COMBINED MATHS
ANANDA ILLANGAKOON

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General Certificate of Education (Adv, Level) Examintaion, November 2025

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Combined Mathematics

Three hours

10 | **E** | **I/II**

∗ Answer **five** questions . **Test paper** 04

1.*a.* The roots of the equation $ax^2+bx+c=0$ are α and β. Where $a\neq 0$. Prove that $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$.

*b.*Show that the equation $\frac{1}{x+\lambda}+\frac{1}{x+\mu}=\frac{1}{2}$ represents the quadratic equation $x^2+(\lambda+\mu-4)x+\lambda\mu-2(\lambda+\mu)=0$.

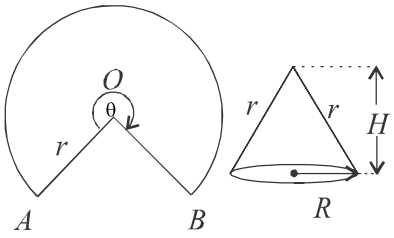
If the roots of this equation are equal in magnitude and opposite in sign , show that the product of the roots is $(\lambda\mu-8)$.

2.*i. a*) Using first principle find the derivative of $y=\sqrt{\sin x}$

b) Let $y=e^{\tan x}$

Show that $\frac{dy}{dx}=y\left[1+[lny]^2\right]$. Hence show that $\frac{d^2y}{dx^2}=[1+lny]^2.\frac{dy}{dx}$

ii. A sector *AOB* with $\theta(0<\theta<2\pi)$ radian is cut from a circular sheet of radius *r* as shown in the diagram.A cone with radius *R* and height *H* is formed by joining the edges *OA* and *OB* together.



a) Show that $R=rx$ and $H=r\sqrt{1-x^2}$, where $x=\frac{\theta}{2\pi}$

b) Show that the volume of the cone *V* is given by $V=\frac{1}{3}\pi r^3x^2\sqrt{1-x^2}$.

For a given *r* , find the value of *x* for which *V* is maximum and deduce that

$\theta=2\left(\frac{2}{3}\right)^{\frac{1}{2}}\pi$.

3.*i.* If $5\theta=\frac{\pi}{2}$ prove that $\sin 3\theta=\cos 2\theta$ .Hence show that $\sin 18^\circ=\frac{\sqrt{5}-1}{4}$.

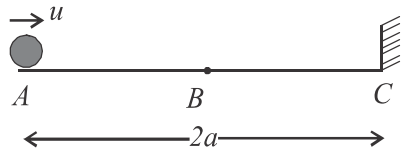
ii. In usual notation **state** "sine" rule and "cosine' rule for a triangle *ABC* .

In a triangle *ABC, CotA,CotB,CotC* lie in Arithmetic progression.Show that a^2,b^2,c^2 also lie in an arithmetic progression.

iii. If $\sin^{-1}\frac{2a}{1+a^2}+\sin^{-1}\frac{2b}{1+b^2}=2\tan^{-1}x$, show that $x=\frac{a+b}{1-ab}$.

see page 2

4. A particle is given an initial velocity u horizontally along the ground from a point A on the ground. The particle moves perpendicular to a smooth vertical wall at C such that $AC = 2a$. (see the diagram) The mid - point of AC is B The part A to B of the path is rough and the part B to C is smooth. When the particle passes the point B its velocity is λu. Where $0 < \lambda < 1$. The particle strikes on the wall and after rebounding it moves in the opposite direction. The coefficient of restitution between the wall and the particle is e.If the particle does not come back to A, sketch the velocity - time graph for the motion of the particle. Hence,



i. show that the particle reaches the point B after a time $\frac{2a}{(1+\lambda)u}$.

ii. show that the retardation of the particle in AB motion is $\frac{u^2(1-\lambda^2)}{2a}$.

iii. if the ratio of the times for B to C and C to B motions is 2 : 3, show that $e = \frac{2}{3}$.

iv. Show that the distance travelled by the particle during the retarding motion in its backward motion is $\frac{4\lambda^2 a}{9(1-\lambda^2)}$.

v. if the total displacement of the particle is $\frac{5a}{9}$, show that $\lambda = \frac{1}{\sqrt{2}}$

5. $ABCDEF$ is a framework formed with six uniform rods each of weight w and length a, smoothly jointing at their ends. The framework is suspended from A, and the regular hexagonal shape is maintained by a light rod joining the points L and M on BC and FE respectively, such that $BL = FM = b$.

A uniform circular lamina of weight w and radius $\frac{a}{\sqrt{3}}$ is placed symmetrically on the rods DC and DE such that its plane is on the same vertical plane of the framework $ABCDEF$.

i.Show that the reaction of D is $\frac{17w}{6\sqrt{3}}$ horizontally.

ii. Show that the thrust of the light rod is $\frac{34w}{3\sqrt{3}}$.

iii. Show that $b = \frac{7a}{34}$.

6.The figure represents a framework consisting of five smoothly jointed light rods. The framerowk is smoothly hinged at a fixed point A and carries a load w at C. The framework is kept in equilibrium in a vertical plane with AC horizontal and AB vertical by a force P applied at B in a direction parallel to AC. Find the magnitude of P and the horizontal and vertical components of the reaction at A.

Draw a stress diagram for the framewrok, using Bow's Notation.Hence, determine the stresses in the rods in terms of w, distinguishing between tensions and thrusts.