TRIGONOMETRY

1. Prove the following identities

1. $\tan A + \cot A = \sec A.\operatorname{cosec} A$

2. $\tan^2 A(1-\sin^2 A) = \sin^2 A$

3. $\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \sin\theta$

4. $(1+\cot^2 A)(1-\sin^2 A) = \cot^2 A$

5. $\sec A = \dfrac{\operatorname{cosec} A}{\sqrt{\operatorname{cosec}^2 A - 1}}$

6. $\sin A\sqrt{1+\tan^2 A} = \sqrt{\sec^2 A - 1}$

7. $\sqrt{\sec^2 A - 1} = \sin A \sec A$

8. $\tan^2 A + \sec^2 B = \tan^2 B + \sec^2 A$

9. $\sec^2 A \operatorname{cosec}^2 A - 2 = \tan^2 A + \cot^2 A$

10. $\sin A \sec A = \tan A$

11. $\cot A \sec A \sin A = 1$

12. $\cot^2 A(1-\cos^2 A) = \cos^2 A$

13. $\cos\theta\sqrt{\cot^2\theta + 1} = \sqrt{\operatorname{cosec}^2\theta - 1}$

14. $(1-\cos^2 A)(1+\tan^2 A) = \tan^2 A$

15. $\dfrac{1}{\sec^2 A} + \dfrac{1}{\operatorname{cosec}^2 A} = 1$

16. $\sin^2 A(1+\cot^2 A) + \cos^2 A(1+\tan^2 A) = 2$

17. $\dfrac{\tan\alpha + \cot\beta}{\cot\alpha + \tan\beta} = \tan\alpha.\cot\beta$

18. $\dfrac{\tan\alpha - \cot\beta}{\tan\beta - \cot\alpha} = \tan\alpha.\cot\beta$

19. $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$

20. $(\sin A + \cos A)(\cot A + \tan A) = \sec A + \operatorname{cosec} A$

21. $\cos\theta(2+\tan\theta)(1+2\tan\theta) = 2\sec\theta + 5\sin\theta$

22. $\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta = \cos^2\beta - \cos^2\alpha$

23. $\sec^2\alpha\tan^2\beta - \tan^2\alpha\sec^2\beta = \tan^2\beta - \tan^2\alpha$

24. $\sin^2 A\tan A + \cos^2 A\cot A + 2\sin A\cos A = \sec A\operatorname{cosec} A$

25 $\sec^2 A.\operatorname{cosec}^2 A = \sec^2 A + \operatorname{cosec}^2 A$

26. $\cos^4 A + \sin^4 A = 1 - 2\sin^2 A + 2\sin^4 A$

27. $\cos^4 A - \sin^4 A = 2\cos^2 A - 1$

28. $\cos^6 A + \sin^6 A = 1 - 3\cos^2 A\sin^2 A$

29. $\cos^3 A - \sin^3 A = (\cos A - \sin A)(1 + \cos A\sin A)$

30. $\operatorname{cosec}^4 A - \cot^4 A = \dfrac{1+\cos^2 A}{1-\cos^2 A}$

31. $\sec^4 A - 1 = 2\tan^2 A + \tan^4 A$

32. $\dfrac{\tan\theta}{\sec\theta - 1} + \dfrac{\tan\theta}{\sec\theta + 1} = 2\operatorname{cosec}\theta$

33. $\dfrac{\sin A}{1-\cos A} = \dfrac{1+\cos A}{\sin A}$

34. $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$

35. $\dfrac{1}{\sec A - \tan A} = \sec A + \tan A$

36. $\dfrac{\cot A}{1-\tan A} + \dfrac{\tan A}{1-\cot A} = \sec A\operatorname{cosec} A + 1$

37. $\dfrac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta} = \dfrac{1+\cos\theta}{\sin\theta}$

38. $\dfrac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta} = \dfrac{1-\cos\theta}{\sin\theta}$

39. $\dfrac{\sin\theta+1-\cos\theta}{\sin\theta-1+\cos\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

40. $\dfrac{2\sin\theta\cos\theta - \cos\theta}{1-\sin\theta-\cos^2\theta+\sin^2\theta} = \cot\theta$

Ananda illangakoon

41. $\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}=2\cos ecA$

42. $(\sin A+\cos A)(1-\sin A.\cos A)=\sin^3 A+\cos^3 A$

43. $\sqrt{\frac{1-\sin A}{1+\sin A}}=\sec A-\tan A$

44. $\frac{\cos ecA}{\cos ecA-1}+\frac{\cos ecA}{\cos ecA+1}=2\sec^2 A$

45. $\frac{\cos ecA}{\cot A+\tan A}=\cos A$

46. $\frac{1}{\cot A+\tan A}=\sin A\cos A$

47. $(\sec A+\cos A)(\sec A-\cos A)=\tan^2 A+\sin^2 A$

48. $\frac{1-\tan A}{1+\tan A}=\frac{\cot A-1}{\cot A+1}$

49. $\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\sin^2 A}{\cot^2 A}$

50. $\frac{\sec A-\tan A}{\sec A+\tan A}=1-2\sec A.\tan A+2\tan^2 A$

51. $\sec^4 A-\sec^2 A=\tan^4 A+\tan^2 A$

52. $\cot^4 A+\cot^2 A=\cos ec^4 A-\cos ec^2 A$

53. $\sqrt{\cos ec^2 A-1}=\cos A\cos ecA$

54. $\frac{\cot A\cos A}{\cot A+\cos A}=\frac{\cot A-\cos A}{\cot A\cos A}$

55. $(\sin A+\cos A)(\cot A+\tan A)=\sec A+\cos ecA$

56. $(1+\cot A-\cos ecA)(1+\tan A+\sec A)=2$

57. $\frac{1}{\cos ecA-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\cos ecA+\cot A}$

58. $\left[\frac{1}{\sec^2\alpha-\cos^2\alpha}+\frac{1}{\cos ec^2\alpha-\sin^2\alpha}\right]\cos^2\alpha\sin^2\alpha=\frac{1-\cos^2\alpha\sin^2\alpha}{2+\cos^2\alpha\sin^2\alpha}$

59. $\sin^8 A-\cos^8 A=(\sin^2 A-\cos^2 A)(1-2\sin^2 A\cos^2 A)$

60. $\frac{\cos A\cos ecA-\sin A\sec A}{\cos A+\sin A}=\cos ecA-\sec A$

61. $\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}$

62. $(\tan\alpha+\cos ec\beta)^2-(\cot\beta-\sec\alpha)^2=2\tan\alpha\cot\beta(\cos ec\alpha+\sec\beta)$

63. $2\sec^2\alpha-\sec^4\alpha-2\cos ec^2\alpha+\cos ec^4\alpha=\cot^4\alpha-\tan^4\alpha$

64. $(\sin\alpha+\cos ec\alpha)^2+(\cos\alpha+\sec\alpha)^2=\tan^2\alpha+\cot^2\alpha+7$

65. $(\cos ecA+\cot A)\cos A-(\sec A+\tan A)\sin A=(\cos ecA-\sec A)(2-\sin A\cos A)$

66. $(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\cos ec^2 A}-\frac{\cos ecA}{\sec^2 A}$

2. **Express the following in the simplest form.**

1. $\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$
2. $\sin(30+\alpha).\cos(30-\alpha)+\cos(30+\alpha).\sin(30-\alpha)$
3. $\cos A\cos A+\sin A\sin A$
4. $\cos A\cos A-\sin A\sin A$
5. $\cos 15^\circ\cos 45^\circ-\sin 15^\circ\sin 45^\circ$
6. $\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$
7. $\sin 3A\cos 2A-\sin 2A\cos 3A$
8. $\frac{1}{\sqrt{2}}(\cos A+\sin A)$
9. $\frac{\cos A+\sin A}{\cos A-\sin A}$
10. $\frac{1+\tan(A-45^\circ)}{1-\tan(A-45^\circ)}$
11. $\frac{\tan A-\tan(A-60^\circ)}{1+\tan A.\tan(A-60^\circ)}$
12. $\frac{1+\tan\alpha.\tan\beta}{\tan\alpha-\tan\beta}$
13. $\cos\left[45^\circ+\frac{A}{2}\right]\sin 45^\circ-\sin\left[45^\circ+\frac{A}{2}\right]\cos 45^\circ$
14. $\frac{\tan(A-B)+\tan B}{1-\tan(A-B)\tan B}$

