VECTORS

1).$ABCDEF$ is a regular hexagon. If $\overrightarrow{AB}=\underline{a}$ and $\overrightarrow{BC}=\underline{b}$, find the vectors $\overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AF}, \overrightarrow{AE}, \overrightarrow{CE}$ in terms of $\underline{a}$ and $\underline{b}$.

2).$ABCD$ is a parallelogram. Show that

i. $\overrightarrow{AC}+\overrightarrow{DB}=2\overrightarrow{DC}$ ii. $\overrightarrow{AC}-\overrightarrow{BD}=2\overrightarrow{AB}$

3).$ABCDE$ is a pentagon. Show that $\overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{ED}+\overrightarrow{DC}=2\overrightarrow{AC}$.

4).$ABCD$ is a parallelogram. The mid - points of the sides AD and CD are E and F respectively. By expressing the vectors $\overrightarrow{BE}$ and $\overrightarrow{BF}$ in $\overrightarrow{BA}$ and $\overrightarrow{BC}$ vectors show that $\overrightarrow{BE}+\overrightarrow{BF}=\frac{3}{2}\overrightarrow{BD}$.

5).$ABCD$ is a quadrilateral. Show that $\overrightarrow{AB}+\overrightarrow{DC}=\overrightarrow{AC}+\overrightarrow{DB}$.

6).The mid - points of the sides AB and AC of the triangle ABC are D and E respectively. Show that $\overrightarrow{BE}+\overrightarrow{DC}=\frac{3}{2}\overrightarrow{BC}$.

7).The mid - points of the sides of the triangle ABC are D, E, and F. Show that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}$.

8).The mid - points of the sides of the quadrilateral $ABCD$ are P, Q, R, S. Prove that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=\overrightarrow{OP}+\overrightarrow{OQ}+\overrightarrow{OR}+\overrightarrow{OS}$.

9).The mid - points of the sides AB and BC of the triangle ABC are E and D respectively. Prove that $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{BC}=4\overrightarrow{ED}$.

10).Relative to a vector origon O, the position vectors of the points A and B are $\underline{a}$ and $\underline{b}$. The points P and Q are on OA and OB such that $OP:PA=2:1$ and $OQ:QB=1:2$.

1.Find the following vectors in terms of $\underline{a}$ and $\underline{b}$.

i. $\overrightarrow{OP}$ *ii.* $\overrightarrow{OQ}$ *iii.* $\overrightarrow{PQ}$

2.The point R is such that $OR:AR=2:1$. Find $\overrightarrow{RB}$.

3.Show that RB and PQ are parallel, find the ratio $RB:PQ$.

4.The line QP is produced to S such that $QP=PS$. Find the vectors $\overrightarrow{PS}$ and $\overrightarrow{AS}$ in terms of $\underline{a}$ and $\underline{b}$.

5.Show that the points B, A and S are collinear.

11).The points O, A, and B are not collinear. Let $\overrightarrow{OA}=\underline{a}$, $\overrightarrow{OB}=\underline{b}$. C is a point such that $\overrightarrow{OC}=\underline{a}+\underline{b}$. P is the mid - point of BC. Show that $\overrightarrow{OP}=\frac{1}{2}(\underline{a}+2\underline{b})$.

12)*i*.The centroid of the triangle ABC is G. O is any point. Prove that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=3\overrightarrow{OG}$.

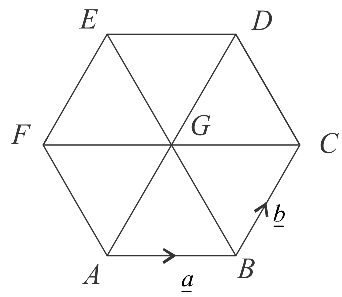
ii.$ABCDEF$ is a regular hexagon. Prove that $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=3\overrightarrow{AD}$.

13).Given that $\overrightarrow{AB}=\underline{a}$, $\overrightarrow{CA}=\underline{b}$ and $\overrightarrow{CD}=2\underline{a}$. Point E is on CB such that $\overrightarrow{CE}=2\overrightarrow{EB}$.

i.Find the vectors represented by $\overrightarrow{CB}, \overrightarrow{EB}, \overrightarrow{DB}, \overrightarrow{AE}, \overrightarrow{ED}$.

ii.Show that the points *A, E, D* are collinear.

14).*a*).*ABCDEF* is a regular hexagon. Prove that $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AG}$.
Where *G* is the centroid of the hexagon.



b). *ABCDEF* is a regular hexagon. Let $\overrightarrow{AB}=\underline{a}$ and $\overrightarrow{BC}=\underline{b}$. Find the vectors determined by the other four sides taken in order. Also express the vecotors $\overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AF}, \overrightarrow{AE}, \overrightarrow{CE}$ in terms of $\underline{a}$ and $\underline{b}$.

c) *ABCDEF* is a regular hexagon. $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{EA}+\overrightarrow{FA}=4\overrightarrow{AB}$

15).Prove by vectorial methods, that the straight line joining the middle points of two sides ofa triangle is parallel to the third side and is half of its length.

16).*a*).Prove that the straight line joining the middle points of two non-parallel sides of a trapezium is parallel to the parallel sides and half of their sum.

b).Prove that the straight line joining the middle points of the diagonals of a trapezium is parallel to parallel sides and half of their difference.

17).*a*). Prove that the quadrilateral formed by joining the mid-points of the sides of any quadrilateral taken in order is a parallelogram.

b).Prove that the lines joining the mid-points of opposite sides of a quadrilateral bisect each other.

18). From an origin O the points A, B, C have position vectors $\underline{a}, \underline{b}, 2\underline{b}$ respectively. The points O, A , B are not collinear. The mid point of AB is M and the points of trisection of AC nearer to A is T. Draw a diagram to show O,A,B,C,M,T. Find, interms of $\underline{a}$ and $\underline{b}$, the position vectors of M and T. Use your results to prove that O,M, T are collinear, and find the ratio in which M divides OT.

19)*i*.The position vectors of the three points *A, B, C* are $\underline{a}, \underline{b}$ and $3\underline{a}, -2\underline{b}$ respectively. Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in terms of $\underline{a}$ and $\underline{b}$. Hence show that *A, B, C* are collinear points.

ii.The position vectors of the points *A, B, C* are $(2\underline{a}+\underline{b})$, $(\underline{a}+3\underline{b})$ and $(4\underline{a}-m\underline{b})$ respectively. If the three points *A, B, C* are collinear find the value of *m*.

20).In the triangle *OAB*, $\overrightarrow{OA}=\underline{a}$, $\overrightarrow{OB}=\underline{b}$. The points *F* and *E* lie on *OA* and *OB* respectively such that $\frac{OF}{FA}=\frac{2}{3}$, and $\frac{OE}{EB}=\frac{1}{3}$. The lines *AE* and *BF* intersect at *R*. Find $\frac{AR}{RE}$ and $\frac{BR}{RE}$. The line *OR* produced meets *AB* at *D*. Find $\frac{AD}{DB}$.

21).In the parallelogram *OACB*, $\overrightarrow{OA}=\underline{a}$, $\overrightarrow{OB}=\underline{b}$. The mid - point of *AC* is *D*. Find $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{b}$. The lines *AB* and *OD* intersect at *E*.

Find $\frac{AE}{EB}$ and $\frac{OE}{ED}$.

22).In the parallelogram, *ABCD*, the mid - point of *AB* is *X*. The lines *DX* and *AC* meet at *P*. Let $\overrightarrow{AB}=\underline{a}$, $\overrightarrow{AD}=\underline{b}$,

$\overrightarrow{AP}=\lambda\overrightarrow{AC}$, and $\overrightarrow{DP}=\mu\overrightarrow{DX}$,

i.Find $\overrightarrow{AP}$ in terms of $\lambda, \underline{a}, \underline{b}$.

ii.Find $\overrightarrow{AP}$ in terms of $\mu, \underline{a}, \underline{b}$

Hence show that *P* is a trisection point of *AC* and *DX* lines.

