

BASIC MATHS

1) Simplify the following.

$$\begin{array}{lllll} 1. \frac{x}{3} + \frac{y}{4} & 2. \frac{x+1}{5} + \frac{2x+5}{3} & 3. \frac{4x-5}{2} - \frac{3x-1}{6} & 4. \frac{1}{x} + \frac{3}{y} & 5. \frac{5}{m} - \frac{2}{n} \\ 6. \frac{3}{a+2} + \frac{2}{a+1} & 7. \frac{10}{x} + \frac{10}{x+1} & 8. \frac{a}{a+b} + \frac{b}{a-b} & & \end{array}$$

2) Simplify the following.

$$\begin{array}{llll} 1. 1 + \frac{1}{a} + \frac{1}{a^2} & 2. \frac{1}{c^2} - \frac{1}{c} - 1 & 3. x + 1 - \frac{1}{x} & 4. 1 + \frac{1}{b} - \frac{b}{1-b} \\ 5. \frac{1}{y+1} - \frac{2}{y-2} + \frac{1}{2} & 6. 3 - \frac{2}{x-2} + \frac{5}{2x+3} & & \end{array}$$

3) Factorise the following quadratic expressions.

$$1. p^2 + 2p + 1 \quad 2. x^2 + 4x + 4 \quad 3. x^2 + 6x + 9 \quad 4. x^2 + 8x + 16 \quad 5. x^2 + 10x + 25 \quad 6. a^2 + 2a + 1$$

4) Factorise the following.

$$1. 2a^2 + 11a + 5 \quad 2. 3x^2 + 13x + 4 \quad 3. 2p^2 + 7p + 5 \quad 4. 3x^2 + 8x + 4 \quad 5. 2a^2 + 7ab + 6b^2$$

5) Factorise the following.

$$\begin{array}{llll} 1. 9x^2 + 6x + 1 & 2. 4a^2 + 4a + 1 & 3. 16a^2 + 24a + 9 & 4. 4y^2 + 20y + 25 \\ 5. 24x^2 - 29xy - 4y^2 & 6. 3x^2 + 7xy - 10y^2 & & \end{array}$$

6) Factorise the following.

$$\begin{array}{llll} 1. x^2 - (3x+2)^2 & 2. a^2 - 9(x+y)^2 & 3. (5x+1)^2 - (3x-1)^2 & 4. (x+5)^2 - (a-b)^2 \\ 5. 25x^2 - 36y^2 & 6. 100x^2y^2 - 25a^2b^2 & 7. \frac{16x^2}{y^2} - \frac{b^2}{25a^2} & 8. \frac{49b^2}{a^2} - \frac{x^2}{64y^2} \\ 9. \frac{81a^2}{25b^2} - \frac{100x^4}{64y^4} & & & \end{array}$$

7) **Sum and difference of two cubes**

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Factorise the following.

$$\begin{array}{llllll} 1. 8x^3 + 64y^3 & 2. 27x^3y^3 - 8a^6b^6 & 3. (1+a^3) & 4. (x^3-1) & 5. 1000x^3 - 125y^6 & 6. 216x^3 + 125y^6 \\ 7. x^3 + 1 & 8. p^3 - 27 & 9. x^3 - 8 & 10. 1 + 27x^3 & 11. (1-y^3) & 12. (8x^3-1) \\ 13. (27x^3+8) & 14. (64x^3-125) & & & & \\ 15. a^3b^3 - 1 & 16. x^3y^3 + 216 & 17. 125p^3 - 27q^3 & 18. 8x^4 + x & 19. 32a^3 - 4b^3 & 20. 16a^3b^3 - 2a^6 \\ 21. x^4y - xy^4 & 22. x^6 - y^6 & & & & \end{array}$$

8) Find the L.C.M of the following.

$$1. x(x+5), (x+5)(x-3) \quad 2. (3x+1)(4x-3), (4x-3)(x+1) \quad 3. (5x-1)(3x-2), (5x-1)(2x-7)$$

9) Simplify the following till single fraction.

$$1. \frac{2}{x+4} + \frac{x-5}{x^2+7x+12} \quad 2. \frac{1}{x+1} + \frac{3}{x^2-1} \quad 3. \frac{1}{x^2} - \frac{1}{x^2+x} \quad 4. \frac{3}{x^2+x-2} + \frac{2}{x^2+3x+2} \quad 5. \frac{1}{a^2-2a+1} - \frac{3}{a^2-1}$$