3. **Obtain the relation between x and y of the following.**

1. $x=4\sec\theta$, $y=5\tan\theta$
2. $x=a\cos ec\theta$, $y=b\cot\theta$
3. $x=2\tan\theta$, $y=3\cos\theta$
4. $x=1-\sin\theta$, $y=a\sec\theta$
5. $x=2+\tan\theta$, $y=2\cos\theta$
6. $x=a\sec\theta$ *(i).If* $y=b\sin\theta$

4. 1**.** If $\sin\theta=\frac{3}{5}$ and $\cos\phi=-\frac{12}{13}$, where θ and ϕ both lie in the second quadrant, find the values *of*,

(a). $\sin(\theta-\phi)$ *(b).* $\cos(\theta+\phi)$ *(c).* $\tan(\theta-\phi)$

2. If $\cos\theta=\frac{4}{5}$ and $\cos\phi=\frac{12}{13}$, where θ and ϕ both lie in the fourth quadrant, find the values of

(a). $\cos(\theta+\phi)$ *(b).* $\sin(\theta-\phi)$ *(c).* $\tan(\theta+\phi)$

3. If $\cot\alpha=\frac{1}{2}$ and $\sec\beta=\frac{-5}{3}$ where $\pi<\alpha<\frac{3\pi}{2}$ and $\frac{\pi}{2}<\beta<\pi$ find the value of $\tan(\alpha+\beta)$

5. **Prove the following identities**

1. $\frac{1-\cos 2A}{\sin 2A}=\tan A$
2. $\tan\theta+\cot\theta=2\cos ec\,2\theta$
3. $\sec 2A+\tan 2A=\frac{\cos A+\sin A}{\cos A-\sin A}$
4. $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A}=\tan A$
5. $\cos 4A=8\cos^4 A-8\cos^2 A+1$
6. $\cos 3A=4\cos^3 A-3\cos A$
7. $\sin 2A=\frac{2\tan A}{1+\tan^2 A}$
8. $\cot A-\tan A=2\cot 2A$
9. $\frac{1+\cos A}{1-\cos A}=\cot^2\frac{A}{2}$
10. $\frac{\sin 4A}{1+\cos 4A}=\tan 2A$
11. $\frac{1+\cos A}{\sin A}=\cot\frac{A}{2}$
12. $\frac{\cos A-\sin A}{\cos A+\sin A}=\sec 2A-\tan 2A$

13. $4(\cos^6 A+\sin^6 A)=4-3\sin^2 2A$

14. $\dfrac{1+\tan^2\left[\dfrac{\pi}{4}-A\right]}{1-\tan^2\left[\dfrac{\pi}{4}-A\right]}=\operatorname{cosec}2A$

15. $2(\cos^4\theta+\sin^4\theta)=1+\cos^2 2\theta$

16. $\dfrac{\sin A\cos A-\sin B\cos B}{\sin^2 A-\sin^2 B}=\cot(A+B)$

17. $\dfrac{\cos A}{1\pm\sin A}=\tan\left[\dfrac{\pi}{4}\mp\dfrac{A}{2}\right]$

18. $\dfrac{\sec 4A-1}{\sec 2A-1}=\dfrac{\tan 4A}{\tan A}$

19. $\tan\left[\dfrac{\pi}{4}+\theta\right]-\tan\left[\dfrac{\pi}{4}-\theta\right]=2\tan 2\theta$

20. $4(\cos^6 A-\sin^6 A)=3\cos 2A+\cos^3 2A$

21. $\sin^3 2A\cos 6A+\cos^3 2A\sin 6A=\dfrac{3}{4}\sin 8A$

22. $\sin 3\theta=3\sin\theta-4\sin^3\theta$

23. $\dfrac{\sin 2A}{1+\cos 2A}=\tan A$

24. $\dfrac{1-\cos 2A}{1+\cos 2A}=\tan^2 A$

25. $\dfrac{\sin 2A}{1-\cos 2A}=\cot A$

26. $\operatorname{cosec}2A+\cot 2A=\cot A$

27. $\dfrac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}=\tan\dfrac{A}{2}.\cot\dfrac{B}{2}$

28. $\dfrac{\sec 8A-1}{\sec 4A-1}=\dfrac{\tan 8A}{\tan 2A}$

29. $\dfrac{1+\tan^2(45^\circ-A)}{1-\tan^2(45^\circ-A)}=\operatorname{cosec}2A$

30. $\dfrac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

31. $\dfrac{\sin^2 A-\sin^2 B}{\sin A\cos A-\sin B\cos B}=\tan(A+B)$

32. $\dfrac{\cos A+\sin A}{\cos A-\sin A}-\dfrac{\cos A-\sin A}{\cos A+\sin A}=2\tan 2A$

33. $\cot(A+15^\circ)-\tan(A-15^\circ)=\dfrac{4\cos 2A}{1+2\sin 2A}$

34. $\dfrac{\sin\theta+\sin 2\theta}{1+\cos 2\theta+\cos 2\theta}=\tan\theta$

35. $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\dfrac{\theta}{2}$

6. Write the following as a product.

1. $\sin 3A+\sin A$
2. $\cos 5A+\cos 3A$
3. $\sin 4A-\sin 2A$
4. $\cos 7A-\cos A$
5. $\sin 3A-\sin 5A$
6. $\cos A-\cos 5A$
7. $\sin 30^\circ+\sin 60^\circ$
8. $\cos 70^\circ+\cos 50^\circ$

7. Write the following as a sum or a difference.

1. $2\sin^2\theta.\cos\theta$
2. $2\cos 3\theta.\cos 2\theta$
3. $2\cos\theta.\sin 4\theta$
4. $-2\sin 3\theta.\sin\theta$
5. $2\sin 4\theta.\sin 2\theta$
6. $\cos\theta.\cos 4\theta$
7. $2\sin 30^\circ.\cos 60^\circ$
8. $2\cos 20^\circ.\cos 40^\circ$
9. $2\sin 3A.\cos 2A$
10. $2\sin 4A.\cos 7A$

8. Prove the following identities.

1. $\dfrac{\sin 4A+\sin 2A}{\sin 4A-\sin 2A}=\tan 3A\cot A$

2. $\dfrac{\cos 11A-\cos 3A}{\cos 11A+\cos 3A}=-\tan 7A\tan 4A$

3. $\dfrac{\sin A+\sin B}{\cos A+\cos B}=\tan\dfrac{A+B}{2}$

4. $\dfrac{\cos 2B-\cos 2A}{\sin 2B-\sin 2A}=-\tan(A+B).$

5. $\dfrac{\sin 2A+\sin 2B}{\sin 2A-\sin 2B}=\dfrac{\tan(A+B)}{\tan(A-B)}$

6. $\sin\left[\dfrac{\pi}{4}+A\right]-\sin\left[\dfrac{\pi}{4}-A\right]=\sqrt{2}\ \sin A$

7. $\dfrac{\cos(30-A)+\cos(30+A)}{\cos(30+A)-cos(30-A)}=-\cot 30\cot A$

8. $\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$

9. $\cos^2 5A-\cos^2 3A=-\sin 8A\sin 2A$

10. $\dfrac{\sin A-\cos A}{\cos A+\sin A}=\dfrac{\sin\left[A-\dfrac{\pi}{4}\right]}{\sin\left[A+\dfrac{\pi}{4}\right]}$