23). $ABCD$ is a parallelogram. $\vec{AB} = \underline{a}$, $\vec{AD} = \underline{b}$ .The mid-point of BC is E. Find the vector $\overrightarrow{AE}$ .The lines BD and AE intersect at F. Find the ratio $BF{:}FD$. The lines DE produced and AB produced meet at the point G. show that $DG = 2DE$ and $AG = 2AB$.

24).In the parallelogram, $OACB$, given that $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$. The point D is on AC such that $AD{:}DC = 3{:}2$. The lines OD and AB intersect at E.

Express $\overrightarrow{OE} = \lambda\left(\underline{a} + \frac{3}{5}\underline{b}\right)$ and $\overrightarrow{OE} = \underline{a} + \mu(\underline{b} - \underline{a})$. where λ and μ scalars to be determined. Hence find, $OE : ED$ and $AE : EB$.

25).Given that $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, $\overrightarrow{OP} = \frac{4}{5}\overrightarrow{OA}$ and that Q is the mid point of AB, express $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ in terms of $\underline{a}$ and $\underline{b}$. PQ is produced to meet OB produced at R, so that $\overrightarrow{QR} = n\overrightarrow{PQ}$ and $\overrightarrow{BR} = k\underline{b}$. Express $\overrightarrow{QR}$.

i). in terms of n, $\underline{a}$ and $\underline{b}$. *ii*). in terms of k, $\underline{a}$ and $\underline{b}$.
Hence find the value of n and of k.

26).The position vectors of three points A, B and C relative to an origin O are $\underline{p}$, $3\underline{q} - \underline{p}$ and $9\underline{q} - 5\underline{p}$ respectively. Show that the points A, B and C lie on the same straight line and state the ratio $AB : BC$. Given that $OBCD$ is a paralleogram and that E is the point such that $\overrightarrow{DB} = \frac{1}{3}\overrightarrow{DE}$. Find position vectors of D and E relative to O.

27). $\underline{a}$ and $\underline{b}$ are two non-zero , non-parallel vectors. when α and β are two scalars and if $\alpha\,\underline{a} + \beta\,\underline{b} = \underline{0}$, show that $\alpha = 0, \beta = 0$. OAB is a triangle .Given that $\overrightarrow{OA} = \underline{a}, \overrightarrow{OB} = \underline{b}$. The point D is on OB and E is on OA so that $\frac{OD}{DB} = \frac{1}{2}$ and $\frac{OE}{EA} = \frac{2}{3}$. The lines AD and BE intersect at G. Show that $\overrightarrow{OG}$ can be expressed as

$\overrightarrow{OG} = \underline{a} + \lambda\left(\frac{\underline{b}}{3} - \underline{a}\right)$ and $\overrightarrow{OG} = \underline{b} + \mu\left(\frac{2}{5}\underline{a} - \underline{b}\right)$ where λ and μ are scalars. Hence find $\overrightarrow{OG}$.

The line OG produced meet AB at D'. Find $\overrightarrow{OD'}$. Hence find $\overrightarrow{OC}$ such that $OACD'$ is a parallelogram.

28). The points A, B and C have position vectors $\underline{a}, \underline{b}$ and $\underline{c}$ respectively reffered to an origin O.

a). Given that the point X lies on AB produced. So that $AB{:}BX{=}2{:}1$, find $\underline{x}$, the position vector of X, in terms of $\underline{a}$ and $\underline{b}$.

b).If Y lies on BC, between B and C so that $BY{:}YC{=}1{:}3$. Find $\underline{y}$, the position vector of Y, in terms of $\underline{b}$ and $\underline{c}$.

c).Given that Z is the mid-point of AC. Show that X, Y and Z are collinear.

d).Calculate $XY : YZ$

29).In the triangle OAB, $\overrightarrow{OA} = \underline{a}, \overrightarrow{OB} = \underline{b}$. The mid points of OA, OB are D, E respectively. The lines BD and AE meet at G. show that

i) $\overrightarrow{OG} = \frac{\underline{a} + \underline{b}}{3}$ *ii*) $BG : GD = AG : GE = 2:1$

The line OG produced meets AB at F. find $BF : FA$. Hence show that the medians of a triangle are concurrent.

30). The vertices *A, B* and *C* of a triangle have position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively relative to an origin O. The point *P* is on *BC* such that *BP:PC=3:1* the point *Q* is on *CA* such that *CQ:QA=2:3* the point *R* is on *BA* produced such that *BR:AR=2:1*. The position vectors of *P, Q* and *R* are $\underline{p}, \underline{q}$ and $\underline{r}$ respectively.

Show that $\underline{q}$ can be expressed in terms of $\underline{p}$ and $\underline{r}$ and hence or otherwise show that *P, Q* and *R* are collinear. State the ratio of the lengths of the line segments *PQ* and *QR*.

31). Prove that for any two vectors $\underline{a}$ and $\underline{b}$

i). $|\underline{a}| - |\underline{b}| \leq |\underline{a} + \underline{b}| \leq |\underline{a}| + |\underline{b}|$

ii). $|\underline{a} - \underline{b}| \geq |\underline{a}| - |\underline{b}|$

32). The point *P* is on *AB*. The position vectors of *A, B* relative to origin O are $\underline{a}$, $\underline{b}$. If $AP : PB = m : n$ when $m + n = 1$, show that the position vector of *P* is $m\underline{a} + n\underline{b}$.. The sides *BC, CA, AB* or produced of triangle *ABC* are cut by a line at *X, Y, Z*. Show that $\frac{AZ}{ZB} \cdot \frac{BX}{XC} \cdot \frac{CY}{YA} = -1$.

33). In the triangle *OAB*, $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$. The points *C* and *D* are on *OA* and *OB* such that *OC* : *CA* = 2 : 1 and *OD* : *DB* = 3 : 4. The lines *BC* and *AD* intersect at *G*. when λ and μ are two scalars show that

i) $\overrightarrow{OG} = \underline{a} + \lambda(-\underline{a} + \frac{3}{7}\underline{b})$

ii) $\overrightarrow{OG} = \underline{b} + \mu(-\underline{b} + \frac{2}{3}\underline{a})$. Find λ and μ.

Hence find the ratios *AG* : *GD* and *CG* : *GB*.

34). Through the middle point *P* of the slide *AD* of a parallelogram *ABCD* the straight line *BP* is drawn cutting *AC* at *R* and *CD* produced at *Q*. Prove that *QR=2RB*

35). In the triangle *OAB*, the position vectors of *A* and *B* relative to *O* are $\underline{a}$ and $\underline{b}$ respectively. The points *D*, *E* are situated on *OB*, *OA* such that *OD* : *DB* = 5 : 2 and *OE* : *EA* = 3 : 4.

The lines *AD* and *BE* intersect at *G*. Show that $\overrightarrow{OG} = \underline{b} + \lambda\left(\frac{3}{7}\underline{a} - \underline{b}\right)$. Where λ is a scalar.

By writing another expression for $\overrightarrow{OG}$, find $\overrightarrow{OG}$.

The lines *OG* and *AB* meet at *F*. By writing $\overrightarrow{OF} = k\,\overrightarrow{OG}$ and find the ratio. *AF* : *FB*.

Verify that $\frac{AE}{EO} . \frac{OD}{DB} . \frac{BF}{FA} = 1$.

36). *a*). The mid points of the sides *BC, CA, AB* of the triangle *ABC* are *D, E* and *F* respectively. Show that,

i). $2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = 2\overrightarrow{FC}$

ii). $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \underline{0}$

b). $OABC$ is a parallelogram. The position vectors of the points A, B and C relative to O are $\underline{a}, \underline{b}$ and $\underline{c}$ respectively. M is the mid point of BC.