10) Simplify the following

$$1. \frac{8x-16y}{2} \quad 2. \frac{3c}{6ac+9bc} \quad 3. \frac{4a^2-8ab}{a-2b} \quad 4. \frac{2a-2b}{2(a-b)} \quad 5. \frac{4x+20}{3x+15} \quad 6. \frac{y-3}{4y-12}$$

$$7. \frac{12p-3q}{24p-6q} \quad 8. \frac{y^2-y-6}{y-3} \quad 9. \frac{ax+5a}{(x+5)(x+3)} \quad 10. \frac{x^2-1}{x^2+2x+1}$$

11) Find the solutions of the following quadratic equations.

$$1. (x+2)(x+3)=0 \quad 2. (x+4)(x+\frac{3}{7})=0 \quad 3. (x+\frac{5}{2})(x+\frac{6}{7})=0 \quad 4. (x-5)(x+4)=0$$

$$5. (x-3)(x+\frac{5}{11})=0 \quad 6. (x-\frac{1}{2})(x+3)=0 \quad 7. (x-\frac{8}{11})(x+\frac{3}{7})=0 \quad 8. (a-3)(a-7)=0$$

$$9. (a-\frac{5}{12})(a-10)=0 \quad 10. (a-\frac{2}{25})(a-\frac{11}{10})=0 \quad 11. (x+5)^2=0 \quad 12. (x+\frac{3}{4})^2=0$$

$$13. (x-7)^2=0 \quad 14. (x-\frac{5}{6})^2=0$$

12) Find the solutions of the following quadratic equations using factors or using formula or using completing square.

$$1. x^2+6x+8=0 \quad 2. x^2+7x+12=0 \quad 3. 30+11x+x^2=0 \quad 4. 24+10p+p^2=0$$

$$5. x^2+6x+9=0 \quad 6. x^2+16x+64=0 \quad 7. x^2-7x+12=0 \quad 8. p^2+5=6p$$

$$9. n^2-2n+1=0 \quad 10. p^2-10p+25=0 \quad 11. x^2-2x-35=0 \quad 12. x^2-5x=14$$

$$13. y^2+2y-8=0 \quad 14. a^2+7a=44$$

13). Solve the following equations.

$$1. (x-4)^2-9=0 \quad 2. (x+3)^2-12=0 \quad 3. (x-5)^2=9 \quad 4. (x+4)^2=5 \quad 5. (x-2)^2-\frac{9}{4}=0$$

$$6. (x-\frac{4}{3})^2-\frac{5}{4}=0$$

14) **Proportions**

$a:b=c:d$ is written as $\frac{a}{b}=\frac{c}{d}$.

It is read as "a is to b" equal to "c is to d"

Properties of proportions

$$1. \text{ If } \frac{a}{b}=\frac{c}{d} \text{ then } ad=bc.$$

$$2. \text{ If } \frac{a}{b}=\frac{c}{d} \text{ then } i. \frac{a}{c}=\frac{b}{d}. \quad ii. \frac{b}{a}=\frac{d}{c}.$$

$$3. \text{ If } \frac{a}{b}=\frac{c}{d} \text{ then } \frac{a+b}{b}=\frac{c+d}{d}.$$

$$4. \text{ If } \frac{a}{b}=\frac{c}{d} \text{ then } \frac{a-b}{b}=\frac{c-d}{d}.$$

$$5. \text{ If } \frac{a}{b}=\frac{c}{d} \text{ then } \frac{a+b}{a-b}=\frac{c+d}{c-d}$$

15)1. If $x : y = 3 : 5$, find the following.

1. $\frac{x}{y}$

2. $\frac{y}{x}$

3. $\frac{x+y}{y}$

4. $\frac{x-y}{y}$

5. $\frac{x+y}{x-y}$

16) Find $a : b$ for the following proportions.

1. $4a = 3b$ 2. $a = b - \frac{b}{4}$ 3. $3a - 2b = a + 6b$ 4. $\frac{a+b}{a-b} = \frac{7}{3}$

17) Solve the following equations using the properties of proportions.

1. $\frac{2}{a} = \frac{3}{5}$ 2. $\frac{x-2}{2} = \frac{1}{3}$ 3. $\frac{1}{x-2} = \frac{1}{2x}$ 4. $\frac{2x+5}{2x-5} = \frac{1}{2}$

18) Solve the following simultaneous equations.

