COMBINED MATHS
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General Certificate of Education (Adv, Level) Examintaion, November 2025

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Combined Mathematics

10 | E | I/II

Three hours

* Answer **five** questions .

Test paper 05

1.*a*) Let $f(x)=ax^2+bx+c$, where $a>0$. Find *p* , *q* and *r* in terms of *a,b* and *c* such that $f(x)=p(x-q)^2+r$.
Hence, find the condition for the quadratic equation $f(x)=0$ to have real roots and the condition to have coincident roots .

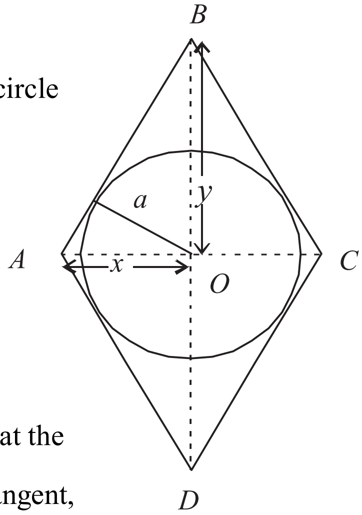
i) If the roots of the quadratic equation $x^2+px+q=0$ are real, show that the roots of $x^2+px+q+(x+a)(2x+p)=0$ are real for all real values of *a*.

ii) If the quadratic equation $\left(1-q+\dfrac{p^2}{2}\right)x^2+p(1+q)x+q(q-1)+\dfrac{p^2}{2}=0$ has coincident roots for $pq\neq 0$ and $q\neq -1$, show that $p^2=4q$.

b) For $k\in\Re$, let $f(x)=4x^3-4x^2+kx-2$. If $(x-2)$ is a factor of $f(x)$, find the value of *k*. Express $f(x)$ in the form of $(x-2)(ax+b)^2$,where *a* and *b* are constants to be determined. For this values of *a* and *b*, find *p*, *q* and *r* such that $(ax+b)^2=p(x-2)^2+qx+r$. **Hence** ,find the remainder when $f(x)$ is divided by $(x-2)^3$.

2.*a*. Find $\displaystyle\lim_{x\to 0}\frac{\cos\left(\frac{\pi}{2}\cos^2 x\right)}{x^2}$.

b. The adjacent figure represents a rhombus *ABCD* drawn so that its four sides touch a circle of radius *a* and centre *O*.Let $OB=y$ $OA=x$



i) Show that $y=\dfrac{ax}{\sqrt{x^2-a^2}}$

ii) Show also that, when the rhombus is drawn as above so as to minimise its area, the rhombus becomes a square and the minimum area is $4a^2$.

c. A tangent is drawn to the curve $y=e^{2x}+x^2$ at the point $x=0$ on the curve. Show that the equation of the tangent is $2x-y+1=0$. Find area of the triangle bounded by this tangent, *y*-axis and perpendicular drawn from the origin to this tangent.

see page 2

3. Show that the coordinates of any point on the perpendicular drawn from $P(\alpha, \beta)$ to the line $ax + by + c = 0$ can be expressed as $(\alpha + at, \beta + bt)$ where t is a parameter. Q is a point on the perpendicular drawn from P (4, 1) to the line $2x + 3y + 4 = 0$ and R is the point on this perpendicular between P and Q such that $PR : RQ = 1 : 3$ and R lies on the line $x - y + 1 = 0$. The point of intersection of the lines $2x + 3y + 4 = 0$ and $x - y + 1 = 0$ is S.

   i. Find the coordinates of Q.

   ii. Find the coordinates of S.

   iii. Find the area of the triangle QRS.

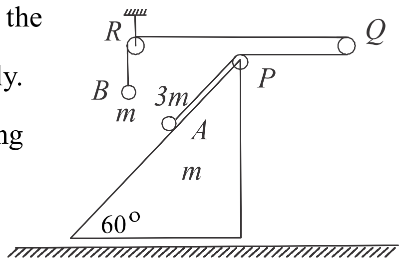
4. Position vectors of *A, B, C* and *D* in *OXY* plane respectively are $\sqrt{3}\underline{i} + \underline{j},\ 4\underline{j}, -\sqrt{3}\underline{i} + 3\underline{j}, -\sqrt{3}\underline{i} + \underline{j}$. Forces of magnitudes $4\sqrt{3}, 10, 2\sqrt{3}\ \sqrt{3}, 2\sqrt{3}Q, \sqrt{3}P$ Newtons act along $\overrightarrow{OA}, \overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{DC}, \overrightarrow{OD}, \overrightarrow{BO}$ respectively.Distances are measured in metres.

   i) Show that this system of forces will never be in equilibrium.

   ii) If $Q = \frac{4}{3}, P = \frac{31}{3}$ show that the system is equivalent to a couple, and show that the magnitude of its moment if its 5 Nm.

   iii) If it is given that $P - Q = 9$, and the system reduces to a single force through *B*, show that $Q = \frac{7}{4}$.
   Show that the system can be reduced to a single force through *AD*, and to a couple .
   Find the magnitude of this force, and the couple.

5.Two particles *A* and *B* with masses **3*m*** and ***m*** are attached to the two ends of a light inextensible string such that *A* is on the face inclined at an angle of 60° to the horizontal of a wedge which is movable along a horizontal table and *B* hangs freely. The string passes through the pulleys *P, Q* and *R*. The part *PQ* and *QR* of the string are horizontal and the part *RB* of the string is vertical.The system is released from rest , find the accelerations of the particles *A, B* and the wedge.



6. A particle *A* is projected with velocity *u* vertically upwards from a point *O* in the space. Let the heighest point reached by the particle *A* be *P* and mid-point of *OP* be *Q*. As the particle *A* passes the point *Q* at the second time, another particle *B* is dropped from *O*. Draw the velocity-time graphs for the motions of the particles *A* and *B* until their collision. Hence show that the time for the collision after releasing the particle *B* is $\frac{u}{2\sqrt{2g}}$.
Find the depth of point of collision below *O*. What happens if the particle *B* is projected from *O* downwards with velocity $\frac{u}{\sqrt{2}}$ instead of dropping from *O* ?