Express in terms of $\underline{a}, \underline{c}$ the position vector of any point X on OM. If X lies on AC, deduce that, OM and AC are trisected by X.

c). The point S is on the side QR of the triangle PQR such that $\frac{RS}{SQ} = \frac{3}{2}$. Show that $3\overrightarrow{PQ} + 2\overrightarrow{PR} = 5\overrightarrow{PS}$.

37).In the triangle ABC, $\overrightarrow{AB} = 2\underline{a}$ and $\overrightarrow{AC} = 2\underline{b}$, E and F are the mid points of AC and AB, BE and CF meet at X.

i). Express $\overrightarrow{BE}$ and $\overrightarrow{CF}$ in terms of $\underline{a}$ and $\underline{b}$

ii). If $\frac{EX}{EB} = \lambda$ and $\frac{FX}{FC} = \mu$ by considering the triangles AFX and AEX obtain two different expressions for $\overrightarrow{AX}$
.

iii).Hence find the values of λ and μ.

iv).Show that X divides the side BE in the ratio 2 : 1.

38).OAB is a triangle. The position vectors of A and B relative to O are $\underline{a}$ and $\underline{b}$ respectively. Point D lies on OA such that $OD:DA = 2:1$. Point E lies on OB such that $OE : EB = 1:3$. The lines BD and AE intersect at G.
Show that,

i). BG : GD = 9 :1 ii). AG : GE = 2 : 3 iii). $\overrightarrow{BG} = \frac{3}{5}\underline{a} - \frac{9}{10}\underline{b}$

iv). $\overrightarrow{AG} = \frac{2}{5}\left(\frac{\underline{b}}{4} - \underline{a}\right)$ v). $\overrightarrow{OG} = \frac{6\underline{a} + \underline{b}}{10}$

39).OAB is a triangle. The position vectors of A and B relative to O are $\underline{a}$ and $\underline{b}$ respectively. Point P is on OA such that $OP:PA=2:1$ and Q is on OB such that $OQ:QB = 1:3$. The lines AQ and BP meet at point R. Find $\overrightarrow{OR}$ in two ways and hence find ratios $AR : RQ$ and $BR : RP$. Find $\overrightarrow{OR}$ only in terms of $\underline{a}$ and $\underline{b}$.

40).i.The position vectors of the points P, Q relative to the orgin O are $\underline{p}$, $\underline{q}$ respectively. Show that the position vector $\underline{r}$ of any variable point R on the line PQ can be writen as $\underline{r} = \underline{p} + \lambda(\underline{q} - \underline{p})$ where λ is a parameter.
$OACB$ is a parallelogram. The sides $\overrightarrow{OA}$, $\overrightarrow{OB}$ represent the vectors $\underline{a}$ and $\underline{b}$. L, M are the mid points of AC and CB. OL and AM intersect at X. Show that $\overrightarrow{OX} = \frac{4}{5}\left(\underline{a} + \frac{\underline{b}}{2}\right)$. CX and OA meet at N. Show that $\overrightarrow{ON} = \frac{2}{3}\underline{a}$.

ii. The median AD of a triangle ABC is bisected at E and BE is produced to meet the side AC in F.

Prove that $AF = \frac{1}{3}AC$ and $EF = \frac{1}{4}BF$.

41).The vectors $\underline{a}$ and $\underline{b}$ are two non - zero, non - parallel vectors. When α, β are scalars, if $\alpha\underline{a} + \beta\underline{b} = \underline{0}$, then show that $\alpha = 0$, $\beta = 0$.

$\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ C is a point such that $\overrightarrow{OC} = \underline{a} + \underline{b}$. P is the midpoint of BC. Then show that $\overrightarrow{OP} = \frac{1}{2}(\underline{a} + 2\underline{b})$.

If the line OP meets AB at R, show that $\overrightarrow{RB} = \underline{b} - k(\underline{a} + 2\underline{b})$. Where k is a scalar. Show also that $AR : RB = 2 : 1$.

42). A and B are two points such that $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. The mid point of OA is M. X is a point on OB such that $OX : XB = 3 : 1$ and Y is a point on AX so that $AY : YX = 4 : 1$.

i). Express $\overrightarrow{OM}$, $\overrightarrow{OX}$ and $\overrightarrow{OY}$ interms of $\underline{a}$ and $\underline{b}$.

ii). Show that $\overrightarrow{BY} = \frac{1}{5}(\underline{a} - 2\underline{b})$

iii). Show that B, Y and M are collinear iv). Show that $BY : YM = 2 : 3$

43). $ABCD$ is a parallelogram. L and M are points on BC and DC, such that $\frac{BL}{LC} = \frac{DM}{MC} = \frac{1}{2}$. AL and BD intersect at P and AM and BD intersect at Q.

Find the ratios $AP : PL$ and $DQ : QB$ and hence show that $PQ = \frac{1}{2} BD$.

44). The position vectors of the points P and Q relative to a fixed origin are $2\underline{a}$ and $2\underline{b}$ respectively. S and R are the mid-points of OP and OQ respectively. SQ and PR meet at X. If $SX : SQ =$ m and $RX : RP = n$ then show that $\overrightarrow{OX} = \underline{a} + m(2\underline{b} - \underline{a}) = b + n(2\underline{a} - \underline{b})$. Hence find the values of m and n.

45).(i) Let $\underline{a}$ and $\underline{b}$ be two non-zero, non-parallel vectors. If $\alpha\underline{a} + \beta\underline{b} = \underline{0}$ Where α and β are two scalars, show that $\alpha = 0$ and $\beta = 0$.

(ii) O, A, B are three non-collinear points so that $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ points P, Q are such that $\overrightarrow{OQ} = \frac{\underline{a}}{2}$ and

$\overrightarrow{QP} = \frac{|\underline{a}|}{2|\underline{b}|}\underline{b}$. Express $\overrightarrow{OP}$ and $\overrightarrow{PA}$ in terms of $\underline{a}$ and $\underline{b}$, and deduce that,

(i) $\underline{a} = \overrightarrow{OP} + \overrightarrow{PA}$ (ii) $\underline{b} = \frac{|\underline{b}|}{|\underline{a}|}\left(\overrightarrow{OP} - \overrightarrow{PA}\right)$

46). In triangle OAB the position vectors of A and B relative to origin O are $\underline{a}$ and $\underline{b}$ respectively. Points D and E are on OB and OA such that $OD : DB = 5 : 2$ and $OE : EA = 3 : 4$. The lines AD and BE intersect at G.

When λ is a scalar, show that $\overrightarrow{OG} = \underline{b} + \lambda\left[\frac{3\underline{a}}{7} - \underline{b}\right]$.

Write down another expression for $\overrightarrow{OG}$. Find scalars involving in them. Express $\overrightarrow{OG}$ interms of $\underline{a}$ and $\underline{b}$.

47).*i*. Position vectors of two points P and Q relative to a point O are $\underline{p}$ and $\underline{q}$ respectively. Show that the position vector of any point on the line PQ is $t\underline{p} + (1-t)\underline{q}$ .Where t is a parameter. $\underline{a}$ and $\underline{b}$ are two non-parallel vectors. The position vectors of points A, B, C and D are $\underline{a}$, $2\underline{b}$, $2\underline{a}$ and $3\underline{b}$ respectively. Show that the position vector of point of intersection of AB and CD is $2(3\underline{b} - \underline{a})$.

ii..Relative to an origin O the position vectors of the points A, B are $\underline{a}$ and $\underline{b}$. Show that the vector equation of the line AB is $\underline{r} = \underline{a} + t(\underline{b} - \underline{a})$.

If the position vectors of C, D are $\frac{\underline{a}}{2}, \frac{3\underline{b}}{2}$. Find the position vector of the intersection of AB and CD.