1. $2x - y = 3$ 2. $2a - b = 1$ 3. $x + 3y = 8$ 4. $7a - b = 2$ 5. $a - 2b = 3$ 6. $x + 2y = 7$
 $x + y = 9$ $2a + b = 9$ $x - 2y = 3$ $6a - b = 0$ $2a + b = 1$ $2x + 3y = 12$

7. $3x + 5y = 34$ 8. $3x + 9 = y$ 9. $\frac{1}{x} + \frac{2}{y} = 2\frac{1}{3}$ 10. $\frac{2}{x} + \frac{5}{y} = 5$ 11. $3x - y = 1$
 $x + y = 12$ $13 + 2x = 3y$ $\frac{2}{x} + \frac{3}{y} = 3\frac{2}{3}$ $\frac{1}{x} - \frac{7}{y} = 12$ $\frac{2}{x} + \frac{3}{y} = 2\frac{1}{2}$

12. $\frac{x+2}{5} = \frac{8-y}{4} = \frac{x+y}{7}$ 13. $\frac{x+y}{3} = \frac{3y-x}{5} = \frac{x+2y+2}{6}$ 14. $\frac{3x-2}{5} = \frac{7-y}{2}, 3x-2y=6$

15. $\frac{2x}{3} + \frac{3y}{2} = 4x - 5y - 23 = -1$

19) Solve the following simultaneous equations.

1. $x = 2$ 2. $a = 3$ 3. $a = -3$
 $x + 2y = 4$ $2a - b = 8$ $2b - 3c = 19$
 $x + y + 3z = 15$ $2a - 2b + 3c = 25$ $a + 2b + 3c = 10$
4. $x - 3y = 9$ 5. $2a + b = 8$ 6. $x + 2y = 8$
 $x + 2z = 9$ $a + 3c = 14$ $x + 3y + z = 6$
 $2x + y + 3z = 13$ $2a - 2b + 3c = 23$ $x + 2y - 3z = 23$

20) Solve the following simultaneous equations.

1. $x + y - z = 4$: $2x + y + z = 9$: $3x - y - z = 6$ 2. $a + b + c = 8$: $a + 2b - c = 14$: $2a - b - 3c = 9$
3. $a + b + c = 5$: $a - 2b - c = -4$: $2a + 3b + 2c = 11$ 4. $2x + y + z = 11$: $x - 2y - z = 6$: $3x + 2y - z = 8$

5. $x + y + z = 6$: $2x - y - 2z = 5$: $5x + 2y + 2z = 21$ 6. $x - \frac{y}{3} = 4$: $y - \frac{z}{4} = 1$: $z - \frac{x}{5} = 7$

7. $x + y + z = x - y = 8$: $z + x = 2$ $8x + y = 10$: $y + z = 8$: $z + x = 4$

9. $x - \frac{x}{2} + \frac{y}{3} + \frac{z}{2} = 7$: $\frac{x}{4} + \frac{y}{2} + z = 7$: $\frac{x}{8} + \frac{y}{3} + \frac{z}{2} = 4$ 10. $a + c = 2b$: $9a + 3c = 8b$: $2a + 3b + 5c = 36$

11. $\frac{a+b}{2} = \frac{b+c}{4} = \frac{c+a}{3}$: $2a + b + c = 30$ 12. $a + b = 3$: $2a + 3b = 3c + 7$: $3a - 2c = 8 - 2b$

21) Solve the following equations.

1. $2x + y = 4$

$x^2 + y^2 = 5$

2. $3x - y = 8$

$3x^2 - xy - y^2 = -9$

3. $x + y = 1$

$2x^2 - xy + y^2 = 53$

4. $x - 2y = 2$

$x^2 + 3xy = 4$

5. $\frac{x}{2} + \frac{y}{3} = 1$

$4xy = 3$

6. $x + 2y = 13$

$xy = 15$

22) Simplify as far as possible.

1. $5\sqrt{6} + 3\sqrt{6}$

2. $4\sqrt{5} - 3\sqrt{5}$

3. $8\sqrt{2} + 2\sqrt{3} - 3\sqrt{2} - 6\sqrt{2} + 5\sqrt{3}$

4. $2\sqrt{7} + 3\sqrt{6} + 2\sqrt{6} - 5\sqrt{7} + 8\sqrt{7}$

5. $7\sqrt{28} - \sqrt{700} + \sqrt{63}$

6. $\sqrt{20a^2} - \sqrt{5a^2}$

7. $2\sqrt{90} + 3\sqrt{40} - \sqrt{160}$

8. $\sqrt{45} - 9\sqrt{5} + \sqrt{80}$

9. $3\sqrt{7a} + 4\sqrt{28a}$

23) Simplify.

