such that $a \neq 0$ and $a+b+c \neq 0$, and let $f(x)=ax^2+bx+c$.

Show that 1 **is not** a root of the equation $f(x)=0$.

Let α and β be the roots of $f(x)=0$.

Show that $(\alpha-1)(\beta-1)=\frac{1}{a}(a+b+c)$ and that the quadratic equation with $\frac{1}{\alpha-1}$ and $\frac{1}{\beta-1}$ as the roots is given by $g(x)=0$, where $g(x)=(a+b+c)x^2+(2a+b)x+a$.

Now , let $a>0$ and $a+b+c>0$.Show that the minimum value m_1 of $f(x)$ is given by $m_1=-\frac{\Delta}{4a}$, where $\Delta=b^2-4ac$. Let m_2 be the minimum value of $g(x)$. Deduce that $(a+b+c)m_2=am_1$.

Hence , show that $f(x)\geq 0$ for all $x\in\Re$ if and only if $g(x)\geq 0$ for all $x\in\Re$.

89.The roots of $x^2+px+q=0.$ are α,β. Where p and q are real. If $\lambda=\alpha+\beta^2$ and $\mu=\beta+\alpha^2$, find the equation with the roots λ and μ. When α and β are imaginary, Prove that λ and μ are real only if $p=-1$.

When this happens prove also that $\lambda=\mu=1-q$.

90.If $t+\frac{1}{t}=T+\frac{1}{T}$, show that $t=T$ or $t=\frac{1}{T}$. The roots of the equation $px^2+qx+r=0$ are α,β.

Let $\lambda=\frac{\alpha}{\beta}$. Show that $\lambda+\frac{1}{\lambda}=\frac{(q^2-2pr)}{pr}$. Hence, if the roots of $a_1x^2+b_1x+c_1=0$ are α_1,β_1 and the roots of $a_2x^2+b_2x+c_2=0$ are $\alpha_2\ \beta_2$, when it is given that $a_1b_2^2c_2=a_2b_1^{\ 2}c_2$, show that $\lambda_1+\frac{1}{\lambda_1}=\lambda_2+\frac{1}{\lambda_2}$.

where $\lambda_1=\frac{\alpha_1}{\beta_1}$ and $\lambda_2=\frac{\alpha_2}{\beta_2}$. Also show that $\frac{\alpha_1}{\beta_1}=\frac{\alpha_2}{\beta_2}$ *or* $\frac{\alpha_1}{\beta_1}=\frac{\beta_2}{\alpha_2}$.

91.Let $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ Where $a,b,c\in\Re$.

Show that the above equation contains the quadratic equation $3x^2-2(a+b+c)x+ab+bc+ca=0$.

Show also that the discriminant of this equation can be given as $2\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$.

Hence show that this equation has real coincident roots if and only if $a=b=c$.

92.Let $f(x)=ax^2+bx+c,$ where $a,b,c\in\Re$ with $a\neq 0$. Show that the roots of $f(x)=0$ are real distinct, real coincident and imaginary according as $af\left(\frac{-b}{2a}\right)\begin{matrix}<\\=\\>\end{matrix}0$. If the roots of $f(x)=0$ are real distinct, prove that

i.the roots of the equation $2a^2x^2+2abx+b^2-2ac=0$ are imaginary.

ii. the roots of the equation $a^2x^2+(2ac-b^2)x+c^2=0$ are real .

93.Let $a,b\in\Re$. Write down the discriminant of the equation $3x^2-2(a+b)x+ab=0$ in terms of a and b, and **hence**, show that the roots of this equation are real.

Let α and β be these roots. Write down $\alpha+\beta$ and $\alpha\beta$ in terms of a and b.

Now, let $\beta=\alpha+2$. Show that $a^2-ab+b^2=9$ and **deduce** that $|a|\leq\sqrt{12},$ and find b in terms of a.

