COMBINED MATHS
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General Certificate of Education (Adv, Level) Examintaion, November 2025

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Combined Mathematics

Three hours

10 | E | I/II

∗ Answer **five** questions . **Test paper** 06

1.*a*. The roots of the quadratic equation $x^2+ax+b=0$ are α and β. Obtain the quadratic equation

$\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ as its roots. Hence deduce the quadratic equation whose roots are $2\beta^2+\frac{5}{\alpha^2}$, $2\alpha^2+\frac{5}{\beta^2}$.

b. Find λ,μ,υ such that

$$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{\lambda}{1-ax}+\frac{\mu}{1-bx}+\frac{\upsilon}{1-cx}.$$

Hence deduce $\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-a)(b-c)}+\frac{c^2}{(c-a)(c-b)}=1.$

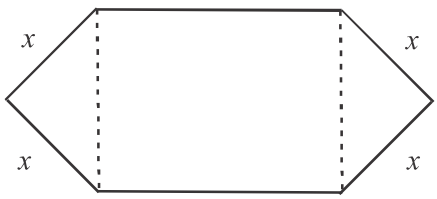
2 *a*. Using the first principle , find the first derivative of $y=\frac{1}{(1-x)}$.

b. Given that $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$.

Prove that

i. $(1-x^2).\frac{dy}{dx}=1+xy$ and *ii*. $(1-x^2).\frac{d^2y}{dx^2}-3x.\frac{dy}{dx}=y$. Find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=0$.

c. A wire frame shown in the diagram is formed with a wire of length *l*. The wire frame consists of two equilateral triangles and its middle part is a rectangle. If the length of a side of the triangle is *x* , show that the area of the frame is $A=\frac{lx+(\sqrt{3}-4)x^2}{2.}$.



Find the value of *x* which maximises the area *A*.

If $l=182(cm)$, deduce the maximum value of the area of the wire frame.

3.*a*) Let $f(\theta)=\cos\theta\sin^2\left(\frac{\theta}{2}\right)$, where $0<\theta<\pi$. Find *a*, *b* and *c* such that $f(\theta)=a-b(\cos\theta-c)^2$. In a triangle *ABC*, in usual notation, if $\cos A\sin^2\frac{A}{2}+\cos B\sin^2\frac{B}{2}+\cos C\sin^2\frac{C}{2}=\frac{3}{8}$, show that *ABC* is an equilateral triangle.

b) For $-\frac{\pi}{2}<x<\frac{\pi}{2}$, let $f(x)=\frac{1+\cot x}{1+\cot^2 x}$.

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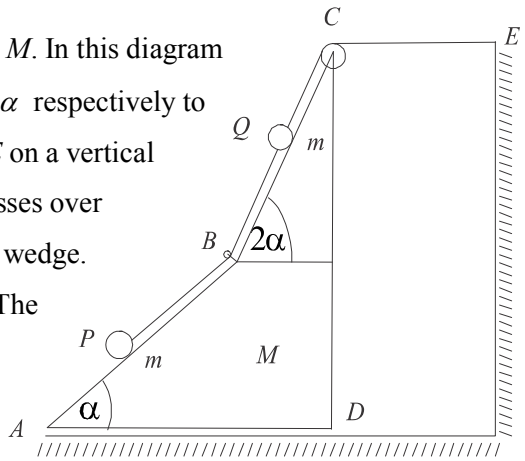
Find A, B and $\alpha\left(0<\alpha<\frac{\pi}{2}\right)$ such that $f(x)=A\cos(2x+\alpha)+B$.

Hence, draw the graph of $y=2f(x)$ for $-\frac{\pi}{2}<x<\frac{\pi}{2}$.

4)The diagram shows a vertical cross section of a smooth wedge of mass M. In this diagram AD and CE are horizontal, and AB and BC are inclined at angles α and 2α respectively to the horizontal. One end of an inextensible light string is fixed to a point E on a vertical wall and the other end is attached to a particle P of mass m. The string passes over smooth pulley C and passes below smooth pulley B which are fixed to the wedge. Another particle Q of mass m is attached to a point in between B and C.The wedge can freely move on a smooth horizontal plane.

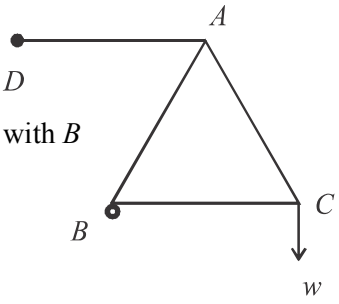


Write down all the equations to find the acceleration of the wedge relative to the floor, accelerations of P and Q particles relative to the wedge and the tensions in BQ and QC portions.

Show that the acceleration of the wedge is $\frac{m(\sin\alpha+\sin 2\alpha)g}{[M+4m-2m(\cos\alpha+\cos 2\alpha]}$.

5.*a*. Four equal uniform rods *AB, BC, CD* and *AD* each of weight *w* are smoothly jointed to form a framework *ABCD*. The framework is suspended from *A* and its square form is maintained by means of a light rod connecting the joints *B* and *D*. Show that the thrust in the light rod is *w* and find the reaction at the joint *C*.

b. Four light rods *AB, BC, AC* and *AD* of equal length are smoothly jointed to form a framework as shown in the figure. The framework is smoothly hinged at *D* . *AD* and *BC* rods are horizontal. A weight *w* is suspended at *C*. The framework rests in a vertical plane with *B* resting on a smooth peg. Using Bow's notation draw a stress diagram and hence find the reactions of *B* and *D* and stresses of all rods indicating tension and thrust.



6.*a*.A particle is projected at an angle α to the horizontal. If the horizontal range is *R* and the time of flight is *T*, show that $\tan\alpha=\frac{9T^2}{2R}$.

b. A motor car of mass 1200*kg* moves on a level road which produces frictional force of 1.5(*N*) per kilogram. If the maximum velocity of the motor car is $72(kmh^{-1})$, find the power of the motor car. Now the motor car with the same power moves up a hill of inclination 1 is to 25 subject to the same friction. When the motor car moves with the velocity $45kmh^{-1}$ find its retardation $(g=10ms^{-2})$.

c.A uniform sphere of weight *w* and radius *r* is in equilibrium on a smooth inclined plane of inclination 30^{o} to the horizontal with a string of length *l* joining the sphere and a point on the inclined plane. Show that the tension in the string is $\frac{w(r+l)}{2\sqrt{l^2+2rl}}$ and find the reaction on the sphere by the plane.