COMBINED MATHS
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General Certificate of Education (Adv, Level) Examintaion, November 2025

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Combined Mathematics

Three hours

10 | E | I/II

∗ Answer **five** questions .

Test paper 02

1.*a.* Sketch the graph of $y=|2x-8|$.

Hence sketch the graph of $y=-|2x-8|$. On the same diagram, sketch the graphs of $y=4-|2x-8|$ and $y=|2x-10|$. Hence find the values of x satisfying the inequality $|2x-10|+|2x-8|\le 4$.

b. Find partial fractions of $\dfrac{1}{(x-2)(x^2-5x+6)}$.

2. *a.* Let $f(x)=\dfrac{4x+3}{1+x^2}$.

Show that $f'(x)=\dfrac{2(2+x)(1-2x)}{(1+x^2)^2}$.

Sketch the graph of $y=f(x)$ indicating turning points.

b. If $y=e^{-2x}\cos 3x$,

Show that $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+13y=0$.

c. If $y=5\sin x-3\cos x$, find the equation of the tangent drawn at $x=0$.

3 *a.* Show that $\sin^2\alpha+\cos^2(\alpha+\beta)+2\sin\alpha\sin\beta\cos(\alpha+\beta)=\cos^2\beta$

b. Let $f(x)=\dfrac{2(1+\sqrt{3}\tan x)}{1+\tan^2 x}$

Express $f(x)$ in the form $A\sin(2x+\alpha)+B$ where $A(>0)$, B and α $(0<\alpha<\dfrac{\pi}{2})$ are constants to be determined.

Sketch the graph of $y=f(x)$ for $\dfrac{-\pi}{2}<x<\dfrac{\pi}{2}$.

see page 2

4. Two straight parallel walls are at a distance a apart. A particle is projected with velocity u along the smooth horizontal ground from a foot of a wall, perpendicular to it. The coefficient of restitution between the particle and each wall is e. $0 < e < 1$.

i. Draw the vellocity time graph of the motion of the particle up to the third collision.

ii. Hence show that the time up to third impact is $\frac{a}{u}\left[1+\frac{1}{e}+\frac{1}{e^2}\right]$

iii. Show that the total time up to n^{th} collision is $\frac{a}{u}\left[\frac{e^n-1}{e^n-e^{n-1}}\right]$

(You may use the formula $S_n = \frac{a(1-r^n)}{1-r}$ for the sum of n terms of a geometric progression)

5. A point A is a height h vertically above a point O on the ground. A particle is projected with velocity v making an angle α to the horizontal from the point A. Taking the horizontal and vertical axes through O as X and Y axes respectively, show that the equation of the path of the particle is $y = h - \frac{gx^2}{2v^2} + x\tan\alpha - \frac{gx^2}{2v^2}\tan^2\alpha$.

Particle falls the ground at a point B, a distance d from O.

when $v^2 = gh$, show that $d^2\tan^2\alpha - 2dh\tan\alpha + d^2 - 2h^2 = 0$.

Deduce that $d \le \sqrt{3}h$

Also, if the angle of projection of the above projectile is $\frac{\pi}{6}$, deduce further that the horizontal range of the particle is $\sqrt{3}h$.

6.*a.* Let $A \equiv (0,4)$, $B \equiv (4,10)$ and $C \equiv (7,8)$ be three points in OXY cartisian coordinate plane. The unit vectors along $\overrightarrow{OX}$ and $\overrightarrow{OY}$ be $\underset{\sim}{i}$ and $\underset{\sim}{j}$ respectively. Find the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$. Hence find $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Using these vectors show that the triangle ABC is a right angled triangle.

b. In the triangle ABC, the point E is situated on BC such that $BE:EC = 2:3$. F is on AC such that $CF:FA = 3:4$. The point G is on AB produced so that $GB:GA = 1:2$. Relative to an origin $\boldsymbol{O}$ the position vectors of the points A, B, C are $\underset{\sim}{a}, \underset{\sim}{b}$, and $\underset{\sim}{c}$ respectively.

Prove the following results.

i) $\overrightarrow{OE} = \frac{3}{5}\underset{\sim}{b} + \frac{2}{5}\underset{\sim}{c}$ *ii)* $\overrightarrow{OF} = \frac{3}{7}\underset{\sim}{a} + \frac{4}{7}\underset{\sim}{c}$ *iii)* $\overrightarrow{OG} = 2\underset{\sim}{b} - \underset{\sim}{a}$

Find $\overrightarrow{FE}$ and $\overrightarrow{FG}$ interms of $\underset{\sim}{a}, \underset{\sim}{b}$, and $\underset{\sim}{c}$,