48). Q is the point on the side AB nearer to B which trisects AB of the triangle OAB. P is a point on OQ such that $OP : OQ = 2 : 5$.The line AP produced meets OB at R. Show that $\overrightarrow{OP} = \frac{2}{15}(\underline{a} + 2\underline{b})$ and write down $\overrightarrow{AP}$ interms of $\underline{a}$ and $\underline{b}$. Where $\underline{a}$ and $\underline{b}$ are the position vectors of A and B respectively, relative to O.Find the value of scalar k so that $\overrightarrow{OA} + k\overrightarrow{AP}$ is independent of $\underline{a}$. By expressing the position vector $\underline{r}$ of R interms of $\underline{b}$ and show that $OR : OB = 4 : 13$.

49). ABC is a triangle $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, $\overrightarrow{OC} = \underline{c}$. Where O is a fixed point. Point Q is in the triangle such that AQ, BQ, CQ produced meet the sides BC, CA, AB, at X, Y, Z respectively.

If $\frac{BX}{XC} = \frac{1}{2}$ and $\frac{XQ}{QA} = \frac{3}{2}$, prove that $15\overrightarrow{OQ} = 9\underline{a} + 4\underline{b} + 2\underline{c}$ Find the ratio $AY : YC$.

50). A, B, C are three non-collinear points. $\overrightarrow{AB} = \underline{b}$, $\overrightarrow{AC} = 3\underline{a}$ .Point D is situated such that $\overrightarrow{AD} = 3\underline{a} + \underline{b}$. E is on CD such that $DE : EC = 1 : 2$. Show that $\overrightarrow{AE} = \frac{1}{3}(9\underline{a} + 2\underline{b})$. The lines BC and AE intersect at F. If λ is a constant, show that $\overrightarrow{FB} = \underline{b} - \lambda(9\underline{a} + 2\underline{b})$. Prove also that $FB : CF = 3 : 2$.

51). A, B, C points are such that $\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{AC} = \underline{b}$. Point E is on BC such that $BE : EC = k : 1$. Point F is on AE . CF produced meets AB at H and BF produced meets AC at L. If $AH : HB = 3k : 1$ and $CL : LA = 2k : 1$.
Prove that $(1+k)\overrightarrow{AE} = \underline{a} + k\underline{b}$ and $(1+2k+6k^2)\overrightarrow{AF} = 6k^2\underline{a} + \underline{b}$
Deduce that $6k^3 = 1$.

52). The position vectors of P and Q are $\underline{a}$ and $\underline{b}$. Find the position vector of point R which divides PQ in the ratio 3 : 2. The position vectors of points S and T are $2\underline{a}$ and $\frac{(9\underline{b} - 14\underline{a})}{5}$. Show that the points R, S, T are collinear. Also find the ratio R divides the line ST.

53).In the triangle OAB, $\overrightarrow{OA} = 2\underline{a}$ and $\overrightarrow{OB} = 3\underline{b}$, C is the midpoint of OA and D is the point on OB such that $OD : DB = 2 : 1$. OD and BC meet at E.

(*a*) Express $\overrightarrow{AD}$ and $\overrightarrow{CB}$ in terms of $\underline{a}$ and $\underline{b}$.

(*b*) By expressing $\overrightarrow{OE}$ in the form $\lambda\underline{a} + \mu\underline{b}$ in two different ways, prove that E is the midpoint of BC.

54). O, A, B and C are four points such that $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = 2\underline{b}$ and $\overrightarrow{OC} = 6\underline{a} - 2\underline{b}$ show that A, B and C are collinear.

55). The four points O, A, B and C are such that $\overrightarrow{OA} = \underline{a}$ $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = k\underline{a} - \underline{b}$. Given that A, B and C are collinear, find the value of k.

56). OAB are three points such that $\overrightarrow{OA} = 2\underline{a}$ and $\overrightarrow{OB} = \underline{b}$.$C$ is the midpoint of OA and OB is produced to D such that $BD = OB$. E is a point on CD such that $CE : ED = 1:2$. Find $\overrightarrow{AB}$, $\overrightarrow{CD}$ and $\overrightarrow{AE}$ in terms of $\underline{a}$ and $\underline{b}$. and prove that A, E and B are collinear.

57). The medians BE and CF of the triangle ABC intersect at G. Given that

$\overrightarrow{AB} = 2\underline{a}$ and $\overrightarrow{AC} = 2\underline{b}$,

(a) express $\overrightarrow{BE}$ and $\overrightarrow{CF}$ in terms of $\underline{a}$ and $\underline{b}$.

(b) expressing $\overrightarrow{AG}$ in the form $k\underline{a} + m\underline{b}$ in two different ways, prove that G divides each median in the ratio 1 : 2

58). $ABCD$ is a trapezium in which AB is parallel to and half DC. The diagonals AC and BD intersect at E. Given that $AB = \underline{p}$ and $BC = \underline{q}$ express $\overrightarrow{AC}$ and $\overrightarrow{BD}$ in terms of $\underline{p}$ and $\underline{q}$ and find the ratio in which E divides AC.

59). $ABCD$ is a parallelogram in which E and F are the midpoints of AB and CD respectively. AF and CE intersect the diagonal BD at X and Y. Using vectors, show that the points X and Y trisect the diagonal BD .

60). P, Q and R are points on the sides AB, BC and CA, respectively, of a triangle ABC such that $AP : PB$ = 1:2, $BQ:QC$ = 2:3 and $CR:RA$ = 3:1. BR and CP meet at X. Given that $\overrightarrow{AP} = \underline{a}$ and $\overrightarrow{AR} = \underline{b}$, find $\overrightarrow{AQ}, \overrightarrow{BR}, \overrightarrow{CP}$ and $\overrightarrow{AX}$ in terms of $\underline{a}$ and $\underline{b}$ and show that AP, BQ and CR are concurrent.

61). The sides $\overrightarrow{AD}, \overrightarrow{AB}$ of the parrallelogram represent the vectors $\underline{a}$ and $\underline{b}$ respectively.The point E is on DC and F is on BC such that $DE:EC=1:2$ and $BF:FC=2:1$ respectively. DF and AE intersect at K. Show that $10\overrightarrow{AK} = 3(3\underline{a}+\underline{b})$ If CK and AD intersect at H,.show that $7\overrightarrow{AH} = 6\underline{a}$

62).Let the points $A \equiv (-2,1)$, $B \equiv (1,3)$, $C \equiv (1,-1)$. Find the position vectors of the points A, B and C. Also find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$. Hence find the lengths of the sides of the triangle ABC.

63).Let $\overrightarrow{OA} = 7\underline{i} + 10\underline{j}$, $\overrightarrow{OB} = 4\underline{i} + 5\underline{j}$, $\overrightarrow{OC} = 2\underline{i} - \underline{j}$. Find

i. $\left|\overrightarrow{OA} + \overrightarrow{OB}\right|$ *ii.* $\left|\overrightarrow{OA} - \overrightarrow{OC}\right|$ *iii.* $\left|3\overrightarrow{OA} - 2\overrightarrow{OB}\right|$

64).The position vector of the point P is $12\underline{i} + 5\underline{j}$. Find

*i.*The distance to P from O

*ii.*unit vector in the direction $\overrightarrow{OP}$.

*iii.*The vector of size 26 in the direction $\overrightarrow{OP}$.

*iv.*The angle OP makes with (+) X-axis.

65).The position vectors of the points A and B are $\underline{i} + 2\underline{j}$ and $3\underline{i} - 4\underline{j}$ respectively. Find

i. $\overrightarrow{AB}$ *ii.* $\left|\overrightarrow{AB}\right|$ *iii.*unit vector in the direction $\overrightarrow{AB}$

*iv.*The vector of size 98 in the direction of $\overrightarrow{AB}$

*v.*The angles AB makes with X and Y axes

66).Let $\overrightarrow{AB} = (\lambda - 1)\underline{i} + (2\lambda + 1)\underline{j}$. find minimum of $\left|\overrightarrow{AB}\right|$ and corresponding value of λ.