94.Let $p \in R$ and $0 < p \le 1$. Show that 1 is not a root of the equation $p^2x^2 + 2x + p = 0$. Let α and β be the roots of this equation. Show that α and β are both real. Write down $\alpha + \beta$ and $\alpha\beta$ in terms of *p*, and show that

$$\frac{1}{(\alpha - 1)} \cdot \frac{1}{(\beta - 1)} = \frac{p^2}{p^2 + p + 2}.$$

Show also that the quadratic equation whose roots are $\frac{\alpha}{\alpha - 1}$ and $\frac{\beta}{\beta - 1}$ is given by

$(p^2 + p + 2)x^2 - 2(p+1)x + p = 0$ and that both of these roots are positive. (2019)

95.Let $k > 1$. Show that the equation $x^2 - 2(k+1)x + (k-3)^2 = 0$ has real distinct roots. Let α and β be these roots. Write down $\alpha + \beta$ and $\alpha\beta$ in terms of *k*, and find the values of *k* such that both α and β are positive.

Now, let $1 < k < 3$. Find the quadratic equation whose roots are $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$, in terms of *k*. (2021)

96.Let $f(x) = x^2 + px + c$ and $g(x) = 2x^2 + qx + c$, where $p, q \in \mathbb{R}$ and $c > 0$. It is given that $f(x) = 0$ and $g(x) = 0$ have a common root α. Show that $\alpha = p - q$.
Find *c* in terms of *p* and *q*, and deduce that

(i) if $p > 0$, then $p < q < 2p$,

(ii) the discriminant of $f(x) = 0$ is $(3p - 2q)^2$.

Let β and γ be the other roots of $f(x) = 0$ and $g(x) = 0$ respectively. Show that $\beta = 2\gamma$.
Also, show that the quadratic equation whose roots are β and γ is given by $2x^2 + 3(2p - q)x + (2p - q)^2 = 0$.(2020)

97. Let $k \in \mathbb{R} - \{-3\}$.

Find the set of values of real constant *k*, so that the roots of the equation $(k+3)x^2 - 2(k+1)x + 2k - 1 = 0$ are real. Also find the set of values of *k*, for which the roots of the equation above are real and opposite in signs. If the roots of this equation are α and β, find the equation, in terms of *k*. whose roots are $(\alpha - 1)$ and $(\beta - 1)$.

98.Let the roots of the equation $ax^2 + bx + c = 0$ are α and β. Find in terms of *a, b* and *c* the condition required for both of these roots are positive.
Let $f(x) = 3x^2 - (p-4)x - (2p+1)$. When the roots of the equation $f(x) = 0$ are real , show that *p* does not take any value between -14 and -2 . Also find the value of *p* so that the roots of $f(x) = 0$ are equal. Further , find the quadratic equation whose roots are greater than by 2 of the roots of $f(x) = 0$.

99.The roots of the equation $10x^2 + 4x + 1 = 2\lambda x(2 - x)$ are α and β. Where λ is a real constant.

*i.*Find the equation with the roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

*ii.*Find the range of the values of λ so that α and β are real. Deduce the values of λ, for $\alpha = \beta$.

100. Let the roots of the equation $ax^2 + bx + c = 0$ are α and β, where $a \neq 0$ and $\alpha > \beta$. Write down values of $\alpha + \beta$ and $\alpha\beta$. Show that $\alpha - \beta = \frac{\sqrt{\Delta}}{a}$. where $\Delta = b^2 - 4ac$. Show that the roots of the equation $(k^2 - 2k)x^2 + 2(k^2 + 2)x + k^2 + 2k + 4 = 0$ are real. where $k \neq 0, 2$ and $k \in \mathbb{R}$.

101. Let $f(x)=x^2+(3\lambda-1)x+2\lambda^2-\lambda$.

i. Show that for all $\lambda\in\mathbb{R}$, $f(x)=0$ has real roots.

ii. Find in terms of λ, the coordinates of the vertex of the graph of $y=f(x)$. Hence, when $\lambda=1$ sketch the graph of $y=f(x)$. .

iii. Let $\lambda=2$. If $g(x)=f(1-x)$ sketch the graph of $y=g(x)$

102.*a.* The equation $\dfrac{p}{2x}=\dfrac{a}{(x+c)}+\dfrac{b}{(x-c)}$ has equal roots. If the corresponding values of p are p_1 and p_2 $(p_1>p_2)$

Show that $p_1-p_2=4\sqrt{ab}$.

b. Sketch the graphs of $y=x^2-x-2$ and $y=2x-1$ in the same diagram. Using the graph deduce that only one root of the equation $x^2-x-2-(2x-1)=0$ lies between the roots of $x^2-x-2=0$.