11. $\dfrac{\sin 2A-\sin 2B}{\sin 2A+\sin 2B}=\dfrac{\tan(A-B)}{\tan(A+B)}$

12. $\dfrac{\sin 3x+\sin 5x}{\sin 4x+\sin 6x}=\dfrac{\sin 4x}{\sin 5x}$

13. $\dfrac{\sin 3A-\sin 7A+\sin 11A-\sin 15A}{\cos 3A-\cos 7A+\cos 11A-\cos 15A}=-\cot 9A$

14. $\dfrac{\cos A-\cos B}{\sin A+\sin B}=\tan\dfrac{B-2}{2}$

15. $\dfrac{\sin A-\sin B}{\sin A+\sin B}=\cot\dfrac{A+B}{2}\tan\dfrac{A-B}{2}$

16. $\dfrac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A}=\tan 3A$

17. $\cos 2A\cos 3A-\cos 2A\cos 7A=\sin 5A\sin 4A$

18. $1+2\cos 2A+\cos 4A=4\cos^2 A\cos 2A$

19. $\dfrac{\sin 6A\cos 3A-\sin 8A\cos A}{\sin 3A\sin 4A-\cos 2A\cos A}=\tan 2A$

20. $\dfrac{\sin 3A+2\sin 5A|+\sin 7A}{\sin A+2\sin 3A+\sin 5A}=\cos 2A+\sin 2A\cot 3A$

21. $\dfrac{\cos 2A+\cos 2B}{\cos 2B-\cos 2A}=\cot(A+B)\cot(A-B)$

22. $\dfrac{\sin A\sin 2A+\sin 3A\sin 6A}{\sin A\cos 2A+\sin 3A\cos 6A}=\tan 5A$

23. $\dfrac{\cos 5A+2\cos 7A+\cos 9A}{\cos 3A+2\cos 5A+\cos 7A}=\cos 7A\cos 5A$

24. $\sin 3A+\cos A=2\sin\left[\dfrac{\pi}{4}+A\right]\cos\left[2A-\dfrac{\pi}{4}\right]$

25. $\sin A\sin(A+2B)-\sin B\sin(B+2A)=\sin(A+B)\sin(A-B)$

26. $\sin\theta+\sin 2\theta+\sin 3\theta=\sin 2\theta(1+2\cos\theta)$

27. $\sin\dfrac{A}{2}\sin\dfrac{5A}{2}+\sin\dfrac{3A}{2}\sin\dfrac{9A}{2}=\sin 4A\sin 2A$

28. $\cos 2\theta+\cos 4\theta+\cos 6\theta+\cos 120\theta=4\cos 3\theta\cos 4\theta\cos 5\theta$

29. $\dfrac{\sin A+\sin 2A+\sin 4A+\sin 5A}{\cos A+\cos 2A+\cos 4A+\cos 5A}=\tan 3A$

30. $\dfrac{\cos 6\theta-\cos 4\theta}{\sin 6\theta+\sin 4\theta}=-\tan\theta$

31. $\dfrac{\sin A+\sin 3A}{\cos A+\cos 3A}=\tan 2A$

33. $\dfrac{\sin 7A-\sin A}{\sin 8A-\sin 2A}=\cos 4A\sec 5A$

34. $\dfrac{\sin A+\sin 2A}{\cos A-\cos 2A}=\cot\dfrac{A}{2}$

35. $\dfrac{\sin 5A-\sin 3A}{\cos 3A+\cos 5A}=\tan A$

Ananda illangakoon

9. Prove the following .

1. $\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$

2. $\cot 4x(\sin 5x+\sin 3x)=\cot x(\sin 5x-\sin 3x)$

3. $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$

4. $(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\left(\dfrac{x-y}{2}\right)$

5. $\sin 3x+\sin 2x-\sin x=4\sin x\cos\dfrac{x}{2}\cos\dfrac{3x}{2}$

6. $\dfrac{\cos 4x\sin 3x-\cos 2x\sin x}{\sin 4x\sin x+\cos 6x\cos x}=\tan 2x$

7. $\dfrac{\cos 2x\sin x+\cos 6x\sin 3x}{\sin 2x\sin x+\sin 6x\sin 3x}=\cot 5x$

8. $\sin 10^0\sin 30^0\sin 50^0\sin 70^0=\dfrac{1}{16}$

9. $\sin 20^0\sin 40^0\sin 60^0\sin 80^0=\dfrac{1}{16}$

10. $\cos 10^0\cos 30^0\cos 50^0\cos 70^0=\dfrac{3}{16}$

11. $\sin 2x(\tan x+\cot x)=2$

12. $\cos ec2x+\cot 2x=\cot x$

13. $\cos 2x+2\sin^2 x=1$

14. $(\sin x-\cos x)^2=1-\sin 2x$

15. $\cot x-2\cot 2x=\tan x$

16. $\left(\cos^4 x+\sin^4 x\right)=\dfrac{1}{2}(2-\sin^2 2x)$

17. $\dfrac{\cos^3 x-\sin^3 x}{\cos x-\sin x}=\dfrac{1}{2}\left(2+\sin 2x\right)$

18. $\dfrac{1-\cos 2x+\sin x}{\sin 2x+\cos x}=\tan x$

19. $\cos x\cos 2x\cos 4x\cos 8x=\dfrac{\sin 16x}{16\sin x}$

10. Find sin and cos values of the following angles.

1. $7\frac{1}{2}$	2. *15*	3. $22\frac{1}{2}$	4. *75*	5. *105*
6. *120*	7. *150*	8. 18^0	9. 72^0	10. 36^0

11. Prove the following .

1. $2\sin 22\dfrac{1^0}{2}\cos 22\dfrac{1^0}{2}=\dfrac{1}{\sqrt{2}}$

2. $2\cos^2 15^0-1=\dfrac{\sqrt{3}}{2}$

3. $8\cos^3 20^0-6\cos 20^0=1$

4. $3\sin 40^0-4\sin^3 40^0=\dfrac{\sqrt{3}}{2}$

5. $\sin^2 24^0-\sin^2 6^0=\dfrac{\left(\sqrt{5}-1\right)}{8}$

6. $\sin^2 72^0-\cos^2 30^0=\dfrac{\left(\sqrt{5}-1\right)}{8}$

7. $\tan 6^0\tan 42^0\tan 66^0\tan 78^0=1$

8. $\sin\dfrac{\pi}{10}+\sin\dfrac{13\pi}{10}=-\dfrac{1}{2}$

9. $\left(\cos^2 48^0-\sin^2 12^0\right)=\dfrac{\sqrt{5}+1}{8}$

10. $\left(\sin^2 72^0-\sin^2 60^0\right)=\dfrac{\left(\sqrt{5}-1\right)}{8}$

11. $\cos 6^0\cos 42^0\cos 66^0\cos 78^0=\dfrac{1}{16}$

12. If $A+B+C=180$ prove the following .

1. $Sin2A+Sin2B-Sin2C=4CosACosB\sin C$

2. $Cos2A+Cos2B+Cos2C=-1-4CosACosBCosC$

3. $Cos2A+Cos2B-Cos2C=1-4SinASinBCosC$

4. $SinA+SinB+SinC=4Cos\dfrac{A}{2}Cos\dfrac{B}{2}Cos\dfrac{C}{2}$

5. $SinA+SinB-SinC=4Sin\dfrac{A}{2}Sin\dfrac{B}{2}Cos\dfrac{C}{2}$

6. $CosA+CosB+CosC=1+4Sin\dfrac{A}{2}Sin\dfrac{B}{2}Sin\dfrac{C}{2}$

7. $Sin^2A+Sin^2B-Sin^2C=2SinASinBCo\,sC$

8. $Cos^2A+Cos^2B+Cos^2C=1-2CosACosBCo\,sC$

9. $Cos^2A+Cos^2B-Cos^2C=1-2SinASinBCo\,sC$

TRIGONOMETRY

1) ***Sketch the following trigonometric graphs.***

1) $y = \sin\theta$ 2) $y = \cos\theta$ 3) $y = \tan\theta$ 4) $y = \sin 2\theta$ 5) $y = \cos 3\theta$ 6) $y = \tan 2\theta$

7) $y = \sin\frac{\theta}{2}$ 8) $y = \cos\frac{\theta}{3}$ 9) $y = \sin(-\theta)$ 10) $y = \cos(-\theta)$ 11) $y = \tan(-\theta)$ 12) $y = \sin\theta + 1$

13) $y = \cos\theta - 2$ 14) $y = 2\sin\theta$ 15) $y = 3\cos\theta$ 16) $y = 2\sin\theta + 1$ 17) $y = 3\cos\theta - 1$

18) $y = 2\sin\left(2\theta + \frac{\pi}{3}\right) - 1$ 19) $y = 3\cos\left(3\theta - \frac{\pi}{4}\right) + 2$ 20) $y = \sin(2\theta - \frac{\pi}{3})$ 21) $y = \cos(2\theta + \frac{\pi}{3})$

22) Sketch the graph of $y = 2\cos(\theta - \frac{\pi}{6}) + 3$, in the range $-\frac{\pi}{3} \le \theta \le \frac{13\pi}{6}$.