67).Relative to O, the position vectors of the points A and B are $\underline{a}$ and $\underline{b}$ respectively. where $\underline{a} = 3\underline{i} - \underline{j}$, $\underline{b} = \lambda(\underline{i} + 2\underline{j})$. Find $\left|\overrightarrow{AB}\right|$. Also find the minimum value of $\left|\overrightarrow{AB}\right|$ and corresponding value of λ.

68).Let $\underline{a} = \underline{i} - 2\underline{j}$ and $\underline{b} = -3\underline{i} + \underline{j}$. If the vector $\underline{a} + \lambda\,\underline{b}$ is parallel to $-\underline{i} - 3\underline{j}$, find the value of λ .

69).Given that $\underline{a} = 2\underline{i} - \underline{j}$, and $\underline{b} = \underline{i} + \underline{j}$.

i. If $\underline{a} + \lambda\underline{b}$ is parallel to the direction of $\underline{i}$ find the value of λ .

ii. If $\mu\underline{a} + \underline{b}$ is parallel to the direction of $\underline{j}$, find the value of μ .

70). The position vectors of the the points *A* and *B* relative to an origin O are $\underline{a}$ and $\underline{b}$ respectively. The point *C* is situated such that $\overrightarrow{OC} = \underline{a} + 2\underline{b}$. The mid- point of *BC* is *D*. Show that $2\overrightarrow{OD} = \underline{a} + 3\underline{b}$. if the lines *OD* and *AB* intersect at *E*, show that *AE:EB = 3:1.*

71). (i) If $\underline{a} = \underline{i} - 2\underline{j}$, $\underline{b} = 4\underline{i}$ and $\underline{c} = 3\underline{i} - \underline{j}$ find $\underline{a} + \underline{b} + \underline{c}$ and the unit vector along $\underline{a} + \underline{b} + \underline{c}$.

(ii) *A, B, C* are three points on *OXY* plane. Where *O* is the vector origin such that $\overrightarrow{OA} = 2\underline{j}$, $\overrightarrow{OB} = -\underline{i} + 5\underline{j}$, $\overrightarrow{OC} = 2\underline{i} + 4\underline{j}$. Find $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ and hence show that *ABC* is an isoscelles triangle.

72). If $\overrightarrow{OA} = \underline{i} + 3\underline{j}$, $\overrightarrow{OB} = 2\underline{i} + 4\underline{j}$, $\overrightarrow{OC} = 5\underline{i} + 4\underline{j}$. Find,

(i) $\left|\overrightarrow{OA} + \overrightarrow{OB}\right|$ (ii) $\left|\overrightarrow{AB} + \overrightarrow{BC}\right|$ (iii) $\left|\overrightarrow{AB} - \overrightarrow{AC}\right|$

73). If $\overrightarrow{OA} = \underline{i} + 2\underline{j}$, $\overrightarrow{OB} = 3\underline{i} - \underline{j}$, $\overrightarrow{OC} = -\underline{i} + 5\underline{j}$. Find $\overrightarrow{AB}$ and $\overrightarrow{CA}$.
Hence show that *A, B, C* are collinear.

74). Three points *A, B, C* are such that
$\overrightarrow{OA} = 2\underline{i} + 3\underline{j}$, $\overrightarrow{OB} = 6\underline{i} + 6\underline{j}$, $\overrightarrow{OC} = \underline{i}$. Where *O* is the origin. Find $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$. Hence find the length of the sides of the triangle *ABC*.

75). (i) Relative to an origin *O*, if *A* = (1, -2) and $\overrightarrow{AB} = 2\underline{i} + 3\underline{j}$, where *B* is a point, find coordinates of *B*.

(ii) If $\underline{a} = 3\underline{i} + \alpha\,\underline{j}$ and unit vector along $\underline{a}$ is $\dfrac{\underline{i} + 2\underline{j}}{\sqrt{5}}$ find α .

(iii) If $\underline{u}$ is a unit vector on *OXY* plane making an angle α with *X* axis, state $\underline{u}$ in cartisian form.

76).The coordinates of the points *A, B, C* are *(0, 4), (4, 10), (7, 8)* respectively. If the unit vecors along *X* and *Y* axes are $\underline{i}$, $\underline{j}$ respectively, find interms of $\underline{i}$, $\underline{j}$ the vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$. Hence show that the triangle *ABC* is a right angled triangle.

77) (i) $\overrightarrow{OA} = \underline{i} + 3\underline{j}$, $\overrightarrow{OB} = 2\underline{i} + 4\underline{j}$, $\overrightarrow{OC} = 5\underline{i} + 4\underline{j}$. Find

(i) $\left|\overrightarrow{OA} + \overrightarrow{OB}\right|$ (ii) $\left|\overrightarrow{AB} + \overrightarrow{BC}\right|$ (iii) $\left|\overrightarrow{AB} - \overrightarrow{AC}\right|$

(ii) If $\overrightarrow{OA} = \underline{i} + 2\underline{j}$, $\overrightarrow{OB} = 3\underline{i} - \underline{j}$, $\overrightarrow{OC} = -\underline{i} + 5\underline{j}$ find $\overrightarrow{AB}$ and $\overrightarrow{CA}$. Hence show that *A, B, C* are collinear.

(iii) If $\underline{a} = 2\underline{i} + 3\underline{j},\ \underline{b} = \underline{i} + 5\underline{j},\ \underline{c} = \underline{a} + 2\underline{b}$

(a) Find unit vector along $\underline{c}$.

(b) If $\underline{a}, \underline{b}$ are the position vectors of A and B, then find position vector of R on AB such that $AR : RB = 3 : 2$.

78). In the triangle ABC, the position vectors of A, B, C are $\underline{a} = 3\underline{i} + 4\underline{j},\ \underline{b} = \underline{i} + \underline{j},\ \underline{c} = 10\underline{i}$ respectively.

(i) Find the length of the median AD.

(ii)Find the position vector of the centroid of the triangle.

79).Side of a square OABC is 2a. The unit vectors along OA, OC are $\underline{i}, \underline{j}$. The mid points of AB and BC are L,M. OL and AM intersect at P and BP and OA intersect at N. When λ and μ are scalars. Show that,

i). $\overrightarrow{OP} = \lambda(2a\underline{i} + a\underline{j})$

ii). $\overrightarrow{OP} = 2ai + \mu(-a\underline{i} + 2a\underline{j})$

Show by finding λ, μ that $ON = \frac{2}{3}OA$

80).$ABCD$ is a rectangle with $AB = 2a$, $AD = a$. The mid points of BC and CD are E and F. The unit vectors along AB and AD are $\underline{i}, \underline{j}$. If the lines AE, BF intersect at P, when λ and μ are scalars show that,

i). $\overrightarrow{AP} = \lambda\left(2a\underline{i} + \frac{a}{2}\underline{j}\right)$

ii). $\overrightarrow{AP} = 2a\underline{i} + \mu\left(-a\underline{i} + a\underline{j}\right)$

Hence find λ and μ obtain $AP:PE$ and $BP:PF$

81).$OABC$ is a rectangle. $OA = a, OC = b$. The unit vectors along OA and OC are $\underline{i}$ and $\underline{J}$ respectively. D is a point on CB such that $\frac{CD}{DB} = \frac{3}{2}$. OD and CA intersect at E. The BE produced meets OC at F.

i. Express $\overrightarrow{OD}$ in terms of $\underline{i}$ and $\underline{j}$

ii. Express $\overrightarrow{OE}$ in terms of $\underline{i}$ and $\underline{j}$ iii.Express $\overrightarrow{CA}$ in terms of $\underline{i}$ and $\underline{j}$

iv. Hence by finding $\overrightarrow{CE}$, by the triangle OCE express $\overrightarrow{OE}$.

v. Using (ii) and (iv) results obtain $\frac{OE}{ED}$ and $\frac{CE}{EA}$.