103. The roots of the equation $x^2+2kx+k+2=0$ are α and β. Where *k* is a constant. Write down $\alpha+\beta$ and $\alpha\beta$ in terms of *k*. Show that $(\alpha-\beta)^2=4(k^2-k-2)$.

Hence show that there exist two equations as above so that the difference of the roots is 4 and find their equations.

when it is given $k\neq-2$, Show that the quadratic equation $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$ as roots is

$(k+2)x^2+2k(4k^2-3k-6)x+(k+2)^2=0$ and hence find the equation with the roots $1+\dfrac{\alpha^2}{\beta}$ and $1+\dfrac{\beta^2}{\alpha}$.

104. The roots of the equation $x^2-2x+3=0$ are α and β. Write down $\alpha+\beta$ and $\alpha\beta$. Hence

i. Show that $\alpha^2+\beta^2=-2$

ii. Find the value of $\alpha^3+\beta^3$

iii. Show that $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$

iv. Find the quadratic equation with the roots $\alpha^3-\beta$ and $\beta^3-\alpha$.

105.*i.* Find the set of values of p, so that for all real values of x, the expression $(p-3)x^2-4px+p-3$ to be negative.

ii. Let $g(x)=3x^2-2\lambda x+3$. Find the value of λ for $g(x)$ to be a perfect square.

iii. Let $f(x)=x^2+(k-2)x-2k$. Where *k* is a real constant. Show that , *f*(*x*) has factors for any value of *k*.

Let $g(x)=f(x-k)-2x$. Show that $g(x)$ has real roots. If $(x-7)$ is a factor of $g(x)$, find the value of *k* and sketch the graph of $y=g(x)$.

106. The roots of the equation $x^2+ax+b=0$ are α,β.

Find the roots of the following equations in terms of α and β.

i. $bx^2-(a^2-2b)x+b=0$

ii. $b^2x^2-(a^2-2b)x+1=0$

iii. $bx^2-a^2x+a^2=0$

107.*a.* If and α,β are the roots of the equation, $ax^2+bx+c=0$ find the value of $(\alpha-\beta)^2$ in terms of *a, b* and *c*.
Obtain the roots of the equation $(c-b+a)x^2+(b-2a)x+a=0$ in terms of α,β.

bi.If there is a common root for the equations $ax^2 + a^2x + 1 = 0$ and $bx^2 + b^2x + 1 = 0$, show that the quadratic equation $abx^2 + x + a^2b^2 = 0$ is satisfied by the other roots of them.

ii.Show that for real *x*, there is no real value of the expression $\frac{x^2 + 2x - 1}{2x - 1}$ between 1 and 2.

108.Let *f*(*x*) be a quadratic function in the form $f(x) = ax^2 + bx + c$. Here *a, b, c* are real constants and $a \neq 0$.

Show that $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} + \frac{f(x)}{a}$.

Hence **deduce** that if the equation $ax^2 + bx + c = 0$ has real roots $b^2 - 4ac \geq 0$.

Show that the quadratic equation $qx^2 - 2p\sqrt{p}x + p^2 = 0$ has real roots **if and only if** $p \geq q$. Here $p, q \in \mathbb{R},\ p \neq 0,\ q \neq 0$.

109.Let $f(x) = (3-k)x^2 - kx + 1 \ for \ 0 < k < 3,$

i.Write down the discriminent of $f(x) = 0$ in terms of *k*. By this find the range of values of *k* for the roots of $f(x) = 0$ to be real.

ii.Let the roots of $f(x) = 0$ are α, β. Write $\alpha + \beta$ and $\alpha\beta$ in terms of *k*. Show that if α and β are real then both α and β are positive. Show that quadratic equation with $\alpha + 2$ and $\beta + 2$ as roots is $(3-k)x^2 - 3(4-k)x + 13 - 2k = 0$.

110.Let the roots of the equation $x^2 - (2k+3)x + k(k+5) + 2 = 0$ be $\alpha + 2, \beta + 2$ for $k \in \mathbb{R}$.

i.Find the quadratic equation, in terms of *k*, whose roots are α and β.

ii.Find the range of values of *k* for which α and β are real.

iii.Find the range of values of *k* for which α and β are both real and negative.