23)Sketch the graph of $y = 3\sin(2\theta - \frac{\pi}{3}) + 2$ in the interval $-\frac{\pi}{12} \le \theta \le \frac{7\pi}{6}$.

24)Sketch the graph of $y = 2\cos(2\theta - \frac{\pi}{3}) + 1$; $in\ \ 0 \le \theta \le 360^{o}$

2) ***Find general solutions of the following.***

(i) $\sec^2\theta + \tan^2\theta = 7$ (ii) $\tan\theta + \cot\theta = 2$ (iii) $2\cos\theta = \sin(\theta + 30^{o})$ (iv) $\cos 2x + 3\sin x = 2$

(v) $\cos 2x = \sin x$ (vi) $\cot x + \tan x = 2\mathrm{cosec}\, x$ (vii) $\sec^2 2x = 1 - \tan 2x$

3) Find general solutions of the following.

(i) $4\sin^2 x = 1$ (ii) $2\cos^2 x = 1$ (iii) $\cot^2 x = 3$

4) Find the general solutions of

$(a)\ \sin 2x + \sin 4x + \sin 6x = 0$ $(b)\ \sin x + \sin 3x + \sin 5x = 0$

5) Find the general solutions of $2\cos^2 x + 3\sin x = 0$

6) Find the general solutions of

(i) $\cos 4x = \cos 2x$ (ii) $\cos 3x = \sin 2x$ (iii) $\sin 3x + \cos 2x = 0$

Ananda illangakoon

7) Solutions of the equation $a\cos\theta + b\sin\theta = c$

Find general solutions of the following

(i) $\sqrt{3}\cos x - \sin x = 1$ (ii) $\sec x - \tan x = \sqrt{3}$ (iii) $\cos x + \sin x = 1$ (iv) $\cos x - \sin x = -1$

8.Find the general solutions of the following.

(a). $\sqrt{3}\cos\theta + \sin\theta = 1$ (b). $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$ (c). $\cos\theta + \sin\theta = \sqrt{2}$ (d). $\cos 2\theta = \sin\theta + \cos\theta$

(e). $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$

9. Solve following equations.

(i). $2\cos^2\theta + 6\sin\theta.\cos\theta + 10\sin^2\theta = 1$

(ii). $9\cos^2 x + 24\sin x\cos x + 16\sin^2 x = \frac{25}{4}$

(iii). $6\sin^2 x - \sin x\cos x - \cos^2 x = 3$

10.*i*. Show for all values of θ that the expression

$2\cos^2\theta + 6\sin\theta\cos\theta + 10\sin^2\theta$ takes values between 1 and 11.

ii. Find the minimum and maximum values of $9\cos^2 x + 24\sin x\cos x + 16\sin^2 x$

iii. Show that the expression $\frac{\cos x + 2\sin x + 1}{\cos x + \sin x}$ cannot assume values between 1 and 2.

11. Sketch the graph of the following.

1. $y = \sqrt{3}\cos x + \sin x + 1$, in $0 \le x \le 2\pi$.
2. $y = \sin x - \cos x + 1$, in $-\pi \le x \le +\pi$.
3. $y = \sqrt{3}\cos 2x + \sin 2x - 1$, in $0 \le x \le \pi$.
4. $y = 11\cos^2 x + 16\sin x\cos x - \sin^2 x$, in $0 \le x \le \pi$.
5. $y = 5\cos^2 x + 18\sin x\cos x + 29\sin^2 x$, in $0 \le x \le \pi$.
6. $y = \cos\theta + \sin\theta + 1$, in $-2\pi \le x \le 2\pi$
7. $y = 5\cos^2 x - 24\sin x\cos x - 5\sin^2 x$, in $0 \le x \le \pi$.
8. $y = \cos\theta + \sqrt{3}\sin\theta + 2$, in $-\pi \le \theta \le \pi$.

12.Express $11\cos^2 x + 16\cos x\sin x - \sin^2 x$ in the form $a + b\cos(2x - \alpha)$. Where *a, b,* α are constants to be determined.

Let $f(x) = 11\cos^2 x + 16\cos x\sin x - \sin^2 x$.

Find the values of x in the range $0 \le x \le 180^0$ such that

(i). $f(x) = 0$

(ii). $f(x)$ attains the minimum.

(iii). $f(x)$ attains the maximum.

Hence sketch the rough graph of $f(x)$ in $0 \le x \le 180^0$.

13.Show that for all real values of θ the expression $a\cos\theta + b\sin\theta + c$ is such that

$c - \sqrt{a^2 + b^2} \le a\cos\theta + b\sin\theta + c \le c + \sqrt{a^2 + b^2}$. Deduce that for any real value of θ, $\cos\theta + \sqrt{3}\sin\theta + 2$ is not negative and sketch its graph in $-\pi \le \theta \le \pi$.

14.Express $f(x)=\cos^2 x-2\sin x\cos x-\sin^2 x$ in the form $R\cos(2x+\alpha)$.

Find the minimum and the maximum values of $f(x)$. Sketch the graph of $y=f(x)$ in $\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}$. Hence find the range of k of the equation $f(x)=k$, to have

(i). two real roots.

(ii). three real roots.

(iii). no real roots.

15.Sketch the graph of $f(\theta)=3\cos^2\theta-4\sin\theta\cos\theta-3\sin^2\theta$ in $0\le\theta\le 2\pi$. Hence find the range of k, for the equation $f(\theta)=k$ has

(i). five real root. (ii). two roots. (iii). four roots.

16.Show for all real values of x that $8\left(\cos^6 x+\sin^6 x\right)=5+3\cos 4x$. Sketch the graph of $y=\cos^6 x+\sin^6 x$ in $-\pi/2\le x\le \pi/2$. Hence deduce the range of k, for the equaton $\cos^6 x+\sin^6 x=k$ to have,

(i). no solutions. (ii). only two solutions.

(iii). only three solutions. (iv). only four solutions.

17. Prove the following.

(i). $\sin^{-1}(-x)=-\sin^{-1}x$ (ii). $\cos^{-1}(-x)=\pi-\cos^{-1}x$ (iii). $\tan^{-1}(-x)=-\tan^{-1}x$

(iv). $\sin^{-1}x+\cos^{-1}x=\pi/2$ (v). $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ (vi). $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

18) **Prove the following identities**.

(i) $2\tan^{-1}\frac{1}{2}=\cos^{-1}\frac{3}{5}$ (ii) $2\tan^{-1}\frac{3}{4}=\cos^{-1}\frac{7}{25}$ (iii) $4\cot^{-1}2+\tan^{-1}\frac{24}{7}=\pi$ (iv) $\sin(2\tan^{-1}x)=\frac{2x}{1+x^2}$

(v) $\tan^{-1}x+\tan^{-1}\frac{1}{x}=\frac{\pi}{2}$ (vii) $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]=\frac{1}{2}(3-\sqrt{5})$ (vii) $\cos^{-1}\frac{15}{17}+2\tan^{-1}\frac{1}{5}=\cos^{-1}\frac{140}{221}$

(viii). $2\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}\left(\frac{3}{4}\right)$ (ix). $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{4}$ (x). $\tan^{-1}(3)+\tan^{-1}(2)=\frac{3\pi}{4}$

19). **Prove the following identities**..