1. $\sqrt{5} \times \sqrt{8}$

2. $\sqrt{8} \times \sqrt{4}$

3. $3\sqrt{5} \times 2\sqrt{5}$

4. $2\sqrt{3} \times 3\sqrt{33}$

5. $\sqrt{\frac{7}{8}} \times \sqrt{\frac{8}{7}}$

6. $\frac{\sqrt{10}}{\sqrt{5}}$

7. $\frac{\sqrt{75}}{\sqrt{3}}$

8. $\frac{\sqrt{99}}{8\sqrt{11}}$

9. $\frac{3\sqrt{60}}{4\sqrt{15}}$

10. $\frac{\sqrt{80} - 3\sqrt{5}}{\sqrt{5}}$

State the following numbers with rational denominators.

11. $\frac{1}{\sqrt{3}}$

12. $\frac{4}{\sqrt{8}}$

13. $\frac{\sqrt{3}}{\sqrt{11}}$

14. $\frac{5\sqrt{5}}{6\sqrt{3}}$

15. $\frac{5}{2 + \sqrt{3}}$

16. $\frac{7}{5 - \sqrt{7}}$

17. $\frac{3 + \sqrt{2}}{3 - \sqrt{2}}$

18. $\frac{3 - \sqrt{7}}{2 + \sqrt{7}}$

19. $\frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} + \sqrt{3}}$

20. $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$

24). Expand the following using "Pascal's" triangle.

1. $(a + b)^4$

2. $(a - b)^5$

3. $(2x - y)^6$

4. $(3x + 2y)^7$

5. $\left(\frac{x}{2} - 3y\right)^4$

6. $\left(\frac{x}{3} + \frac{y}{2}\right)^5$

7. $(2ab - 3xy)^5$

8. $\left(\frac{3x}{2} + \frac{2y}{3}\right)^4$

9. $\left(\frac{2a}{3} - \frac{1}{2}b\right)^5$

10. $(x + y + z)^3$

11. Show that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.

INDICES AND LOGARITHMS

i. Laws Of Indices

i. $a^m \times a^n = a^{m+n}$

v. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

ix. $a^{\frac{1}{2}} = \sqrt{a}$

ii. $\frac{a^m}{a^n} = a^{m-n}$

vi. $a^n = \frac{1}{a^{-n}}$

iii. $(a^m)^n = a^{mn}$

vii. $a^0 = 1 (a \neq 0)$

iv. $(ab)^n = a^n b^n$

viii. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

ii. Laws Of Logarithms

$$\begin{aligned}
 i. \log_a^{(MN)} &= \log_a^M + \log_a^N & ii. \log_a^{\left(\frac{M}{N}\right)} &= \log_a^M - \log_a^N & iii. \log_a M^r &= r \log_a^M & iv. \log_a^b &= \frac{1}{\log_b^a} \\
 v. \log_a^b &= \frac{\log_c^b}{\log_c^a} & vi. a^{\log_a^x} &= x & vii. a^{\log_b^x} &= x^{\log_b^a} & viii. \log_{a^m}^{b^n} &= \frac{n}{m} \cdot \log_a^b & ix. \log_a^a &= 1 \\
 x. \log_a^1 &= 0
 \end{aligned}$$

1. a. Prove that $\frac{1}{\log_x^{(xyz)}} + \frac{1}{\log_y^{(xyz)}} + \frac{1}{\log_z^{(xyz)}} = 1$

b. Prove that $(\log_{b^2}^a) \cdot (\log_{c^2}^b) = \frac{1}{4} \log_c^a$

c. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$ prove that $abc=1$ and $a^a b^b c^c = 1$

2. a. If $\log_4^x = a$, $\log_{12}^x = b$ prove that $\log_3^4 = \frac{b}{a-b}$ and $\log_3^{48} = \frac{a+b}{a-b}$

b. If a, b, x are three positive numbers. show that $\log_{ab}^x = \frac{\log_b^x}{1 + \log_b^a}$

c. If $p = \log_a^{(bc)}$, $q = \log_b^{(ca)}$, $r = \log_c^{(ab)}$ Show that $pqr = p + q + r + 2$

