

## Polynomial Functions.(Tute 1)

### Long Division

1. Show that the polynomial function  $f(x) = x^3 + 2x^2 + x + 2$  is divided by  $(x+1)$  the quotient is  $(x^2 + x)$  and the remainder is 2. Show also that  $f(-1) = 2$ .
2. Let  $f(x) = 2x^3 + x^2 + x + 3$ . Let  $f(x)$  is divided by  $(2x-1)$ . Then show that the quotient is  $(x^2 + x + 1)$  and the remainder is 4. Also find  $f\left(\frac{1}{2}\right)$ . what can you conclude by this result.
3. The polynomial function  $f(x)$  is defined as  $f(x) = 2x^3 - 2x^2 + 8x + 3$ . Show that when  $f(x)$  is divided by  $(2x-2)$  the quotient is  $(x^2 + 4)$  and remainder is 11. Determine the value  $a$  so that  $f(a)$  gives the remainder above.
4. Show that when  $f(x) = x^3 + 2x^2 + 2x + 2$  is divided by  $(x^2 + 1)$  the quotient is  $(x + 2)$  and remainder is  $x$ .
5. Show that when  $f(x) = x^4 + 2x$  is divided by  $(x^2 - 1)$  the remainder is  $(2x + 1)$  and the quotient is  $(x^2 + 1)$ . Is it possible to find a relationship among  $f(x)$ ,  $(x^2 - 1)$ ,  $(x^2 + 1)$  and  $(2x + 1)$ . If so state this relationship.
6. Show that when  $f(x) = x^4 + x^3 + x^2 + 2x + 3$  is divided by  $(x^2 + x - 1)$  the quotient is  $(x^2 + 2)$  and remainder is 5. Justify the result you stated in question 5 above.

### Use of algorithm of division to divide a polynomial function by another polynomial function.

7. Show that when  $x^3 + 2x^2 + 3x - 2$  is divided by  $(x^2 + x)$ , the quotient is  $(x + 1)$  and the remainder is  $2x - 2$ .
8. Show that when  $x^4 + x^2 + x - 2$  is divided by  $(x^2 - 1)$  the quotient is  $(x^2 + 2)$  and the remainder is  $x$ .
9. Let  $f(x) = x^3 + 4x^2 + x - 2$ .  $f(x)$  is divided by  $(x^2 + x - 2)$ . Show that the quotient and remainder are  $(x + 3)$  and 4 respectively.
10. Let  $f(x) = x^3 + 2x^2 + 6x + 9$ .  $f(x)$  is divided by  $(x^2 + 4)$ . Show that the quotient and remainder are  $(x + 2)$  and  $(2x + 1)$  respectively.

### Remainder theorem

11. Let  $f(x) = x^3 - ax^2 + 3x - b$ . When  $f(x)$  is divided by  $(x-1)$  and  $(x+1)$  the remainders are 1 and  $-7$  respectively. show that  $a = 2$  and  $b = 1$ .
12. The polynomial  $f(x)$  is degree 3 with leading coefficient 1. when  $f(x)$  is divided by  $(x-2)$ ,  $(x-1)$  and  $(x+2)$ , the corresponding remainders are 25, 8 and 17 respectively. Show that  $f(x) = x^3 + 4x^2 - 2x + 5$ .
13. Let  $f(x) = x^3 + ax^2 + bx + c$ . When  $f(x)$  is divided by  $(x^2 + x)$  and  $(x-1)$  the remainders are  $(x-1)$  and 6 respectively. Find the values of  $a$ ,  $b$  and  $c$ .  $[a = 3, b = 3, c = -1]$ .

**Factor theorem and its converse**  
**Use converse of factor theorem.**

14. Let  $f(x) = x^3 - 4x^2 + x + 6$ .

Show that  $(x+1)$ ,  $(x-2)$  and  $(x-3)$  are factors of  $f(x)$ .

15. Show that  $(2x-1)$ ,  $(x+2)$  and  $(x+1)$  are factors of  $f(x) = 2x^3 + 5x^2 + x - 2$ .

16. Let  $f(x) = x^3 + 2x^2 - 5x - 6$ . three linear factors of  $f(x)$ .

Hence write down  $f(x)$  in factor form.

**Use of factor theorem**

17. Let  $f(x) = x^3 + ax^2 + bx + c$ . If  $(x+2)$ ,  $(x+3)$  and  $(x+1)$  are factors of  $f(x)$ , then find the values of  $a$ ,  $b$  and  $c$ . [ $a = 6$ ,  $b = 11$ ,  $c = 6$ ].

18. Let  $f(x) = ax^3 + bx^2 - cx + 1$ . If  $(2x-1)$ ,  $(x+1)$  and  $(x-1)$  are factors of  $f(x)$ , then find the values of  $a$ ,  $b$  and  $c$  [ $a = 2$ ,  $b = -1$ ,  $c = 2$ ]

**Use of remainder theorem and factor theorem**

19.  $f(x)$  is a polynomial function in  $x$  of degree 4.  $(x+2)$  and  $(x-3)$  are factors of  $f(x)$ . When  $f(x)$  is divided by  $(x-1)$  and  $(x+1)$  the remainders are  $-12$  and  $8$  respectively. Also  $f(x) - 6$  is divided by  $x$ .

Show that  $f(x) = x^4 + x^3 - 9x^2 - 11x + 6$ .

20.  $f(x)$  is a polynomial function of degree 3.  $(x+1)$  is a factor of  $f(x)$ . When it is divided by  $(x+2)$ ,  $(x-1)$  and  $(x+3)$ , the remainders are  $1$ ,  $12$  and  $-20$  respectively. Show that  $f(x) = 2x^3 + 5x^2 + 4x + 1$ .

Show also that when  $f(x)$  is divided by  $(x^2 - 4)$ , the quotient is  $(2x + 5)$  and the remainder is  $3(4x + 7)$ .

Hence show that when  $f(x)$  is divided by  $(x^2 - 3)$  the quotient is  $(2x + 5)$  and remainder is  $2(5x + 8)$ .