(i). $\sin^{-1}3/5+\sin^{-1}8/17=\sin^{-1}77/85$ (ii). $\sin^{-1}5/13+\sin^{-1}7/25=\sin^{-1}(253/325)$

(iii). $\cos^{-1}4/5+\tan^{-1}3/5=\tan^{-1}(27/11)$ (iv). $\cos^{-1}4/5+\cos^{-1}12/13=\cos^{-1}33/65$

(v). $\cos^{-1}x=2\sin^{-1}\sqrt{\frac{1-x}{2}}=2\cos^{-1}\sqrt{\frac{1+x}{2}}$ (vi). $\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=\tan^{-1}\frac{2}{9}$

(vii). $\tan^{-1}\frac{2}{3}=\frac{1}{2}\tan^{-1}\frac{12}{5}$ (viii). $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=\frac{\pi}{4}$

(ix). $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$ (x). $3\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{20}=\frac{\pi}{4}-\tan^{-1}\frac{1}{1985}$

(xi). $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\pi}{4}$ (xii). $\tan^{-1}\frac{120}{119}=2\sin^{-1}\frac{5}{13}$ (xiii). $\tan^{-1}\frac{m}{n}-\tan^{-1}\frac{m-n}{m+n}=\frac{\pi}{4}$

20) **Solve the following equations.**

i. $\tan^{-1}x+\tan^{-1}2x=\frac{\pi}{4}$ ii. $\tan^{-1}x+\tan^{-1}(2)=\tan^{-1}4$ iii. $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\tan^{-1}(32)$

iv. $\sin^{-1}\left(\frac{x}{x-1}\right)+2\tan^{-1}\left(\frac{1}{x+1}\right)=\frac{\pi}{2}$ v. $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

21). **Solve the following equations.**

(i). $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$ (ii). $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{4}{7}$ (iii). $\tan^{-1}x+\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{2}$

(iv). $2\tan^{-1}x-\tan^{-1}\left(\frac{2x}{7}\right)=\frac{\pi}{4}$ (v). $\tan^{-1}\left(\frac{2x}{x^2-1}\right)+\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)=\frac{2\pi}{6},\ (x>1)$ (vi). $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}2\sqrt{\frac{2}{3}}$

(vii).If $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$. Show that $x=\frac{1}{\sqrt{2}}$ or $x=-2\sqrt{2}$.

22) Show that $\sin^{-1}\left(\frac{2x}{1+x^2}\right)=2\tan^{-1}x$ .Hence Show that the Solutions of

$\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$ is $x=\frac{a+b}{1-ab}$

23) Show that $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=2\tan^{-1}x$. Hence show that the solution of the equation

$\sin^{-1}\left(\frac{2a}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is $x=\frac{a-b}{1+ab}$.

24) Prove that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$. Show that the only solution of the equation $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$ is $x=\frac{\sqrt{3}}{2}$.

25) Show that the solutions of the equation $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ are $x=0,\ \frac{1}{2}$.

26) Show that the solutions of the equation $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$ are $x=0$ or $x=\frac{1}{2}$.

27) Show that the solutions of the equation $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$ is $x=\frac{1}{2}\sqrt{\frac{3}{7}}$.

28.Prove the following.

(i). If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ then $yz+zx+xy=1$.

(ii). If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{3}$, then $x^2+xy+y^2=\frac{3}{4}$.

(iii). If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$, then $x^2+y^2=1$.

(iv). If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ then $x^2+y^2+z^2+2xyz=1$.

29) In usual notation state **sine** and **cosine** rule.

1) For any $\Delta\ ABC$, prove that $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}.\cot\frac{A}{2}$

2) $a(\sin B-\sin C)+b(\sin C-\sin A)+c(\sin A-\sin B)=0$

3) $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$ 4) $\frac{\sin(B-C)}{\sin(B+C)}=\frac{b^2-c^2}{a^2}$ 5) $\frac{(a-b)}{c}\cos\frac{C}{2}=\sin\frac{(A-B)}{2}$

6) $\frac{(b^2-c^2)}{a^2}.\sin 2A+\frac{(c^2-a^2)}{b^2}.\sin 2B+\frac{(a^2-b^2)}{c^2}.\sin 2C=0$ 7) $\frac{b-c}{b+c}=\frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$

8) $a(\cos C-\cos B)=2(b-c)\cos^2\frac{A}{2}$ 9) $a\cos A+b\cos B+c\cos C=2a\sin B.\sin C$

10) $a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0$ (11). $(b-c)\cot\frac{A}{2}+(c-a)\cot\frac{B}{2}+(a-b)\cot\frac{C}{2}=0$

30). Using "Sine" rule and "Cosine" rule prove that

(i). $\frac{b^2-c^2}{a^2}\sin 2A+\frac{c^2-a^2}{b^2}\sin 2B+\frac{a^2-b^2}{c^2}\sin 2c=0$ (ii). $\frac{\sin(B-C)}{\sin A}=\frac{b^2-c^2}{a^2}$

31). In usual notation state and prove "Sine" rule for a triangle ABC.

Prove that (i). $a=(b-c)\cos\frac{A}{2}.\operatorname{cosec}\frac{B-C}{2}$ (ii). $\cot\left(\frac{B-C}{2}\right)=\frac{b+c}{b-c}\tan\frac{A}{2}$.

Hence deduce that $a^2=b^2+c^2-2bc\cos A$.

(iii). If $c=\frac{\pi}{3}$, Prove that $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$.

32). (i). Write down general solution of $\tan 3\theta+\tan 4\theta=0$

Prove that the roots of the equation $x^3-12x^2+35x-7=0$ are

$\tan^2\frac{\pi}{7},\tan^2\frac{2\pi}{7},\tan^2\frac{3\pi}{7}$

(ii). Prove that, in usual notaton, for any triangle ABC

$$(a-b)\cos\frac{C}{2}=c\sin\left(\frac{A-B}{2}\right)$$

(iii). In usual notation, for the triangle ABC if , $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$ show that

(a) $\frac{\sin A}{7}=\frac{\sin B}{6}=\frac{\sin C}{5}$ (b) $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

33. (i). Find general solutions of $8\cot^2 2\theta-5\tan\theta=1$

(ii). For any triangle ABC, prove that . $b\cos A+a\cos B=C$

Write down another two similar relations for the triangle ABC. Hence show that

(a) $\frac{\cos A}{a}+\frac{\cos B}{b}=\frac{c}{b}$

(b) $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$

(c) $\cos C=\frac{a^2+b^2-c^2}{2abc}$

(d). In the triangle *PQR,* if *PQ=PR=x, AR=y*. Show that $Q\hat{P}R=\cos^{-1}\left(1-\frac{y^2}{2x^2}\right)$.

33. In usual notation, for the triangle ABC $a=4, b=5, c=6$ Find the value of $\cos A$ and hence find $\sin A$,

find $\frac{\sin A}{a}$ and show that

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=\frac{\sqrt{7}}{16}$$

34.Prove in usual notation that the area Δ of the triangle *ABC* is $\Delta=\frac{1}{2}bc\sin A$ write down another two results for Δ Hence deduce "Sine" rule for the triangle *ABC*.

35).In a triangle *ABC* , if $\cos A=\frac{\sin B}{2\sin C}$, show that the triangle is isosceles.

36) In a triangle *ABC* if $a\cos A=b\cos B$, Show that the triangle is either isosceles or right angled.

37) In a Δ *ABC* if $\frac{\cos A}{a}=\frac{\cos B}{b}$, Show that the triangle is isosceles.

