COMBINED MATHS
ANANDA ILLANGAKOON

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General Certificate of Education (Adv, Level) Examintaion, November 2025

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Combined Mathematics

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Three hours

∗ Answer **five** questions . **Test paper** 01

1. Let $f(x)=x^2 + px+ q$, such that $q-p+1\neq 0$. Show that -1 is not a root of $f(x)=0$.
Let the roots of $f(x)=0$ are α, β.

Show that $(\alpha+1)(\beta+1)=q-p+1$ and $\dfrac{1}{\alpha+1}+\dfrac{1}{\beta+1}=\dfrac{2-p}{q-p+1}$.

Hence show that the quadratic equation $\dfrac{1}{\alpha+1}$ and $\dfrac{1}{\beta+1}$ as its roots is $(q-p+1)\,x^2+(p-2)\,x+1=0$.

Let the minimum value of $f(x)$ be l_1. Show that $l_1=-\dfrac{\Delta}{4}$, where $\Delta=p^2-4q$.

Let $h(x)=x^2+\dfrac{(p-2)}{(q-p+1)}\,x+\dfrac{1}{q-p+1}$.

If the minimum value of $h(x)$ is l_2, show that $(q-p+1)^2\,l_2=l_1$.

Hence prove that for all $x\in\Re$, $f(x)\geq 0$ if and only if $h(x)\geq 0$ for all $x\in\Re$.

2. *a*. Show that $\displaystyle\lim_{x\to-\pi}\frac{x^2-\pi^2}{\sin(\sin x)}=2\pi$.

b. When *t* is a parameter, *x* and *y* are defined as $x=e^t\sin t,\ y=e^t\cos t$.

Show that $(x+y)^2\dfrac{d^2y}{dx^2}=2\left(x\dfrac{dy}{dx}-y\right)$.

c. The equation of a curve is given by $y=\dfrac{ax+b}{x(x+2)}$. Where *a* and *b* are constants.Curve has a stationary point at (1, -2). Find *a* and *b*.

3. (*i*) State, in usual notation for any triangle *ABC*, the "*sine*" rule and "*cosine*" rule.

In usual notation for the triangle *ABC* if $\dfrac{b+c}{11}=\dfrac{c+a}{12}=\dfrac{a+b}{13}$

show that , $\dfrac{\sin A}{7}=\dfrac{\sin B}{6}=\dfrac{\sin C}{5}$

see page 2

(ii). Show that $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.

4. A particle A of mass m moving with velocity u on a smooth horizontal table collides directly with a stationary particle B of mass $3m$. The coefficient of restitution between A and B is e.

   $i.$ Show that after impact the particles A and B move with velocity $\frac{(3e-1)u}{4}$ and $\frac{(1+e)u}{4}$ respectively.

   Afterwards B collides with a vertical wall which is perpendicular to the direction of motion of B and rebounds. The coefficient of restitution between B and wall is $\frac{1}{2}$.

   $ii.$ If A and B collide at the second time while they move to the same direction, show that $\frac{1}{3}<e<\frac{3}{5}$

   $iii.$ If $e=\frac{1}{2}$ show that the impulse exerted by B on the wall is $\frac{27mu}{16}$.

5. A particle is projected from a point on a cliff of height h with initial velocity u at an angle $\alpha\left[<\frac{\pi}{2}\right]$ to the horizontal.

   $i.$Show that, after a time $\frac{u}{g}\operatorname{cosec}\alpha$ the particle moves along a direction perpendicular to its initial direction of motion.

   $ii.$ Show that, at this instant the particle is at a vertical height $\frac{u^2}{g}\left(1-\frac{1}{2}\operatorname{cosec}^2\alpha\right)$ from the level of point of projection.

   $iii.$ Show that the velocity of the particle at this moment is $u\cot\alpha$.

6. $a.$ A car of mass 1200 kg moves with the engine shut off, down a straight road of inclination α to the horizontal. where $\sin\alpha=\frac{1}{30}$, at a certain constant speed. Taking the acceleration due to gravity $g=10ms^{-2}$, find in newtons, the resistance to the motion of the car. Find the power of the engine in kilowatts, when the car ascends the same road, under the same resistance with an acceleration $\frac{1}{6}ms^{-2}$, at the instant when its speed is $15ms^{-1}$.

   $b.$A particle P of mass $2m$, hanging freely from a horizontal ceiling by a light inextensible string of length l, is in equilibrium. Another particle of mass m moving in a horizontal direction with velocity u collides with the particle P and coalesces to it. The string remains taut after the collision and the composite particle just reaches the ceiling. Show that $u=\sqrt{18gl}$.