VECTOR DOT PRODUCT (Scalar Product)

82).i. $\underline{a}, \underline{b}, \underline{c}$ are mutually perpendicular vectors of equal magnitude. Show that $\underline{a} + \underline{b} + \underline{c}$ is equally inclined to $\underline{a}, \underline{b}, \underline{c}$.

ii. Prove that in usual notation for any triangle ABC

$a)$ $a = b\cos C + c\cos B$

$b)$ $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

83). Using vector dot product prove that the altitudes of a triangle are concurrent.

84). Find λ such that $\underline{a} = 2\underline{i} + 3\underline{j}$ perpendicular to $\underline{b} = \lambda\underline{i} + 5\underline{j}$.

85). Let A (1,2) ,B(3,3) and C(2,5) be the vertices of a triangle.Find the position vectors of A,B,C in terms of $\underline{i}$ and j. Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and lengths of the sides AB and AC. Hence find the angle between AB and AC.

86). If the dot product of a vector with the vectors $-\underline{i}+\underline{j}$ and $4\underline{i}+3\underline{j}$ are 1 and 17 respectively, find the vector.

87). If $|\underline{a}| = 3$ $|\underline{b}| = 1$ $|\underline{c}| = 4$ and $\underline{a}+\underline{b}+\underline{c} = \underline{0}$. Show that $\underline{a}.\underline{b}+\underline{b}.\underline{c}+\underline{c}.\underline{a} = -13$.

88). If $\underline{a}+\underline{b}+\underline{c} = \underline{0}$ and $|\underline{a}| = 3$ $|\underline{b}| = 5$ $|\underline{c}| = 7$, Show that the angle between $\underline{a}$ and $\underline{b}$ is 60°

89). Two vectors $\underline{a}$ and $\underline{b}$ are such that $(\underline{a}+\underline{b}).(\underline{a}-\underline{b}) = 0$ show that $|\underline{a}| = |\underline{b}|$.

90). $\underline{a}$ and $\underline{b}$ are two vectors perpendicular to each other. Prove that $|\underline{a}+\underline{b}|^2 = |\underline{a}-\underline{b}|^2$.

91). $\hat{\underline{a}}$ and $\hat{\underline{b}}$ are unit vectors and θ is the angle between them. Show that $\sin\dfrac{\theta}{2} = \dfrac{1}{2}\left|\hat{\underline{a}} - \hat{\underline{b}}\right|$.

92). Find $|\underline{a}|$ and $|\underline{b}|$ if $(\underline{a}+\underline{b}).(\underline{a}-\underline{b}) = 12$ and $|\underline{a}| = 2|\underline{b}|$.

93). Show that $|\underline{a}|\underline{b}+|\underline{b}|\underline{a}$ is perpendicular to $|\underline{a}|\underline{b}-|\underline{b}|\underline{a}$, for any two non-zero vectors $\underline{a}$ and $\underline{b}$.

94). If $\underline{a}+\underline{b}+\underline{c} = \underline{0}$, show that the angle θ between the vectors $\underline{b}$ and $\underline{c}$ is given by

$$\cos\theta = \frac{|\underline{a}|^2 - |\underline{b}|^2 - |\underline{c}|^2}{2|\underline{b}||\underline{c}|}.$$

95). A parallelogram is constructed on the vector $\underline{a} = 3\underline{p} - \underline{q}$ and $\underline{b} = \underline{p} + 3\underline{q}$. Given that $|\underline{p}| = |\underline{q}| = 2$ and the angle between $\underline{p}$ and $\underline{q}$ is $\dfrac{\pi}{3}$. show that lengths of the diagonals are $4\sqrt{7}$ and $4\sqrt{3}$.

96). Using vector method, show that the diagonals of a rhombus bisect each other at right angles.

97). The position vectors of the vertices of a triangle are $\underline{0}$, $\underline{a}$, $\underline{b}$. Show that its area Δ given by $4\Delta^2 = |\underline{a}|^2|\underline{b}|^2 - (\underline{a}.\underline{b})^2$.

98). $\underline{a}, \underline{b}, \underline{c}$ be three vectors of length 3,4,8 respectively.Let $\underline{a}$ be perpendicular to $\underline{b}+\underline{c}$, $\underline{b}$ to $\underline{c}+\underline{a}$, $\underline{c}$ to $\underline{a}+\underline{b}$. Find the length of the vector $\underline{a}+\underline{b}+\underline{c}$.

99). Using vector dot product

i) Prove that the angle in a semi-circle is a right angle.

ii) Prove that the mid-point of the hypotenuse of a right- angled triangle is equidistant from its vertices.

iii) Prove the pythagorian theorem.

iv) Prove that the perpendicular bisectors of the sides of a triangle are concurrent.

100). $\underline{r} = \underline{a} + \lambda \underline{b}$ is the equation of a straight line. Where $\underline{b}$ is the unit vector in the direction of the line. Prove that the equation of the line through the origin perpendicular to the given line is $\underline{r} = \mu[\underline{a} - (\underline{a}.\underline{b})\underline{b}]$. Where μ is a real parameter.Also show that the perpendicular distance of first line from the origin is $\left[|\underline{a}|^2 - (\underline{a}.\underline{b})^2\right]^{\frac{1}{2}}$.

101) *i*.If $|\underline{a}| = 10$, $|\underline{b}| = 20$, and angle between $\underline{a}$ and $\underline{b}$ is 120° find $\underline{a}\,.\,\underline{b}$.

ii.using distributive law show that

a. $(\underline{a} + \underline{b}).(\underline{a} + \underline{b}) = |\underline{a}|^2 + 2(\underline{a}.\underline{b}) + |\underline{b}|^2$

b. $(\underline{a} - \underline{b}).(\underline{a} - \underline{b}) = |\underline{a}|^2 - 2(\underline{a}.\underline{b}) + |\underline{b}|^2$

102).simplify the following.

i. $(2\underline{a} + \underline{b}).(3\underline{a} + 4\underline{b})$ *ii*. $(5\underline{a} + \underline{b}).(\underline{a} - 2\underline{b})$ *iii*. $2\underline{a}.(3\underline{b} - 2\underline{a})$.

103).Let $\underline{a}$ and $\underline{b}$ be any two vectors. The vectors $(\underline{a} + \underline{b})$ and $\underline{a}$ are perpendicular to each other. If $|\underline{b}| = \sqrt{2}\,|\underline{a}|$ show that $(2\underline{a} + \underline{b})$ and $\underline{b}$ are perpendicular to each other.

104). $|\underline{a}| = 4, |\underline{b}| = 10$ and the angle between $\underline{a}$ and $\underline{b}$ is 60°. Find the angle between the vectors $(\underline{a} + \underline{b})$ and $(\underline{a} - \underline{b})$.

105) .Let $|\underline{a}| = 5, |\underline{b}| = 12$ and the angle between $\underline{a}$ and $\underline{b}$ is 30°. Find

i. $(2\underline{a} + 3\underline{b}).(\underline{a} - \underline{b})$ *ii*. $\underline{a}\,.(5\underline{a} + \underline{b})$

106). Find $\underline{a}.\underline{b}$ for the following

i. $\underline{a} = 2\underline{i} + 5\underline{j}$, $\underline{b} = \underline{i} + 4\underline{j}$ *ii*. $\underline{a} = (2\underline{i} - 3\underline{j})$, $\underline{b} = (7\underline{i} + \underline{j})$ *iii*. $\underline{a} = (4\underline{i} - 5\underline{j})$, $\underline{b} = (2\underline{i} + 11\underline{j})$

107).Find the angle between $\underline{a}$ and $\underline{b}$.

i. $\underline{a} = 2\underline{i} + 5\underline{j}$, $\underline{b} = \underline{i} + 4\underline{j}$ *ii*. $\underline{a} = 2\underline{i} - 3\underline{j}$, $\underline{b} = 7\underline{i} + \underline{j}$ *iii*. $\underline{a} = (4\underline{i} - 5\underline{j})$, $\underline{b} = (2\underline{i} + 11\underline{j})$

108). *i*.The angle between the vectors $(3\underline{i} - \underline{j})$ and $(2\underline{i} + \lambda\underline{j})$ is $\frac{\pi}{4}$. Find λ.

ii.If the vectors $(4\underline{i} - 3\underline{j})$ and $(\lambda\underline{i} + 2\underline{j})$ are perpendicular to each other, find λ.

iii. Relative to a vector origin *O*, the position vectors of *A* and *B* are $(2\underline{i} - 3\underline{j})$ and $(-4\underline{i} - \underline{j})$ respectively. Find the area of the triangle *OAB*.