3. a. Prove that $\log_a^b \times \log_b^c \times \log_c^a = 1$

b. Prove that $2 \log_c^{(a+b)} = 2 \log_c^a + \log_c \left(1 + \frac{2b}{a} + \frac{b^2}{a^2} \right)$

c. If $\log_a^{\left(1+\frac{1}{8}\right)} = p$, $\log_a^{\left(1+\frac{1}{15}\right)} = q$ and $\log_a^{\left(1+\frac{1}{24}\right)} = r$. Show that $\log_a^{\left(1+\frac{1}{80}\right)} = p - q - r$

d. If $\log_a^n = x$, $\log_c^n = y$ and $n \neq 1$ then prove that $\frac{x-y}{x+y} = \frac{\log_b^c - \log_b^a}{\log_b^c + \log_b^a}$

4. a. When $a, b, c \in R^+$, prove that $\log_a^b = \frac{1}{\log_b^a}$

b. If $a^2 + b^2 = c^2$, show that $\frac{1}{\log_{b+c}^a} + \frac{1}{\log_{c-b}^a} = 2$

c. Prove that $\log_a^b \cdot \log_b^c \cdot \log_c^a = 1$.

Hence deduce the value of $\log_5^{32} \cdot \log_4^7 \cdot \log_{49}^{125}$

d. Prove that $(\log_{10}^2)(2 \log_4^5 + 1) = 1$

e. If $\log_b^a + \log_c^a = 2 \log_b^a \cdot \log_c^a$ Prove that $a^2 = bc$

5. a. If $a^2 + b^2 = 7ab$ prove that $2 \log \left(\frac{a+b}{3} \right) = \log a + \log b$

b. The terms $\log_z^x, \log_y^z, \log_x^y$ are in arithmetic progression with middle term a and common difference d .

Prove that $ad^2 = a^3 - 1$

c. Prove that $\log_a^N = \log_b^N \cdot \log_a^b = \frac{\log_b^N}{\log_b^a}$

6. a. If $2^x = 3^y = 12^z$, Prove that $xy = z(x + 2y)$.

b. If $n > 1$, then show that the value of

$$\frac{1}{\log_2^n} + \frac{1}{\log_3^n} + \dots + \frac{1}{\log_{53}^n} \text{ is } \frac{1}{\log_{53!}^n}$$

c. Find the value of $\log_3^{11} \log_{11}^{13} \log_{13}^{15} \log_{15}^{27} \log_{27}^{81}$

7. a. If $\frac{\log_{1/2}^x}{b-c} = \frac{\log_{1/2}^y}{c-a} = \frac{\log_{1/2}^z}{a-b}$ then find the value of $x^a y^b z^c$

b. If $\log_a bc = x, \log_b ca = y, \log_c ab = z$

then find the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

c. If $\log_5^2, \log_5^{(2^x+1)}$ and $\log_5^{\left(2^x + \frac{5}{2}\right)}$ are in arithmetic progression, then find x .

8. a. if $a^2 + 4b^2 = 12ab$ then find $\log(a+2b)$

b. If $\log_{12}^{18} = \alpha$ and $\log_{24}^{54} = \beta$ then find the value of $\alpha\beta + (\alpha - \beta)$

c. If $\log_a bc = x, \log_b ca = y, \log_c ab = z$ then find the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

9. If $x = \log_a \left(\frac{9}{8} \right), y = \log_a \left(\frac{16}{15} \right), z = \log_a \left(\frac{24}{25} \right)$

Prove that

i. $\log_a^2 = 3x + 4y - 2z$ ii. $\log_a^3 = 5x + 6y - 3z$ iii. $\log_a^5 = 7x + 9y - 5z$

10. If $xy^{p-1} = a, xy^{q-1} = b, xy^{r-1} = c,$

prove that $(q-r) \log a + (r-p) \log b + (p-q) \log c = 0$.

11. If $a^x = b^y = (ab)^{xy}$, prove that $x + y = 1$.

12. Prove that $\log_a x^m = \frac{m}{n} \cdot \log_a x$. Deduce that $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \dots + \log_{a^{2009}} x^{2009} = \log_a x^{2009}$.

13. If $\frac{\log z}{a-b} = \frac{\log y}{c-a} = \frac{\log x}{b-c}$, show that $x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1$.

14. Prove that $a^{\log\left(\frac{b}{c}\right)} \cdot b^{\log\left(\frac{c}{a}\right)} \cdot c^{\log\left(\frac{a}{b}\right)} = 1$.