38) In a Δ *ABC* , if $\sin^2 A+\sin^2 B=\sin^2 C$. Show that the triangle is right angled.

39) In a triangle *ABC*, medians *AD* and *CE* are drawn.

$AD=5$, $D\hat{A}C=\frac{\pi}{8}$ and $A\hat{C}E=\frac{\pi}{4}$. Show that the area of the Δ *ABC* is $\frac{25}{3}$ (square units).

40) A circle is inscribed in an equilateral triangle of side *a*. Show that the area of any square inscribed in this circle is $\frac{a^2}{6}$.

41) In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in arithmetic progression, show that the third side can be either $5+\sqrt{6}$ or $5-\sqrt{6}$.

42) A vertical pole subtends an angle $\tan^{-1}\left(\frac{1}{2}\right)$ at a point *P* on the ground. If the angles substended by the upper half and the lower half of the pole at *P* are α and β respectively, show that

(i) $\tan\alpha=\frac{2}{9}$ (ii) $\tan\beta=\frac{1}{4}$.

43) In the triangle *ABC*, $\frac{\sin A}{4}=\frac{\sin B}{5}=\frac{\sin C}{6}$. Prove that $\cos A+\cos B+\cos C=\frac{23}{16}$

44) With usual notation , if in a triangle *ABC,*

$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then prove that $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

45) The sides of a triangle *ABC* are such that $a:b:c=1:\sqrt{3}:2$. show that $A:B:C=1:2:3$

46) Prove that $\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right)$

47) In a triangle *ABC*, prove that

$$\cos^2 A+\cos^2\left(A+\frac{\pi}{3}\right)+\cos^2\left(A-\frac{\pi}{3}\right)=\frac{3}{2}$$

48). *a*.If $5\tan x=\tan(x+\alpha)$, show that $\sin(2x+\alpha)=\frac{3}{2}\sin\alpha$. Find the range of $\sin\alpha$ for *x* to have real solutions.

Hence solve the equation $5\tan x=\tan(x+30^{\circ})$

b.Solve $\tan^{-1}\left(\frac{1}{x-1}\right)-\tan^{-1}\left(\frac{1}{x+1}\right)=\tan^{-1}\frac{5}{3}-\tan^{-1}\frac{1}{3}$ for *x*, so that $2<x<4$.

c.A rod *AB* is against a vertical wall with *A* on the ground making an angle θ with the horizontal. *A* point on the rod at a distance one-third of length of the rod from the ground has an angle of elevation α from a point O on the ground. $(\alpha<\theta)$. *OB* is inclined to the horizontal at an angle β .Show that $2\cot\theta=3\cot\beta-\cot\alpha$.

49).*a*. Find general solutions of

i. $\sqrt{3}\tan x=2\sin x$ *ii*. $\cot^2 x-4\operatorname{cosec} x+4=0$

b.In the triangle *ABC*, if $A=\frac{\pi}{4}, B=\frac{5\pi}{12}$ Show that $a+\sqrt{2}c-2b=0$.

c.using the "cosine" rule for the $\Delta\ ABC$ prove that $\dfrac{(a-b).\cos A+c}{a-b+c\cos A}=\dfrac{a+b}{c}$

50). Let $f(x)=4(\sin^4 x+\cos^4 x)$. Prove that $f(x)=3+\cos 4x$.

Hence or otherwise for $|x|\leq\dfrac{\pi}{2}$, Sketch the graph of $f(x)=4(\sin^4 x+\cos^4 x)$.

Find the area enclosed by $y=f(x),\ x=\pm\dfrac{\pi}{2}$ and $y=2$

51) .*a*.Prove the identity

$\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)=\sec x-\tan x$. Hence find the value of $\tan\dfrac{\pi}{8}$.

Show that $\tan\dfrac{11\pi}{24}=2+\sqrt{2}+\sqrt{3}+\sqrt{6}$.

b.Solve $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$.

52. a). Show that if $0<\theta<\dfrac{\pi}{2}$ then $\sin\theta\tan\theta>2(1-\cos\theta)$

b). Using the expansions of $\sin(A-B)$ and $\cos(A-B)$ show that $\sin\dfrac{\pi}{12}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ and

$\cos\dfrac{\pi}{12}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$ Show that $\tan x=\dfrac{1-\cos 2x}{\sin 2x}$ for $0<x<\dfrac{\pi}{2}$ and deduce that

$\tan\dfrac{\pi}{24}=\sqrt{6}-\sqrt{3}+\sqrt{2}-2$.

c). State the *Sine Rule* for triangle, Prove in the usual notation for a triangle *ABC*, that $\dfrac{a^2-b^2}{c^2}=\dfrac{\sin(A-B)}{\sin(A+B)}$.

53).a). i. By solving the equation $\sin 3\theta=\cos 2\theta$ Show that $\sin 18^0=\dfrac{\sqrt{5}-1}{4}$

ii. Show that $\dfrac{\pi}{4}=2\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{7}$ and $\tan^{-1}\dfrac{1}{3}=\tan^{-1}\dfrac{1}{7}+\tan^{-1}\dfrac{2}{11}$

Deduce that $\dfrac{\pi}{4}=2\tan^{-1}\dfrac{2}{11}+3\tan^{-1}\dfrac{1}{7}$.

b). State the *Sine Rule* and deduce the *Cosine Rule* with the usual notation in a triangle *ABC*, it is given that

$\dfrac{b+c}{5}=\dfrac{c+a}{6}=\dfrac{a+b}{7}$.

Show that,

i. $\dfrac{\sin A}{4}=\dfrac{\sin B}{3}=\dfrac{\sin C}{2}$ ii. $\dfrac{\cos A}{-1}=\dfrac{4\cos B}{11}=\dfrac{2\cos C}{7}$

54). a). State the *Sine rule* in the usual notation.

P is a point inside a triangle ABC such that $\angle PAB = \angle PBC = \angle PCA = \varphi$.

Prove that the area of the triangle ABC is $\frac{abc}{2}\left(\frac{BP}{bc}+\frac{CP}{ac}+\frac{AP}{ab}\right)\sin\varphi$ in the usual notation.

Deduce that $\frac{1}{\sin^2\varphi} = \frac{1}{\sin^2 A}+\frac{1}{\sin^2 B}+\frac{1}{\sin^2 C}$

b). Show that,

i. $2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{5}{12}\right)$ ii. $2\tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{120}{119}\right)$ iii. $\tan^{-1}\left(\frac{120}{119}\right) - \frac{\pi}{4} = \tan^{-1}\left(\frac{1}{239}\right)$

deduce that,

$4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$

55) *a*). State and prove the *Sine rule*

P is a point inside the triangle *ABC* such that $P\hat{A}B = P\hat{B}C = P\hat{C}A = \phi$. Prove that

$\frac{bc}{a}(\cot\phi - \cot A) = \frac{ac}{b}(\cot\phi - \cot B) = \frac{ab}{c}(\cot\phi - \cot C)$ in the usual notation.

b). Let *x, y* and *z* be any three non-negative real numbers such that $x+y+z=\pi$, $\cos x + \cos y = 1$ and $t = \sin x + \sin y$ Show that,

i. $\tan^{-1}(t) = \frac{x+y}{2}$, ii. $0 \le t \le \sqrt{3}$

Hence find the values of *x, y* and *z* when t attains its maximum value.

56). (a). Using the identity $\cos^2\theta + \sin^2\theta = 1$ or otherwise, determine the real constants *a* and *b* such that

$\cos^6\theta + \sin^6\theta = a + b\cos 4\theta$ ·

Hence or otherwise

(i). Sketch the graph of $y = 8(\cos^6 x + \sin^6 x)$.

(ii). find the general solution of the equation $\cos^6 x + \sin^6 x = \frac{5}{4} + \frac{1}{2}\sin 4x$.