109).The vectors $\underline{a}$ and $\underline{b}$ are unit vectors. The vector $\underline{a}+\underline{b}$ is also unit vector.

i.Show that the angle between $\underline{a}$ and $\underline{b}$ is $\frac{2\pi}{3}$.

ii.Also show that $|\underline{a}-\underline{b}|=\sqrt{3}$.

110.In the usual notation, let $\underline{i}$ and $\underline{i}+\underline{j}$ be the position vectors of two points *A* and *B* respectively, with respect to a fixed origin *O*. Also, let *C* be a point on the straight line through *A* parallel to *OB*. Show that $\overrightarrow{OC}=(1+\lambda)\underline{i}+\lambda\underline{j}$, where λ is a real number.

Find the value of λ such that *BC* is perpendicular to *OB*.

111.In the usual notation, the position vectors of two points *A* and *B* with respect to an origin *O*, are $\lambda\underline{i}+\mu\underline{j}$ and $\mu\underline{i}-\lambda\underline{j}$ respectively, where λ and μ are real numbers such that $o<\lambda<\mu$. Show that $A\hat{O}B$ is a right angle. Let *C* be the mid-point of the line segment *AB*. If the vector $\overrightarrow{OC}$ is of magnitude 2 and it makes an angle $\frac{\pi}{6}$ with the unit vector $\underline{i}$, find the values of λ and μ.

112.In the usual notation, let $\underline{a}=3\underline{i}+4\underline{j}$, $\underline{b}=4\underline{i}+3\underline{j}$ and $\underline{c}=a\underline{i}+(1-\alpha)\underline{j}$, where $\alpha\in\Re$.

Find i. $|\underline{a}|$ and $|\underline{b}|$,

ii. $\underline{a}.\underline{c}$ and $\underline{b}.\underline{c}$ in terms of α.

If the angle between $\underline{a}$ and $\underline{c}$ is equal to the angle between $\underline{b}$ and $\underline{c}$, show that $\alpha=\frac{1}{2}$.

113.In the usual notation, let $-\underline{i}+2\underline{j}$ and $2\alpha\underline{i}+\alpha\underline{j}$ be the position vectors of two points *A* and *B* respectively, with respect to a fixed origin *O*, where $\alpha(>0)$ is a constant. Using scalar product, show that $A\hat{O}B=\frac{\pi}{2}$.

Let *C* be the point such that *OACB* is rectangle. If the vector $\overrightarrow{OC}$ lies along the y-axis, find the value of α.

114. The position vectors of two distinct points *A* and *B* with respect to a fixed origin *O*, **not collinear** with *A* and *B*, are $\underline{a}$ and $\underline{b}$ respectively. Let $\underline{c}=(1-\lambda)\underline{a}+\lambda\underline{b}$ be the position vector of a point *C* with respect to *O*, where $0<\lambda<1$.

Express the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$ in terms of $\underline{a}$, $\underline{b}$ and λ.

Hence, show that the point *C* lies on the line segment *AB* and that $AC:CB=\lambda:(1-\lambda)$.

Now, suppose that the line *OC* bisects the angle *AOB*. Show that $|\underline{b}|(\underline{a}.\underline{c})=|\underline{a}|(\underline{b}.\underline{c})$ and hence, find λ.

115.In the usual notation, let $3\underline{i}$ and $2\underline{i}+3\underline{j}$ be the position vectors of two points *A* and *B* respectively, with respect to a fixed origin *O*. Let *C* be the point on the straight line *OB* such that $O\hat{C}A=\frac{\pi}{2}$. Find $\overrightarrow{OC}$ in terms of $\underline{i}$ and $\underline{j}$.

116.Let OAB be a triangle, D be the mid-point of AB and E be the mid-point of OD. The point F lies on OA such that $OF:FA=1:2$. The position vectors of A and B with respect to O are $\underline{a}$ and $\underline{b}$ respectively. Express the vectors $\overrightarrow{BE}$ and $\overrightarrow{BF}$ in terms of $\underline{a}$ and $\underline{b}$.

Deduce that B,E and F are collinear and find the ratio $BE:EF$.

Find the scalar product $\overrightarrow{BF}.\overrightarrow{DF}$ in terms of $|\underline{a}|$ and $|\underline{b}|$ and show that if $|\underline{a}|=3|\underline{b}|$, then $\overrightarrow{BF}$ is perpendicular to $\overrightarrow{DF}$.

117.In the usual notation, let $2\underline{i}+\underline{j}$ and $3\underline{i}-\underline{j}$ be the position vectors of two points A and B, respectively, with respect to a fixed origin O. Find the position vectors of the two distinct points C and D such that

$A\hat{O}C=A\hat{O}D=\dfrac{\pi}{2}$ and $OC=OD=\dfrac{1}{3}AB$.

118. Let $OACB$ be a parallelogram and let D be the point on AC such that $AD:DC=2:1$. The position vectors of points A and B with respect to O are $\lambda\underline{a}$ and $\underline{b}$, respectively, where $\lambda>0$.

Express the vectors $\overrightarrow{OC}$ and $\overrightarrow{BD}$ in terms of $\underline{a},\underline{b}$ and λ.

Now, let $\overrightarrow{OC}$ be perpendicular to $\overrightarrow{BD}$. Show that $3|\underline{a}|^2\lambda^2+2(\underline{a}.\underline{b})\lambda-|\underline{b}|^2=0$ and find the value of λ,

if $|\underline{a}|=|\underline{b}|$ and $A\hat{O}B=\dfrac{\pi}{3}$.

119.In the usual notation, let $\underline{i}+2\underline{j}$ and $3\underline{i}+3\underline{j}$ be the position vectors of two points A and B respectively, with respect to a fixed origin O.Also let C be the point such that $OABC$ is parallelogram.Show that $\overrightarrow{OC}=2\underline{i}+\underline{j}$.

Let $A\hat{O}C=\theta$. By considering $\overrightarrow{OA}.\overrightarrow{OC}$, show that $Cos\theta=\dfrac{4}{5}$.

120.In the usual notation, the position vectors of four points A, B, C and D are

$\underline{a}=-\underline{i}-\underline{j},\ \underline{b}=\underline{i}+4\underline{j},\ \underline{c}=8\underline{i}+\alpha\underline{j}$ and $\underline{d}=4\underline{i}-2\underline{j}$, respectively, where $\alpha\in\mathbb{R}$.

The lines AB and DC are parallel. Show that $\alpha=8$.

The lines AC and BD intersect at a point E with the position vector $\underline{e}$. By considering $\overrightarrow{AE}$ and $\overrightarrow{AC}$, show that

$\underline{e}=(1-\lambda)\underline{a}+\lambda\underline{c}$ for $\lambda\in\mathbb{R}$.

Similarly, also show that $\underline{e}=(1-\mu)\underline{b}+\mu\underline{d}$ for $\mu\in\mathbb{R}$.

Hence find $\underline{e}$ in terms of $\underline{i}$ and $\underline{j}$.

