COMBINED MATHS
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General Certificate of Education (Adv, Level) Examintaion, November 2025

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Combined Mathematics

10 | E | I/II

Three hours

∗ Answer **five** questions . **Test paper** 03

1. Given that $f(x) = x^2 - 2(1+\lambda)x + 4\lambda + 3$, where $\lambda \in \Re$.

   i) Show for the roots of $f(x) = 0$ are in opposite signs that $\lambda < \frac{-3}{4}$.

   ii) Show that for all real values of λ the roots of the equation $f(x) = 3$ are real.

   iii) If λ_1 and λ_2 are two values of λ so that the equation $f(x) = 0$ has coincident roots,

   Show that $|\lambda_1 - \lambda_2| = 2\sqrt{3}$.

   iv) Find the values *a* and *b* interms of λ, so that $f(x) = (x-a)^2 + b$, Find the value of λ such that *f*(*x*) has a minimum at $x = -2$. Find also the minimum of *f*(*x*) at this value of λ .

2.*a.* Show that the remainder when the polynomial *f*(*x*) is divided by (*x*-*a*) is *f*(*a*).
Show that the function $f(x) = x^4 - 2x^2 + 6$ has no a factor in the form (*x* - *k*). Where *k* is a real constant .
Let $g(x) = 3f(x) + \lambda x^2 + \mu x$ .If $(x-1)$ and $(x-2)$ are factors of *g*(*x*), find the values of the constants λ and μ . Hence factorise $g(x)$.
Also find the range of *x* so that the function *g*(*x*) is positive.

*b.*Solve the equation $2\log_x^a + \log_{ax}^a + 3\log_{a^2x}^a = 0$, where $a > 0$, and $a \neq 1$

3. *a*) Let $y = e^{\sin x}$, Show that

   i) $\frac{dy}{dx} = y\cos x$. Hence show that $y\frac{d^2y}{dx^2} + y^2\sin x - \left[\frac{dy}{dx}\right]^2 = 0$

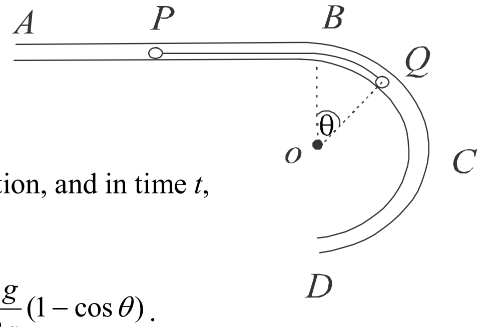
   b) Let $f(x) = \frac{2x^2 - 4x}{(x-3)(x+1)}$; for $x \neq 3, -1$.

   Show that $f'(x) = \frac{-12(x-1)}{(x-3)^2(x+1)^2}$

   sketch the graph of the function $y = f(x)$ indicating the turning points and asymptotes.

see page 2

4. A narrow smooth tube *ABCD* is bent into the form indicated in the figure below. The portion *AB* of the tube is straight. The portion *BCD* is of semi-circular shape with radius *a,* centre O and the diameter *BD* perpendicular to *AB*. The tube is fixed in a vertical plane with *AB* horizontal and uppermost.In side the tube there is a particle *P* of maass *m* and a particle *Q* of mass 3*m* connected by a light inextensible string of length $l\left(>\frac{\pi a}{2}\right)$. Initially, the string is taut, lying *AB,* with the particle *Q* at the point *B*.The particle *Q* is slightly displaced from this position, and in time *t*, radius*OQ* turns through an acute angle θ.



Applying the principle of conservation of energy, show that $\left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{2a}(1-\cos\theta)$.

Hence, or otherwise, show that the acceleration of the particle *P* is $\frac{3g}{4}\sin\theta$. Find the reactiion from the tube on the particle *Q* and the tention in the string, at time *t.*

5.A smooth hemispherical bowl of radius *a* is fixed so that its rim is horizontal and uppermost. A uniform rod *AB* of length 2*l*, and weight 2*w* is in equilibrium with end *A* in contact with the inner surface and end *B* projecting beyond the rim of the bowl with a weight *w* attached there. If the inclination of the rod to the horizontal is θ.

Show that $2l\cos\theta = 3a\cos 2\theta$. Find $\cos\theta$ in terms of *l* and *a*, and show that the lengthof the part of the rod projecting beyond the rim is $\frac{5l-\sqrt{l^2+18a^2}}{3}$.

6.The points $A \equiv (a,0)$, $B \equiv (a,a)$, $C \equiv (0,a)$ where $a > 0$, are in the *OXY* coordinate plane.

Forces of magnitudes $1,2,3,4,9\sqrt{2}$ and $\sqrt{2}$ act along the sides $\overrightarrow{OA}, \overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CO}, \overrightarrow{OB}$ and $\overrightarrow{AC}$ respectively.

Find the magnitude and the direction of the resultant force.

If this resultant force cuts the *OX* axis at *N* , find *ON*. Show that the equation of the line of action of the resultant force is $3y - 4x + 3a = 0$

A couple on the plane of the forces is now added so that the resultant force passes through the origin.

Find the magnitude and the sense of this couple.