(b). Solve the equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

57).(a).State and prove the Cosine rule for a triangle *ABC*, in the usual notation. In the usual notation for a triangle *ABC*, show that

(i). $2\left(\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}\right) = \frac{a^2+b^2+c^2}{abc}$ (ii). if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ then the angle *C* is $\frac{\pi}{3}$.

(b). Express $\sqrt{3}\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$ where *R* and α are real.

Hence, find the general solution of the equation

$\sqrt{3}\cos^2\theta+\left(1-\sqrt{3}\right)\sin\theta\cos\theta-\sin^2\theta-\cos\theta+\sin\theta=0$

(c). Show that $\cos^{-1}(-x)=\pi-\cos^{-1}x$ for $-1\le x\le 1$.

58). (a). State and prove the Cosine rule for a triangle *ABC*, in the usual notation. Deduce that

(i). $\cos A+\cos B+\cos C=\dfrac{a^2(b+c-a)+b^2(c+a-b)+c^2(a+b-c)}{2abc}$

(ii). $\dfrac{1}{a}\cos^2\dfrac{A}{2}+\dfrac{1}{b}\cos^2\dfrac{B}{2}+\dfrac{1}{c}\cos^2\dfrac{C}{2}=\dfrac{(a+b+c)^2}{2abc}$

(b). Find, in radians the general solution of $\sin 2\theta-2\sin\theta-\cos\theta+1=0$

(c). If $\alpha=\tan^{-1}\left(\dfrac{1}{3}\right)$, $\beta=\tan^{-1}\left(\dfrac{1}{4}\right)$ and $\gamma=\tan^{-1}\left(\dfrac{2}{9}\right)$

show that $0<\alpha+\beta+\gamma<\dfrac{\pi}{2}$. Hence, show that $\alpha+\beta+\gamma=\dfrac{\pi}{4}$.

59). (a).State and prove the Sine rule for a triangle *ABC*, in the usual notation. Let $-1<k<1$.For a triangle *ABC*, in the usual notation, prove that if $a-b=kc$ then

(i). $\sin\left(\dfrac{A-B}{2}\right)=k\cos\left(\dfrac{C}{2}\right)$ (ii). $\dfrac{k\sin A}{1-k\cos B}=\dfrac{a}{b}\tan\left(\dfrac{A-B}{2}\right)$

(b). Find the general solution of the equation $\sqrt{3}(\sin x+\cos x)^2=\cos 2x$.

(c). Solve for *x*: $\tan^{-1}x+\tan^{-1}\left(\dfrac{x}{2}\right)+\tan^{-1}\left(\dfrac{x}{3}\right)=\dfrac{\pi}{2}$.

60). (a).For a triangle *ABC*, prove in the usual notation that $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$. Deduce that

$a=(b-c)\cos\dfrac{A}{2}\operatorname{cosec}\dfrac{B-C}{2}$.

(b). Show that for any real value of θ the expression $\tan\theta-2\tan\left(\theta-\dfrac{\pi}{4}\right)$ cannot take any value between -7 and 1.

(c).Express $5\cos^2\theta+18\cos\theta\sin\theta+29\sin^2\theta$ in the form of $a+b\cos(2\theta+\alpha)$, where *a* and *b* are constants and α is an angle independent of θ. Hence or otherwise find the general solution of the equation

$8(\cos x+\sin x)^2+2(\cos x+5\sin x)^2=19$.

61). (a). Prove the identity

$\cos\alpha+\cos\beta-\cos\gamma-\cos(\alpha+\beta+\gamma)=4\cos\dfrac{1}{2}(\alpha+\beta)\sin\dfrac{1}{2}(\beta+\gamma)\sin\dfrac{1}{2}(\gamma+\alpha)$

(b). Let $f(x)=2\sin^2\dfrac{x}{2}+2\sqrt{3}\sin\dfrac{x}{2}\cos\dfrac{x}{2}+4\cos^2\dfrac{x}{2}$

Express $f(x)$ in the form $a\sin(x+\theta)+b$ where $a(>0)$, *b* and $\theta\left(0<\theta<\dfrac{\pi}{2}\right)$ are constants to be determined. Deduce that $1 \le f(x) \le 5$.

Sketch the graph of $y = f(x)$ for $-\frac{\pi}{6} \le x \le \frac{11\pi}{6}$.

(c). Let $p > 2q > 0$. The sides *BC. CA* and *AB* of a triangle *ABC* are of lengths $p+q$, p and $p-q$ respectively. Show that $\sin A - 2\sin B + \sin C = 0$ and deduce that $\cos\frac{A-C}{2} = 2\cos\frac{A+C}{2}$.

62). (a).Let $f(x) = \frac{1-\tan x}{1+\tan^2 x}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Express $f(x)$ in the form $A\cos(2x+\alpha)+B$, where $A(>0)$, B and $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ are constants to be determined. Hence solve the equation $f(x) = \frac{2+\sqrt{2}}{4}$.

Using the first expression given for $f(x)$, show that $f(x) = \frac{2+\sqrt{2}}{4}$ can be written as

$2\tan^2 x + 4k\tan x - k^2 = 0$ where $k = 2-\sqrt{2}$ Deduce that $\tan\frac{\pi}{24} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$.

Also sketch the graph of $y = 2f(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(b).In the usual notation, state the Sine rule for a triangle. Let *ABC* be a triangle. In the usual notation it is given that $a:b:c = 1:\lambda:\mu$ where λ and μ are constants.

Showthat $\mu^2(\sin 2A + \sin 2B + \sin 2C) = 4\lambda \sin^3 C$.

63). (a). State the Sine rule for a triangle.

Show that $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin^3 C} = \frac{4ab}{c^2}$ for a triangle *ABC*, in the usual notation.

(b). Let $f(x) = \sin x \cos\left(x + \frac{\pi}{4}\right)$. Express $f(x)$ in the form $a\cos(bx - \alpha) + c$ where *a, b, c* and $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ are constants to be determined.

Let $g(x) = 4f(x) + \sqrt{2}$. Sketch the graph of $y = g(x)$ for $\frac{\pi}{8} \le x \le \frac{9\pi}{8}$.

(c). Solve $\sin(4\sin^{-1} x) = \sin(2\sin^{-1} x)$.

64) .(*a*) Show that $\cos^2(\alpha+\beta) + \cos^2\alpha + \cos^2\beta - 2\cos(\alpha+\beta)\cos\alpha\cos\beta = 1$.

(*b*) Let $f(x) = \cos 2x + \sin 2x + 2(\cos x + \sin x) + 1$. Express $f(x)$ in the form $k(1+\cos x)\sin(x+\alpha)$, where *k* and α are constants to be determined.

Let $g(x)$ be such that $\frac{f(x)}{1+\cos x} = \sqrt{2}\{g(x) - 1\}$, where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

Sketch the graph of $y = g(x)$ and hence show that the equation $f(x) = 0$ has only one solution in the range given above.

(c) In the usual notation , using the Sin Rule for a triangle *ABC,*

show that $a(b-c)\operatorname{cosec}\frac{A}{2}\cot\frac{A}{2} = (b+c)^2\tan\left(\frac{B-C}{2}\right)\sec\left(\frac{B-C}{2}\right)$.

65). *a*).State and prove the *Sine rule* in the usual notation.

The points *A,B* and *C* taken in the ascending order, lie on a straight line inclined at an angle θ to the horizontal. $AB = x$ and *D* is the point vertically above at a height *h* from the point *C. CD* subtends angles α and β at *A* and *B* respectively. Prove that,

i. $h = \dfrac{x\sin\alpha\sin\beta}{\sin(\beta-\alpha)\cos\theta}$

ii. $d = \dfrac{x\sin(\alpha+\theta)\sin\beta}{\sin(\beta-\alpha)}$ where *d* is the height of *D* above the level of *A*,

b). Find,

i. the general solution of the equation $\sin\theta - \cos\theta = 1$

ii. the value of *x* satisfying the equation $\tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{3} = \sin^{-1} x$.