By considering $\overrightarrow{EA}\cdot\overrightarrow{ED}$, find $A\hat{E}D$.

121.In the usual notation, the position vectors of two points A and B with respect to a fixed origin O are $3\underline{i}+2\underline{j}$ and $2\underline{i}+4\underline{j}$, respectively. Show that O, A and B are non - collinear. Let C be the point such that $\overrightarrow{BC}=\lambda\overrightarrow{OA}$, where $\lambda\in\mathbb{R}$. Find $\overrightarrow{OC}$ in terms of $\underline{i},\underline{j}$ and λ. show that if $B\hat{O}C=\dfrac{\pi}{2}$, then $\lambda=-\dfrac{10}{7}$.

122.Let the position vectors of four points *A, B, C* and *D* be $\underline{a}$, $\underline{b}$, $3\underline{a}$ and $4\underline{b}$, respectively with respect to a fixed origin *O*, where $\underline{a}$ and $\underline{b}$ are non - zero and non - parallel vectors. *E* is the point of intersection of *AD* and *BC*. Using the triangle law of addition for the triangle *OAE*,

show that $\overrightarrow{OE} = \underline{a} + \lambda(4\underline{b} - \underline{a})$ for $\lambda \in \mathbb{R}$.

Similarly, show also that $\overrightarrow{OE} = \underline{b} + \mu(3\underline{a} - \underline{b})$ for $\mu \in \mathbb{R}$.

Hence, show that $\overrightarrow{OE} = \frac{1}{11}(9\underline{a} + 8\underline{b})$.

123.In the usual notation, the position vectors of two points *A* and *B*, with respect to a fixed origin *O* are $2\underline{i} - 3\underline{j}$ and $\underline{i} - 2\underline{j}$, respectively. Using $\overrightarrow{AO}.\overrightarrow{AB}$, find $O\hat{A}B$.

Let *C* be the point on *OA* such that $O\hat{C}B = \frac{\pi}{2}$, Find $\overrightarrow{OC}$.

124.Let $\underline{a}$ and $\underline{b}$ be non - zero and non - parallel vectors, and $\lambda, \mu \in \mathbb{R}$.

show that if $\lambda\underline{a} + \mu\underline{b} = \underline{0}$, then $\lambda = 0$ and $\mu = 0$.

Let *ABC* be a triangle. The mid - point of *AB* is *D* and the mid - point of *CD* is *E*. The lines *AE* (extended) and *BC* meet at F. Let $\overrightarrow{AB} = a$ and $\overrightarrow{AC} = b$. Using the triangle law of addition.

show that $\overrightarrow{AE} = \frac{\underline{a} + 2\underline{b}}{4}$.

Explain why $\overrightarrow{AF} = \alpha\overrightarrow{AE}$ and $\overrightarrow{CF} = \beta\overrightarrow{CB}$, where $\alpha, \beta \in \mathbb{R}$.

Considering the triangle *ACF*, show that $(\alpha - 4\beta)\underline{a} + 2(\alpha + 2\beta - 2)\underline{b} = \underline{0}$. Hence, find the values of α and β.

125.In the usual notation, let $2\underline{i} + \underline{j}$ and $3\underline{i} - \underline{j}$ be the position vectors of two points *A* and *B*, respectively, with respect to a fixed origin *O*. Find the position vectors of the two distinct points *C* and *D* such that $A\hat{O}C = A\hat{O}D = \frac{\pi}{2}$ and

$OC = OD = \frac{1}{3}AB$.

126.let *OACB* be a parallelogram and let *D* be the point on *AC* such that $AD : DC = 2:1$. The position vectors of points *A* and *B* with respect to *O* are $\lambda\underline{a}$ and $\underline{b}$, respectively, where $\lambda > 0$. Express the vectors $\overrightarrow{OC}$ and $\overrightarrow{BD}$ in terms of $\underline{a}$, $\underline{b}$ and λ.

Now, let $\overrightarrow{OC}$ be perpendicular to $\overrightarrow{BD}$. Show that $3|\underline{a}|^2\lambda^2 + 2(\underline{a}.\underline{b})\lambda - |\underline{b}|^2 = 0$ and find the value of λ, if $|\underline{a}| = |\underline{b}|$ and

$A\hat{O}B = \frac{\pi}{3}$.

127.Let $\alpha > 0$ and in the usual notation, let $\underline{i} + \alpha\underline{j}$ and $\alpha\underline{i} - 2\underline{j}$ be the position vectors of two points *A* and *B*, respectively, with respect to a fixed origin *O*. Also, let *C* be the point on *AB* such that $AC : CB = 1:2$. It is given that *OC* is perpendicular to *AB*. Find the value of α.

128.Let $\underline{a}$ and $\underline{b}$ be two non-zero, non-parallel vectors. Show that $\alpha\underline{a}+\beta\underline{b}=\underline{0}$ if and only if $\alpha=0$ and $\beta=0$.

Relative to a vector origin O, the position vectors of the four points *A, B C* and *D* are $\underline{a}, \underline{b}$, $2\underline{a}+5\underline{b}$ and $3\underline{a}+2\underline{b}$ respectively. Express $\overrightarrow{AC}$ and $\overrightarrow{BD}$ in terms of $\underline{a}$ and $\underline{b}$. Let *E* be the point of intersection of the lines *AC* and *BD*. Find the ratios $AE : EC$ and $BE : BD$. using vector dot product , show that $\frac{3}{2}AE^2 - \frac{10}{9}.BE^2 = \frac{4}{7}(5|\underline{b}|^2 - 3|\underline{a}|^2)$.

129.Relative to a vector origin *O*, the position vectors of the points *P* amd *Q*, are $\underline{p}$ and $\underline{q}$ respectively. The point *R* lies on *PQ* such that $PR:RQ=2:1$. Show that the position vector of the point *R* is $\frac{1}{3}\underline{p}+\frac{2}{3}\underline{q}$. A point *S* is on this plane such that *RS* and *RQ* are perpendicular to each other If it is given that $\underline{s}=\underline{p}+\underline{q}$, show that $\underline{p}.\,\underline{q}=2|\underline{p}|^2-|\underline{q}|^2$.
Also when $k>0$, if $\underline{p}=2\underline{i}+\underline{j}$ and $\underline{q}=k\underline{i}+2\underline{j}$, find *k*.

130.The triangle *ABC* is an isosceles such that $AC=BC$. The internal angle bisectors of the vertices *A* and *B* meet at the point *D*. Relative to a fixed vector origin *O*, the position vectors of the points *A, B* and *C* are $2\underline{a}+\underline{b}$, $3\underline{a}-5\underline{b}$, and $\underline{a}+3\underline{b}$ respectively. Where $\underline{a}$ and $\underline{b}$ are two non - zero , non parallel vectors. Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$. By using vector dot product show that $3|\underline{a}|^2+60|\underline{b}|^2=28(\underline{a}.\underline{b})$. Hence if the line *CD* produced , meets *AB* at *E*, Show that *CE* is perpendicular to *AB*. Let the mid point of *CE* be *G*. The line *AG* meets *CB* at *F*. Find the ratios *AG* : *GF* and *CF* : *FB*.

131.The position vectors of the points *A* and *B*, relative to a vector origin *O* are $\underline{a}$ and $\underline{b}$ respectively. Point *C* is on the line passing through *O* parallel to *AB*. Find the position vector of *C* so that *OABC* is a parallelogram. Let *D* be a point on *OC* produced and *E* is a point on *OB* so that *OE* : *EB* = 3 : 1. Find the position vector of *D* so that *A, E, D* are collinear and find the ratio $AE:ED$. It is given that $AB=\sqrt{5}$ (units) and 2*OA* = *OB*.

Show that $B\hat{O}A=\cos^{-1}\left[\frac{5(|\underline{a}|^2-1)}{4|\underline{a}|^2}\right]$.