15. a. Solve $7^{2x} - 7^{x+2} + 48 = 0$ b. Solve $5^{2x} - 5^{x+1} + 4 = 0$ c. Solve $9^x - 12(3^x) + 27 = 0$

16. a. Show that the solutions of

$$\log_2(3^{2x-2} + 7) = 2 + \log_2(3^{x-1} + 1) \text{ are } 1 \text{ and } 2.$$

b. Find the value of x which satisfy the equation $3^{\log_5(x-7)} = \log_5(125)$

c. Find the value of x which satisfy the equation

$$\log_2(x^2 - 3) - \log_2(6x - 10) + 1 = 0$$

17. a. If $\log_5^2$, $\log_5^{(2^x+1)}$ and $\log_5^{(2^x+5/2)}$ are in arithmetic progression then find x .

b. If $x = \log_{24}^{12}$, $y = \log_{36}^{24}$, $z = \log_{144}^{36}$ then find the value of $1 + 2xyz$.

18. a. Express $3^{2y} - 3^{y+1} - 3^y + 3$ in terms of z , where $z = 3^y$

Hence solve the equation $3^{2y} - 3^{y+1} - 3^y + 3 = 0$

b. Given that $\log_x u + \log_x v = p$ and $\log_x u - \log_x v = q$ prove that $u = x^{\frac{1}{2}(p+q)}$ and find v .

c. If $\frac{x(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y} = \frac{z(x+y-z)}{\log z}$ then show that $x^y y^x + z^y y^z - 2x^z z^x = 0$

d. If $xy^2 = 4$ and $\log_3(\log_2 x) + \log_{1/3}(\log_{1/2} y) = 1$ then $x = 64$

e. If $\log_3^2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, determine the value x .

19. Find the values of x and y of the following.

i. $x^2 + y = 12$

ii. $xy = 64$

iii. $xy = 8$

$\log_x^y + \log_y^x = 2$

$\log_x^y + \log_y^x = \frac{5}{2}$

$\log_8^x - 2\log_8^y = 1$

20. Solve the following equations

i. $\log_3^x - 4\log_x^3 + 3 = 0$

ii. $\log_3^x + 2\log_x^3 - 3 = 0$

iii. $\log_2^x + 6\log_x^2 - 5 = 0$

iv. $\log_3^{(2-3x)} = \log_9^{(6x^2-19x+2)}$

21. a, b, c are the p^{th}, q^{th} and r^{th} terms respectively of a geometric progression.

Prove that $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$.

22. Prove that,

$$\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c} = 1$$

23. Prove that,

$$i. \quad \log_{27} 81 = \frac{4}{3} \quad ii. \quad \log_2 \left(\frac{2}{3} \right) + \log_4 \left(\frac{9}{4} \right) = 0 \quad iii. \quad \log_{16} 32 = \frac{5}{4}$$

24. If $a = \log_2 3$, $b = \log_3 5$, $c = \log_7 2$. Show that $\log_{140} 63 = \frac{1+2ac}{2c+abc+1}$.

25. i. If a, b, c in geometric progression then show that

$\log_x a, \log_x b, \log_x c$ in arithmetic progression.

ii. If $\log_e 2 \cdot \log_x 625 = \log_{10} 16 \cdot \log_e 10$. show that $x = 5$.

26. i. If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$ then show that $x = 10$

ii. If $\log_{(2x+3)} (6x^2 + 23x + 21) = 4 - \log_{(3x+7)} (4x^2 + 12x + 9)$ then show that $x = -\frac{1}{4}$.

iii. Show that the solutions of the equations

$$x^{\log_x 2} = \log_3 (x+y) \quad \text{and} \quad x^2 + y^2 = 65 \quad \text{are} \quad x = 8, \quad y = 1.$$

27. a. If $2\log_y^x + 2\log_x^y = 5$, show that $\log_y^x$ is either $\frac{1}{2}$ or 2.

Hence find all pairs of values of x and y which satisfy simultaneously the equation above and the equation $xy = 27$.

b. Prove that if $x = \log_{10}^{(a-by)} - \log_{10}^a$ where a and b are constants, then $y = \left(\frac{a}{b} \right) (1 - 10^x)$

c. Given that $\log_2^x + 2\log_4^y = 4$ show that $xy = 16$.

Hence solve for x and y the simultaneous equations,

$$\log_{10} (x+y) = 1, \quad \log_2 x + 2\log_4^y = 4$$