66).(*a*)Write down the trigonometric identity for $\tan(\alpha+\beta)$ in terms of $\tan\alpha$ and $\tan\beta$. Hence, obtain $\tan 2\theta$ in terms of $\tan\theta$, and show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

By substituting $\theta = \frac{5\pi}{12}$ in the last equation, verify that $\tan\frac{5\pi}{12}$ is a solution of $x^3 - 3x^2 - 3x + 1 = 0$.

Given further that $x^3 - 3x^2 - 3x + 1 = (x+1)(x^2 - 4x + 1)$, deduce that $\tan\frac{5\pi}{12} = 2 + \sqrt{3}$.

(*b*) Show that $\tan^2\frac{A}{2} = \dfrac{1-\cos A}{1+\cos A}$ for $0 < A < \pi$.

In the usual notation, using the Cosine Rule for a triangle *ABC*, show that

$(a+b+c)(b+c-a)\tan^2\frac{A}{2} = (a+b-c)(a+c-b)$.

(c) Show that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

67). (*a*) If $f(\theta) = \sin^2\theta + \cos^4\theta$, Show that $f(\theta) = 1 - \frac{1}{4}\sin^2 2\theta$.

Deduce that $\frac{3}{4} \le f(\theta) \le 1$.

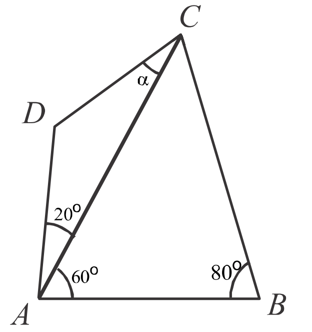
(*b*) Find the general solutions of $\cos 3x.\cos^3 x + \sin 3x.\sin^3 x = 0$

(*c*) **State**, in usual notation, '*sine*' rule for any triangle. Show that

(i) $\dfrac{\cos^2 B-\cos^2 C}{b+c}+\dfrac{\cos^2 C-\cos^2 A}{c+a}+\dfrac{\cos^2 A-\cos^2 B}{a+b}=0$. (ii) $\dfrac{a^2+b^2-ab\cos c}{a\sin A+b\sin B+c\sin C}=\dfrac{a}{2\sin A}$

68).*a*) *i*.Show that $\dfrac{2\cos(60^o-\theta)-\cos\theta}{\sin\theta}=\sqrt{3}$ for $0^o<\theta<90^o$.

ii.In the quadrilateral *ABCD* shown in the figure, $AB=AD$, $A\hat{B}C=80^o$,$C\hat{A}D=20^o$ and $B\hat{A}C=60^o$.



Let $A\hat{C}D=\alpha$. Using the sine Rule for the triangle *ABC*, show that $\dfrac{AC}{AB}=2\cos 40^o$.

Next, using the Sine Rule for triangle *ADC*, show that $\dfrac{AC}{AD}=\dfrac{\sin(20^o+\alpha)}{\sin\alpha}$.

Deduce that $\sin(20^o+\alpha)=2\cos 40^o\sin\alpha$.

Hence, show that $\cot\alpha=\dfrac{2\cos 40^o-\cos 20^o}{\sin 20^o}$.

Now, using the result in (*i*) above, show that $\alpha=30^o$.

b) Solve the equation $\cos 4x+\sin 4x=\cos 2x+\sin 2x$

69) *a*) Solve $\cos 2\theta+\cos 3\theta=0$ for $0\le\theta\le\pi$.

Write down $\cos 2\theta$ and $\cos 3\theta$ in terms of $\cos\theta$, and show that

$\cos 2\theta+\cos 3\theta=4t^3+2t^2-3t-1$, where $t=\cos\theta$.

Hence, write down the three roots of the equation $4t^3+2t^2-3t-1=0$ and show that the roots of the equation $4t^2-2t-1=0$ are $\cos\dfrac{\pi}{5}$ and $\cos\dfrac{3\pi}{5}$. Deduce that $\cos\dfrac{3\pi}{5}=\dfrac{1-\sqrt{5}}{4}$.

b) Let *ABC* be a triangle and let *D* be the point on *BC* such that $BD:DC=m:n$ .where $m, n>0$. It is given that $B\hat{A}D=\alpha$ and $D\hat{A}C=\beta$. Using the Sine Rule for the triangles *BAD* and *DAC*,

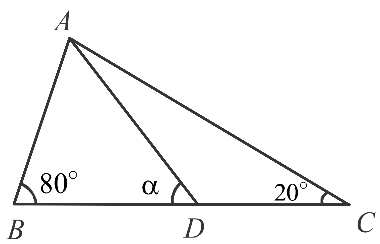
show that $\dfrac{mb}{nc}=\dfrac{\sin\alpha}{\sin\beta}$,, where $b=AC$ and $c=AB$.

70) *a*.Write down $\sin(A-B)$ in terms of sin *A*, cos *A*, sin*B* and cos *B*.

Deduce that

i. $\sin(90^\circ-\theta)=\cos\theta$ and

ii. $2\sin 10^\circ=\cos 20^\circ-\sqrt{3}\sin 20^\circ$.



b.In the usual notation, state the Sine Rule for a triangle *ABC*.

In the triangle *ABC* shown in the figure, $A\hat{B}C=80^\circ$ and $A\hat{C}B=20^\circ$. The point *D* lies on *BC* such that $AB=DC$. Let $A\hat{D}B=\alpha$.

Using the Sine Rule for suitable triangles, show that $\sin 80^\circ\sin(\alpha-20^\circ)=\sin 20^\circ\sin a$.

Explain why $\sin 80^\circ=\cos 10^\circ$ and hence, show that $\tan\alpha=\dfrac{\sin 20^\circ}{\cos 20^\circ-2\sin 10^\circ}$.

Using the result in (*a*) (*ii*) above, deduce that $\alpha = 30^\circ$.

c.Solve the equation $\tan^{-1}\left(\cos^2 x\right) + \tan^{-1}(\sin x) = \frac{\pi}{4}$.

71) *a*.Write down $\sin(A+B)$ in terms of sin *A*, cos *A*, sin *B* and cos*B* and obtain a similar expression for sin(*A*-*B*).
Deduce that
$2\sin A\cos B = \sin(A+B) + \sin(A-B)$ and
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$.

Hence, solve $2\sin 3\theta\cos 2\theta = \sin 7\theta$ for $0 < \theta < \frac{\pi}{2}$.

b.In a triangle *ABC*, the point *D* lies on *AC* such that $BD = DC$ and $AD = BC$. Let $B\hat{A}C = \alpha$ and $A\hat{C}B = \beta$. Using the Sine Rule for suitable triangles, show that $2\sin\alpha\cos\beta = \sin(\alpha + 2\beta)$.

If $\alpha : \beta = 3:2$, using the last result in (*a*) above, show that $\alpha = \frac{\pi}{6}$.

c.Solve $2\tan^{-1} x + \tan^{-1}(x+1) = \frac{\pi}{2}$. Hence, show that $\cos\left(\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)\right) = \frac{3}{\sqrt{10}}$.

72) *a*.Write down $\cos(A+B)$ and $\cos(A-B)$ in terms of cos*A*, cos*B*, sin*A* and sin*B*.

Hence, show that $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

Deduce that $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$.

Solve the equation $\cos 9x + \cos 7x + \cot x(\cos 9x - \cos 7x) = 0$.

b.In the usual notation, state and prove the **Cosine Rule** for a triangle *ABC*.

Let $x \neq n\pi + \frac{\pi}{2}$ for $n \in \mathbb{Z}$. Show that $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

In a triangle *ABC*, it is given that $AB = 20\ cm, BC = 10\ cm$ and $\sin 2B = \frac{24}{25}$.
Show that there are two distinct such triangles and find the length of *AC* for each.

c.Solve the equation $\sin^{-1}\left[\left(1+e^{-2x}\right)^{-\frac{1}{2}}\right] + \tan^{-1}(e^x) = \tan^{-1}(2)$.